GCE AS Further Mathematics (8FM0) – Paper 1
Core Pure Mathematics

Summer 2018 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

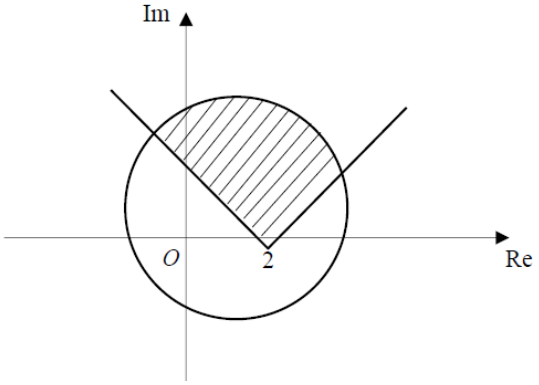
Question 1 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{M}^{-1} = \frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}$	B1	This mark is given for evidence that the determinant is –69 (might be implied by matrix)
		B1	This mark is given for a fully correct matrix
(b)	$\frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}\begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix}$ $x = \frac{(1\times -4)+(13\times 9)+(5\times 5)}{69}$ $y = \frac{(-11\times -4)+(-5\times 9)+(14\times 5)}{69}$ $z = \frac{(-26\times -4)+(7\times 9)+(8\times 5)}{69}$	M1	This mark is given for a method to solve for x, y and z.
	$x = 2$, $y = 1$, $z = 3$	A1	This mark is given for three correct values (accept (2, 1, 3) or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$).
(c)	(2, 1, 3) are the coordinates of the unique point where the three planes meet.	B1	This mark is given for a correct interpretation

Question 2 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$w = 2z + 1 \Rightarrow z = \dfrac{w-1}{2}$	B1	This mark is given for making a connection between z and w by writing $z = \dfrac{w-1}{2}$
	$\left(\dfrac{w-1}{2}\right)^3 - 3\left(\dfrac{w-1}{2}\right)^2 + \left(\dfrac{w-1}{2}\right) + 5 = 0$	M1	This mark is given for substituting $z = \dfrac{w-1}{2}$ into $z^3 - 3z^2 + z + 5 = 0$
	$\dfrac{1}{8}(w^3 - 3w^2 + 1) - \dfrac{3}{4}(w^2 - 2w + 1) + \dfrac{w-1}{2} + 5 = 0$		
	$w^3 - 9w^2 + 19w + 29 = 0$	M1	This mark is given for manipulating into the form $w^3 - pw^2 + qw + r = 0$
	$p = -9$ $q = 19$	A1	This mark is given for finding at least two of p, q and r correctly
	$r = 29$	A1	This mark is given for a fully correct equation

Question 3 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for a circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2
		M1	This mark is given for a "V" shape with the vertex on the positive real axis
		A1	This mark is given for two half lines that meet on the positive real axis and intersect the circle or arc of a circle in the first and second quadrants
		M1	This mark is given for a shaded region between the half-lines and within the circle
		A1	This mark is given for a fully correct diagram including the number 2 marked at the vertex on the real axis, with the correct region shaded and all the previous marks scored.
(b)	$(x-1)^2 + (y-1)^2 = 9, y = x - 2$	M1	This mark is given for a identifying a suitable strategy for finding the x or y coordinate of the point of intersection
	$x = 2 + \frac{\sqrt{14}}{2}, y = \frac{\sqrt{14}}{2},$	A1	This mark is given for correct x and y coordinates for the intersection
	$\lvert w^2 \rvert = \left(2 + \frac{\sqrt{14}}{2}\right)^2 + \left(\frac{\sqrt{14}}{2}\right)^2$	M1	This mark is given for a correct use of Pythagoras to find the required length
	$11 + 2\sqrt{14}$	A1	This mark is given for a correct value

Question 4 (Total 11 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$	M1	This mark is given for finding the direction of W
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\cdot\begin{pmatrix}3\\-5\\-18\end{pmatrix}$	M1	This mark is given for using the model to attempt the scalar product between the direction of W and the normal
	$\sqrt{(2^2+3^2+0^2)}\sqrt{(3^2+(-5)^2+(-18)^2\cos\alpha)} = -9$	A1	This mark is given for correctly finding the scalar product in terms of cos
	$\theta = 90 - \arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$	M1	This mark is given for a complete and correct method for obtaining the acute angle
	$= 7.58°$	A1	This mark is given for finding the correct angle between the pipe and the road
(b)	$\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1	This mark is given for forming the correct parametric form for the pipe W
	$\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}=\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}$	M1	This mark is given for a identifying the need to form the vector connecting C to W
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\cdot\begin{pmatrix}2\\3\\0\end{pmatrix} = 13t+3$	M1	This mark is given for using the model to form the scalar product of their vector C to W and the direction of W to find the value of their parameter
	$t = -\frac{3}{13} \Rightarrow C$ to W is $-\frac{6}{13}\mathbf{i} + \frac{4}{13}\mathbf{j} - 3\mathbf{k}$	A1	This mark is given for a correct vector for C to W
	$d = \sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2} = \sqrt{\frac{121}{13}}$	M1	This mark is given for a correct use of Pythagoras to find the shortest distance between the point and the pipe
	3.05 m	A1	This mark is given for correctly finding the shortest length of the pipe needed

