Advanced Subsidiary Paper 1: Pure Mathematics

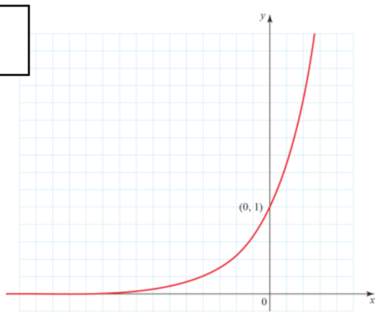
PAPER C Mark Scheme

1	States or implies the formula for differentiation from first principles. $\mathrm{f}(x)=5x^3$ $\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$	**B1**
	Correctly applies the formula to the specific formula and expands and simplifies the formula. $\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{5(x+h)^3-5x^3}{h}$ $\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{5\left(x^3+3x^2h+3xh^2+h^3\right)-5x^3}{h}$ $\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{15x^2h+15xh^2+5h^3}{h}$	**M1**
	Factorises the '*h*' out of the numerator and then divides by *h* to simplify. $\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{h\left(15x^2+15xh+5h^2\right)}{h}$ $\mathrm{f}'(x)=\lim\limits_{h\to 0}\left(15x^2+15xh+5h^2\right)$	**A1**
	States that as $h\to 0$, $15x^2+15xh+5h^2\to 15x^2$ o.e. so derivative = $15x^2$ *	**A1***
		(4 marks)

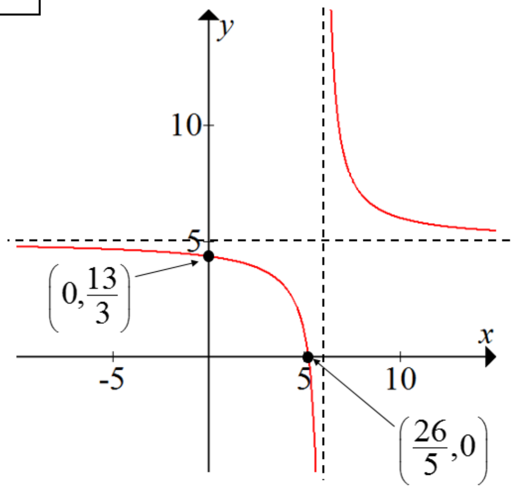
NOTES: Use of δx also acceptable.

Students must show a complete proof (without wrong working) to achieve all 4 marks.

Not all steps need to be present, and additional steps are also acceptable.

2 	Graph has correct shape and does not touch *x*-axis.	**M1**
	The point (0, 1) is given or labelled.	**A1**
		(2 marks)
Translation 1 unit right (or positive *x* direction) or by $\begin{pmatrix}1\\0\end{pmatrix}$		**B1**
Translation 5 units up (or positive *y* direction) or by $\begin{pmatrix}0\\5\end{pmatrix}$		**B1**
		(2 marks)
		Total 4 marks

3a	States that $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$	**M1**
	States $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ or $\overrightarrow{PQ} = -\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$	**M1**
	States $\overrightarrow{PQ} = \frac{3}{5}(-\mathbf{a} + \mathbf{b})$ or $\overrightarrow{PQ} = \frac{3}{5}\overrightarrow{AB}$	**A1**
	Draws the conclusion that as $\overrightarrow{PQ}$ is a multiple of $\overrightarrow{AB}$ the two lines *PQ* and *AB* must be parallel.	**A1**
		(4 marks)
3b	$PQ = \frac{3}{5} \times 10\text{ cm} = 6\text{ cm}$ cao	**B1**
		(1 mark)
		Total 5 marks

4		
	Asymptote drawn at $x = 6$	**B1**
	Asymptote drawn at $y = 5$	**B1**
	Point $\left(0,\frac{13}{3}\right)$ labelled. Condone $\frac{13}{3}$ clearly on y axis.	**B1**
	Point $\left(\frac{26}{5},0\right)$ labelled. Condone $\frac{26}{5}$ clearly on x axis.	**B1**
	Correctly shaped graph drawn in the correct quadrants formed by the asymptotes.	**B1**
		(5 marks)

