A2 Practice Paper F

1. $$\frac{6(x+7)}{(5x-1)(2x+5)} = \frac{A}{5x-1} + \frac{B}{2x+5}$$

$$6x+42 = A(2x+5) + B(5x-1)$$

$x = -\frac{5}{2}$, $6\left(\frac{-5}{2}\right)+42 = B\left(5\left(\frac{-5}{2}\right)-1\right)$

$$27 = -\frac{27}{2}B$$

$$\underline{B = -2}$$

$x = \frac{1}{5}$, $6\left(\frac{1}{5}\right)+42 = A\left(2\left(\frac{1}{5}\right)+5\right)$

$$\frac{216}{5} = \frac{27}{5}A$$

$$\underline{8 = A}$$

$$\frac{8}{5x-1} - \frac{2}{2x+5}$$

2. Proof by contradiction:
There exists an integer a and b for which $25a + 15b = 1$

$$5a + 3b = \frac{1}{5}$$

If a and b are integers 5a and 3b are integers. This contradicts $5a+3b = \frac{1}{5}$ as $\frac{1}{5}$ is not an integer therefore there do not exist integers a and b such that $25a + 15b = 1$.

3 a) $x = \cos 2t \qquad y = \sin t$

$\cos 2t = 1 - 2\sin^2 t$

$x = 1 - 2y^2$

$1 = -4y \dfrac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{1}{4y} = \frac{-1}{4\sin t} = -\frac{1}{4}\operatorname{cosec} t$$

b) when $t = -\dfrac{5\pi}{6}$ $\qquad x = \cos 2\left(\dfrac{-5\pi}{6}\right) = \dfrac{1}{2}$

$y = \sin\left(\dfrac{-5\pi}{6}\right) = \dfrac{-1}{2}$

$$\frac{dy}{dx} = \frac{-1}{4\sin\left(\frac{-5\pi}{6}\right)} = \frac{1}{2}$$

Gradient of normal $= -2$

$y - y_1 = m(x - x_1)$

$y - -\dfrac{1}{2} = -2\left(x - \dfrac{1}{2}\right)$

$2y + 1 = -4\left(x - \frac{1}{2}\right)$

$2y + 1 = -4x + 2$

$2y = -4x + 1$

$y = -2x + \frac{1}{2}$

4. $\int \cot 3x = \int \frac{\cos 3x}{\sin 3x}$; $y = \sin 3x$

$\frac{dy}{dx} = 3\cos 3x.$

$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x)$

$= \frac{1}{3} \ln |\sin 3x| + c$

$\boxed{\int \frac{3\cos 3x}{\sin 3x} = \ln \sin 3x + c}$ → we want $\frac{1}{3}$ of this.

5. A $(-2, 0, -4)$
B $(-2, 4, -6)$
C $(3, 4, 4)$

$|\vec{AB}| = \sqrt{0^2 + 4^2 + (-2)^2}$
$= \sqrt{20}$

$|\vec{AC}| = \sqrt{5^2 + 4^2 + 8^2}$
$= \sqrt{105}$

$|\vec{BC}| = \sqrt{5^2 + 0^2 + 10^2}$
$= \sqrt{125}$

$\left.\begin{array}{l} |\vec{AB}|^2 + |\vec{AC}|^2 = |\vec{BC}|^2 \\ a^2 + b^2 = c^2 \end{array}\right]$ ∴ Right-angled triangle.

6.a) $p(x) = x^2$ $q(x) = 5 - 2x$

$pq(x) = (5-2x)(5-2x)$
$= 25 + 4x^2 - 20x.$

$qp(x) = 5 - 2(x^2)$
$= 5 - 2x^2$

6a) continued...

$$25 + 4x^2 - 20x = 5 - 2x^2$$
$$6x^2 - 20x + 20 = 0$$
$$3x^2 - 10x + 10 = 0.$$

b) $b^2 - 4ac < 0$

$(-10)^2 - (4 \times 3 \times 10) < 0.$

$100 - 120 < 0$

No real solutions.