Question 5 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	Rotation,	B1	This mark is given for identifying the transformation as a rotation
	120° anti-clockwise	B1	This mark is given for the correct angle (allowing equivalents in degrees or radians)
	about the origin	B1	This mark is given for identifying the origin as the centre of rotation
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	This mark is given for showing the correct matrix in the correct form
(c)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} =$	M1	This mark is given for a multiplying the matrices in the correct order
	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	A1	This mark is given for the correct matrix
(d)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$	M1	This mark is given for translating the problem into a matrix multiplication to obtain at least one equation in k or in x and y
	$-\frac{\sqrt{3}}{2}x + \frac{1}{2}k = 1$ or $\frac{1}{2} + \frac{\sqrt{3}}{2}k = k$ or $x = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y$ or $y = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$	A1	This mark is given for a obtaining one correct equation
	$-\frac{\sqrt{3}}{2} + \frac{1}{2}k = 1$ or $x = -\frac{\sqrt{3}}{2}x + \frac{1}{2}k$ $\Rightarrow k = 2 + \sqrt{3}$	A1	This mark is given for a correct value for k in any form
	$\frac{1}{2} + \frac{\sqrt{3}}{2}k = k$ or $y = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$ $\Rightarrow k = 2 + \sqrt{3}$	B1	This mark is given for a checking the answer by solving both equations to obtain $2 + \sqrt{3}$ both times, or for substituting $2 + \sqrt{3}$ into the other equation to confirm its validity

Question 6 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(3r-2)^2 = 9r^2 - 12r + 4$	B1	This mark is given for a correct expansion
	$\sum_{r=1}^{n}(9r^2 - 12r + 4) =$ $9 \times \frac{1}{6}n(n+1)(2n+1) - 12 \times \frac{1}{2}n(n+1)\ldots$	M1	This mark is given for substituting at least one of the standard formulae into their expanded expression
	$9 \times \frac{1}{6}n(n+1)(2n+1) - 12 \times \frac{1}{2}n(n+1) + 4n$	A1	This mark is given for a fully correct expression
	$= \frac{1}{2}n\,[3(n+1)(2n+1) - 12(n+1) + 8]$	M1	This mark is given for an attempt to factorise $\frac{1}{2}n$, having used at least one standard formula correctly
	$= \frac{1}{2}n\,[6n^2 - 3n - 1]$	A1	This mark is given for a obtaining the printed result with no errors seen
(b)	$\sum_{r=5}^{n}(3r-2)^2 =$ $\frac{1}{2}n\,(6n^2 - 3n - 1) - \frac{1}{2}(4)(6(4)^2 - 3 \times 4 - 1)$	M1	This mark is given for a using the result from part (a) by substituting $n = 4$ and subtracting from the result in part (a) to find the first sum in terms of n.
	$\sum_{r=1}^{28} r\cos\frac{r\pi}{2} =$ $0 - 2 + 0 + 4 + 0 - 6 + 0 + 8 + 0 - 10 + 0 + 12\ldots$	M1	This mark is given for identifying the periodic nature of the second sum by calculating terms (this may be implied by a sum of 14).
	$3n^3 - \frac{3}{2}n^2 - \frac{1}{2}n - 166 + 103 \times 14 = 3n^3$ $\Rightarrow 3n^2 + n + 2552 = 0$	A1	This mark is given for a using the sum and the given result to form the correct three-term quadratic
	$3n^2 + n - 2552 = 0$	M1	This mark is given for solving the three-term quadratic
	$n = 29$	A1	This mark is given for obtaining $n = 29$ only (or obtaining $n = 29$ and $-\frac{88}{3}$ and rejecting $-\frac{88}{3}$)

Question 7 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Complex roots are $\alpha \pm \beta$i	B1	This mark is given for recognising that the other roots must form a conjugate pair
	$\alpha + \beta$i + $\alpha - \beta$i + 3 = –1	M1	This mark is given for using the sum of the roots to find a value for α
	$\alpha = -2$	A1	This mark is given for the correct value for α
	$\frac{1}{2} \times 2\beta \times 5 = 35 \Rightarrow \beta = 7$	M1	This mark is given for using the value for α and the given area to find a value for β
	$q = -3(-2 + 7\text{i})(-2 - 7\text{i})$	M1	This mark is given for using an appropriate method to find p or q
	$q = -159$	A1	This mark is given for finding a correct value for either p or q
	$3p + q = -36 \Rightarrow p = \frac{-36 - q}{3} = 41$	A1	This mark is given for correct values for p and q