5		
	Correctly shows that either f(3) = 0, f(−2) = 0 or $f\left(-\frac{1}{2}\right) = 0$	**M1**
	Draws the conclusion that (x – 3), (x + 2) or (2x + 1) must therefore be a factor.	**M1**
	Either makes an attempt at long division by setting up the long division, or makes an attempt to find the remaining factors by matching coefficients. For example, stating $(x-3)(ax^2+bx+c)=2x^3-x^2-13x-6$ or $(x+2)(rx^2+px+q)=2x^3-x^2-13x-6$ or $(2x+1)(ux^2+vx+w)=2x^3-x^2-13x-6$	**M1**
	For the long division, correctly finds the the first two coefficients. For the matching coefficients method, correctly deduces that a = 2 and c = 2 or correctly deduces that r = 2 and q = −3 or correctly deduces that u = 1 and w = –6	**A1**
	For the long division, correctly completes all steps in the division. For the matching coefficients method, correctly deduces that b = 5 or correctly deduces that p = −5 or correctly deduces that v = –1	**A1**
	States a fully correct, fully factorised final answer: (x – 3)(2x + 1)(x + 2)	**A1**
		(6 marks)

NOTES: Other algebraic methods can be used to factorise h(x).

For example, if (x – 3) is known to be a factor then

$2x^3-x^2-13x-6=2x^2(x-3)+5x(x-3)+2(x-3)$ by balancing (M1)

$=(2x^2+5x+2)(x-3)$ by factorising (M1)

$=(2x+1)(x+2)(x-3)$ by factorising (A1)

6a	Attempt is made at expanding $(p+q)^5$. Accept seeing the coefficients 1, 5, 10, 10, 5, 1 or seeing $(p+q)^5 = {}^5C_0 p^5 + {}^5C_1 p^4 q + {}^5C_2 p^3 q^2$ $+ {}^5C_3 p^2 q^3 + {}^5C_4 p q^4 + {}^5C_5 q^5$ o.e.	**M1**
	Fully correct answer is stated: $(p+q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$	**A1**
		(2 marks)
6b	States that *p*, or the probability of rolling a 4, is $\frac{1}{4}$	**B1**
	States that *q*, or the probability of not rolling a 4, is $\frac{3}{4}$	**B1**
	States or implies that the sum of the first 3 terms (or 1 − the sum of the last 3 terms) is the required probability. For example, $p^5 + 5p^4q + 10p^3q^2$ or $1 - (10p^2q^3 + 5pq^4 + q^5)$	**M1**
	$\left(\frac{1}{4}\right)^5 + 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) + 10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$ or $\frac{1}{1024} + \frac{15}{1024} + \frac{90}{1024}$ or $1 - \left(10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 + 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^5\right)$ or $1 - \left(\frac{270}{1024} + \frac{405}{1024} + \frac{243}{1024}\right)$	**M1**
	Either $\frac{53}{512}$ o.e. or awrt 0.104	**A1**
		(5 marks)
		Total **7 marks**

7a	States or implies that $\overrightarrow{BC} = 13\mathbf{i} - 8\mathbf{j}$ o.e.	**M1**
	Recognises that the cosine rule is needed to solve for $\angle BAC$ by stating $a^2 = b^2 + c^2 - 2bc \times \cos A$	**M1**
	Makes correct substitutions into the cosine rule. $\left(\sqrt{233}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{104}\right)^2 - 2\left(\sqrt{45}\right)\left(\sqrt{104}\right) \times \cos A$ o.e.	**M1**
	$\cos A = -\frac{7}{\sqrt{130}}$ or awrt −0.614 (seen or implied by correct answer).	**M1**
	A = 127.9° cao	**A1**
		(5 marks)
7b	States formula for the area of a triangle. Area = $\frac{1}{2}ab\sin C$	**M1**
	Makes correct substitutions using their values from above. Area = $\frac{1}{2}\left(\sqrt{45}\right)\left(\sqrt{104}\right)\sin 127.9...^\circ$	**M1ft**
	Area = 27 (units^2)	**A1ft**
		(3 marks)
		Total 8 marks