7. Proof by contradiction:

Assumption: There are a finite amount of prime numbers.

p_1, p_2, p_3, p_4

$N = (p_1 \times p_2 \times p_3 \times p_4) + 1$

Dividing N by any of the existing prime numbers will leave a remainder of 1. None of the existing prime numbers is a factor of N. Either N is prime or N has a prime factor that is not currently listed.

This contradicts that there is a finite amount of prime numbers and therefore there is an infinite amount of prime numbers.

8. F = area covered by trees
$t = 0$ in 1990
$\frac{dF}{dt} \propto F$
$\frac{dF}{dt} = -kF$

9.a) $\left.\begin{array}{l} a = 100 \\ r = 1{\cdot}05 \\ n = 9 \end{array}\right\}$ $S_n = \frac{a(1-r^n)}{1-r}$

$S_9 = \frac{100(1-1{\cdot}05^9)}{1-1{\cdot}05}$

$= \underline{£1102{\cdot}66}$

b) $\frac{a(1-r^n)}{1-r} > 6000$

$\frac{100(1-1{\cdot}05^n)}{1-1{\cdot}05} > 6000$

$100(1-1{\cdot}05^n) > -300$

$1-1{\cdot}05^n > -3$

$\frac{-1{\cdot}05^n}{-1} > \frac{-4}{-1}$

$1{\cdot}05^n < 4$

$\log 1{\cdot}05^n < \log 4$

$n \log 1{\cdot}05 < \log 4$

$n < \frac{\log 4}{\log 1{\cdot}05}$

c) $\left.\begin{array}{l} a = 50 \\ d = ? \\ n = 29 \end{array}\right\}$ $S_n = \frac{1}{2}n[2a + (n-1)d]$

$6000 = \frac{29}{2}[100 + 28d]$

$\underline{d = £11{\cdot}21}$

10. $\int \cos^2 6x \, dx$.

$\cos 2x = 2\cos^2 x - 1$
$\cos 12x = 2\cos^2 6x - 1$
$\frac{1}{2}(\cos 12x + 1) = \cos^2 6x$

$\int \frac{1}{2}(\cos 12x + 1)\, dx$

$\frac{1}{2}\int \cos 12x + 1 \, dx$

$\frac{1}{2}\left[\frac{1}{12}\sin 12x + x\right] + C$

11.a) $\dfrac{\tan x - \sec x}{1 - \sin x}$.

$\dfrac{\dfrac{\sin x}{\cos x} - \dfrac{1}{\cos x}}{1 - \sin x}$.

$\dfrac{\sin x - 1}{\cos x} \div \dfrac{1 - \sin x}{1}$

$\dfrac{\sin x - 1}{\cos x} \times \dfrac{1}{1 - \sin x}$.

$\dfrac{\overset{\times -1}{\sin x} \overset{\times -1}{-1}}{\underset{\times -1}{\cos x}(1 - \sin x)} = \dfrac{1 - \sin x}{-\cos x(1 - \sin x)}$

$= \dfrac{1}{-\cos x} = -\sec x$

11b) $\frac{\tan x - \sec x}{1 - \sin x} = \sqrt{2}$

$\frac{-1}{\cos x} = \sqrt{2}$

$-1 = \sqrt{2}\cos x.$

$\frac{-1}{\sqrt{2}} = \cos x.$

$x = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$x = \frac{3\pi}{4}, \frac{5\pi}{4}.$

12a) $x = 8(t+10)$ $y = 100 - t^2$

$x = 8t + 80$

$\frac{x-80}{8} = t$

$y = 100 - \left(\frac{x-80}{8}\right)^2$

$y = 100 - \left(\frac{x-80}{8}\right)\left(\frac{x-80}{8}\right)$

$y = 100 - \left(\frac{x^2 - 160x + 6400}{64}\right)$

$y = 100 - \frac{x^2}{64} + \frac{5}{2}x - 100$

$y = -\frac{1}{64}x^2 + \frac{5}{2}x.$

b) $x = 8(t+10) \rightarrow$ width.