Question 8(i) (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$n = 1$, $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^1 = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ $\begin{pmatrix} 4\times1+1 & -8\times1 \\ 2\times1 & 1-4\times1 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ thus result is true for $n = 1$	B1	This mark is given for showing that the result holds for $n = 1$
	Assume true for $n = k$: $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^k = \begin{pmatrix} 4k+1 & -8k \\ 2k & 1-4k \end{pmatrix}$	M1	This mark is given for making a statement that assumes the result is true for some value of n
	$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^{k+1} = \begin{pmatrix} 4k+1 & -8k \\ 2k & 1-4k \end{pmatrix}\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$	M1	This mark is given for an attempt to multiply the assumed result by the original matrix (either way round)
	$= \begin{pmatrix} 5(4k+1)-16k & -8(4k+1)+24k \\ 10k+2(1-4k) & -16k-3(1-4k) \end{pmatrix}$	A1	This mark is given for a correct (unsimplified) matrix
	$= \begin{pmatrix} 4k+5 & -8k-8 \\ 2k+2 & -4k-3 \end{pmatrix}$ $= \begin{pmatrix} 4(k+1)+1 & -8(k+1)k \\ 2(k+1) & 1-4(k+1) \end{pmatrix}$	A1	This mark is given for a correct simplified matrix with no errors
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n.	A1	This mark is given for a correct conclusion

Question 8(ii) (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(ii)	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ thus statement is true for $n = 1$	B1	This mark is given for showing that $f(1) = 21$
	Assume true for $n = k$ so that $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	This mark is given for making a statement that assumes the result is true for some value of n
	$f(k+1) - f(k) = 4^{k+2} + 5^{2k+1} - 4^{k+1} - 5^{2k-1}$ $= 4 \times 4^{k+1} + 25 \times 5^{2k-1} - 4^{k+1} - 5^{2k-1}$	M1	This mark is given for attempting to find $f(k+1) - f(k)$
	$= 3f(k) + 21 \times 5^{2k-1}$	A1	This mark is given for finding a correct expression for $f(k+1) - f(k)$ in terms of $f(k)$
	$f(k+1) = 4f(k) + 21 \times 5^{k-1}$	A1	This mark is given for a correct expression for $f(k+1)$ in terms of $f(k)$
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n.	A1	This mark is given for a correct conclusion

Question 9 (Total 11 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The coordinates of G are (4, 14) and the coordinates of F are, (1, 18) $14 = a(4)^2 + b$ and $18 = a(1)^2 + b$ $16a + b = 14$, $a + b = 18 \Rightarrow 15a = -4$	M1	This mark is given for recognising the curve GF between the points (4, 14) and (1, 18), and substituting into the equation modelling the curve in an attempt to find the values of a and b
	$a = -\frac{4}{15}$, $b = \frac{274}{15}$	A1	This mark is given for inferring, from the data in the model, the values of a and b
(b)	$(\pi \times 4^2 \times 14) + (\pi \times 1^2 \times 10) = 234\pi$	B1	This mark is given for a correct expression for the volume of the two cylindrical parts
	$\pi \int x^2 \ \mathrm{d}y = \frac{\pi}{4} \int (274 - 15y) \ \mathrm{d}y$	B1	This mark is given for using the model to obtain $\pi \int \frac{y-b}{a} \ \mathrm{d}y$
	$= \frac{\pi}{4} \int_{14}^{18} (274 - 15y) \ \mathrm{d}y$	M1	This mark is given for choosing limits appropriate to their model
	$= \frac{\pi}{4} \left[274y - \frac{15y^2}{2} \right]_{14}^{18}$	M1	This mark is given for integrating to obtain an expression of the form $\alpha y + \beta y^2$
		A1	This mark is given for using their model correctly to give $274y - \frac{15y^2}{2}$
	$V = 234\pi + \frac{\pi}{4}(2502 - 2366)$	M1	This mark is given for using the model to find the sum of the cylinders + the integrated volume
	$V = 234\pi + 34\pi = 268\pi \text{ cm}^3$	A1	This mark is given for a correct answer (accept anything which rounds to 842)
(c)	The measurements may not be accurate The equation of the curve may not be a suitable model The bottom of the bottle may not be flat The thickness of the glass may not have been considered	B1	This mark is given for stating an acceptable limitation of the model
(d)	This is not a good estimate as there is a significant difference between the two volumes; $842 - 750 = 92 \text{ cm}^3$ This could be a good estimate as the bottles may not be completely full	B1	This mark is given for comparing the actual volume to their answer to part (b) and making an assessment of the model with a reason