8a	Use of the gradient formula to begin attempt to find *k*. $\frac{k+1-(-2)}{1-(3k-4)}=-\frac{3}{2}$ or $\frac{-2-(k+1)}{3k-4-1}=-\frac{3}{2}$ (i.e. correct substitution into gradient formula and equating to $-\frac{3}{2}$).	**M1**
	$2k + 6 = -15 + 9k$ $21 = 7k$ $k = 3$* (must show sufficient, convincing and correct working).	**A1***
		(2 marks)
8b	Student identifies the coordinates of either *A* or *B*. Can be seen or implied, for example, in the subsequent step when student attempts to find the equation of the line. *A*(5, −2) or *B*(1, 4).	**B1**
	Correct substitution of their coordinates into $y = mx + b$ or $y - y_1 = m(x - x_1)$ o.e. to find the equation of the line. For example, $-2=\left(-\frac{3}{2}\right)(5)+b$ or $y+2=\left(-\frac{3}{2}\right)(x-5)$ or $4=\left(-\frac{3}{2}\right)(1)+b$ or $y-4=\left(-\frac{3}{2}\right)(x-1)$	**M1**
	$y=-\frac{3}{2}x+\frac{11}{2}$ or $3x+2y-11=0$	**A1**
		(3 marks)
8c	Midpoint of *AB* is (3, 1) seen or implied.	**B1**
	Slope of line perpendicular to *AB* is $\frac{2}{3}$, seen or implied.	**B1**
	Attempt to find the equation of the line (i.e. substituting their midpoint and gradient into a correct equation). For example, $1=\left(\frac{2}{3}\right)(3)+b$ or $y-1=\frac{2}{3}(x-3)$	**M1**
	$2x-3y-3=0$ or $3y-2x+3=0$. Also accept any multiple of $2x-3y-3=0$ providing *a*, *b* and *c* are still integers.	**A1**
		(4 marks)
		Total 9 marks

9a	115 (m) is the height of the cliff (as this is the height of the ball when t = 0). Accept answer that states 115 (m) is the height of the cliff plus the height of the person who is ready to throw the stone or similar sensible comment.	**B1**
		(1 mark)
9b	Attempt to factorise the – 4.9 out of the first two (or all) terms. $h(t) = -4.9\left(t^2 - 2.5t\right) + 115$ or $h(t) = -4.9\left(t^2 - \frac{5}{2}t\right) + 115$	**M1**
	$h(t) = -4.9\left(t - 1.25\right)^2 - (-4.9)\left(1.25\right)^2 + 115$ or $h(t) = -4.9\left(t - \frac{5}{4}\right)^2 - (-4.9)\left(\frac{5}{4}\right)^2 + 115$	**M1**
	$h(t) = 122.65625 - 4.9\left(t - 1.25\right)^2$ o.e. (N.B. $122.65625 = \frac{3925}{32}$) Accept the first term written to 1, 2, 3 or 4 d.p. or the full answer as shown.	**A1**
		(3 marks)
9ci	Statement that the stone will reach ground level when $h(t)$ = 0, or $-4.9t^2 + 12.25t + 115 = 0$ is seen.	**M1**
	Valid attempt to solve quadratic equation (could be using completed square form from part **b**, calculator or formula).	**M1**
	Clearly states that t = 6.25 s (accept t = 6.3 s) is the answer, or circles that answer and crosses out the other answer, or explains that t must be positive as you cannot have a negative value for time.	**A1**
		(3 marks)
9cii	hmax = awrt 123 ft A from part b.	**B1ft**
	$t = \frac{5}{4}$ or t = 1.25 ft C from part b.	**B1ft**
		(2 marks)
		Total 9 marks

NOTES: c: Award 4 marks for correct final answer, with some working missing. If not correct B1 for each of *A*, *B* and *C* correct.