$t = -10, x = 0$

$t = 10, x = 160$

width $= \underline{160m}$

c) $y = 100 - t^2$

$\frac{dy}{dt} = -2t$ $\frac{dy}{dx} = 0$ max point $-2t = 0$

$t = 0$, $\underline{h = 100m}$

13. $$\frac{x^3+8x^2-9x+12}{x+6} = Ax^2+Bx+C+\frac{D}{x+6}.$$

$$
\begin{array}{r l}
 & x^2 + 2x - 21 \\
x+6 & \overline{)\,x^3+8x^2-9x+12} \\
 & -\underline{x^3+6x^2} \\
 & \quad 2x^2-9x \\
 & -\underline{2x^2+12x} \\
 & \quad\quad -21x+12 \\
 & \quad\quad -\underline{-21x-126} \\
 & \quad\quad\quad \underline{138}
\end{array}
$$

$$= x^2+2x-21+\frac{138}{x+6}$$

14. $V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ $\quad \frac{dS}{dt} = -12.$ $\quad \frac{dS}{dr} = 8\pi r$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $\quad \frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$-12 = 8\pi r \times \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4\pi r^2}{1} \times \frac{-12}{8\pi r}$

$= \underline{\underline{-6r}}$

$\frac{-12}{8\pi r} = \frac{dr}{dt}.$

15. $\int \sin^3 x = \int \sin x(\sin^2 x)\,dx$

$= \int \sin x\,(1-\cos^2 x)\,dx$

$= \int \sin x - \sin x \cos^2 x\,dx$

$= \int \sin x - \cancel{\sin x}\, u^2 \frac{du}{\cancel{\sin x}}$

$u = \cos x.$

$\frac{du}{dx} = -\sin x$

$\frac{du}{-\sin x} = dx$

$= \int \sin x + u^2\,du$

$= -\cos x + \frac{1}{3}u^3 + c$

$= -\cos x + \frac{1}{3}\cos^3 x + C.$

16a) $h(t) = 40\ln(t+1) + 40\sin\left(\frac{t}{5}\right) - \frac{1}{4}t^2$

$t = 19.3,\ h(19.3) = +0.974$

$h(19.4) = -0.393$

Change of sign and continuous function in the interval $\therefore$ root $[19.3, 19.4]$

b) Newton Raphson : $x_n - \frac{f(x_n)}{f'(x_n)}$

$h'(t) = \frac{40}{t+1} + 8\cos\left(\frac{t}{5}\right) - \frac{1}{2}t$

$h(19.35) = 0.2903$

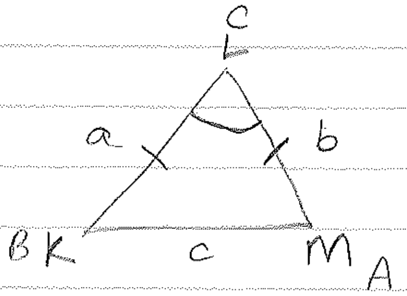
$h'(19.35) = -13.6792$

$x_1 = 19.35 - \frac{0.2903}{-13.6792} = 19.371.$

16c) $h(19.3705) = +0.0100$
$h(19.3715) = -0.00366$

Change of sign and continuous function in the interval $[19.3705, 19.3715] \rightarrow$ root.

17a) $\vec{KL} = 3i + 0j - 6k$



$\vec{LM} = 2i + 5j + 4k$

$|\vec{KL}| = \sqrt{3^2 + 0^2 + (-6)^2}$
$= \sqrt{45}$

$|\vec{LM}| = \sqrt{2^2 + 5^2 + 4^2}$
$= \sqrt{45}$

$\vec{KM} = \vec{KL} + \vec{LM}$
$\vec{KM} = (3+2)i + (0+5)j + (-6+4)k$
$= 5i + 5j - 2k$
$|\vec{KM}| = \sqrt{5^2 + 5^2 + (-2)^2}$
$= \sqrt{54}$

$c^2 = a^2 + b^2 - 2ab\cos C$
$(\sqrt{54})^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - (2 \times \sqrt{45} \times \sqrt{45} \times \cos C)$
$-36 = -90\cos C$
$C = 66.4°$

b) Isosceles triangle
$\frac{180 - 66.4}{2} = 56.8°$