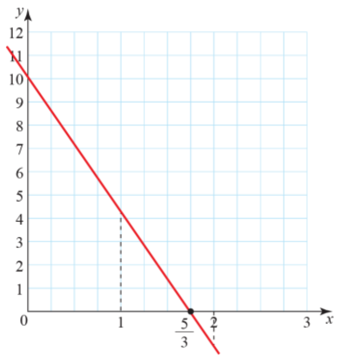
If the student answered part **b** by completing the square, award full marks for part **c**, providing their answer to their part **b** was fully correct.

10a	$\angle A = 45^\circ$ seen or implied in later working.	**B1**
	Makes an attempt to use the sine rule, for example, writing $\frac{\sin 120^\circ}{8x-3} = \frac{\sin 45^\circ}{4x-1}$	**M1**
	States or implies that $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\sin 45^\circ = \frac{\sqrt{2}}{2}$ **NOTE:** Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**A1**
	Makes an attempt to solve the equation for x. Possible steps could include: $\frac{\sqrt{3}}{16x-6} = \frac{\sqrt{2}}{8x-2}$ or $\frac{\sqrt{6}}{16x-6} = \frac{1}{4x-1}$ or $\frac{3}{16x-6} = \frac{\sqrt{6}}{8x-2}$ $\left(8\sqrt{3}\right)x - 2\sqrt{3} = \left(16\sqrt{2}\right)x - 6\sqrt{2}$ or $\left(4\sqrt{6}\right)x - \sqrt{6} = 16x - 6$ or $24x - 6 = \left(16\sqrt{6}\right)x - 6\sqrt{6}$ $6\sqrt{2} - 2\sqrt{3} = x\left(16\sqrt{2} - 8\sqrt{3}\right)$ or $\left(4\sqrt{6}\right)x - \sqrt{6} = 16x - 6$ or $12x - 3 = \left(8\sqrt{6}\right)x - 3\sqrt{6}$	**M1ft**
	$x = \frac{6\sqrt{2} - 2\sqrt{3}}{16\sqrt{2} - 8\sqrt{3}}$ or $x = \frac{6 - \sqrt{6}}{16 - 4\sqrt{6}}$ or $x = \frac{3\sqrt{6} - 3}{8\sqrt{6} - 12}$ o.e.	**A1ft**
	Makes an attempt to rationalise the denominator by multiplying top and bottom by the conjugate. Possible steps could include: $x = \frac{\left(3\sqrt{2} - \sqrt{3}\right)}{\left(8\sqrt{2} - 4\sqrt{3}\right)} \times \frac{\left(8\sqrt{2} + 4\sqrt{3}\right)}{\left(8\sqrt{2} + 4\sqrt{3}\right)}$ $x = \frac{48 + 12\sqrt{6} - 8\sqrt{6} - 12}{128 - 48}$ $x = \frac{36 + 4\sqrt{6}}{80}$	**M1ft**
	States the fully correct simplifed version for x. $x = \frac{9 + \sqrt{6}}{20}$ *	**A1***
	NOTE: Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**(7 marks)**
10b	States or implies that the formula for the area of a triangle is $\frac{1}{2}ab\sin C$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}bc\sin A$	**M1**
	$\frac{1}{2}\left(4\left(\frac{9+\sqrt{6}}{20}\right) - 1\right)\left(8\left(\frac{9+\sqrt{6}}{20}\right) - 3\right)\left(\sin 15 \text{ or } awrt\, 0.259\right)$ or $\frac{1}{2}\left(awrt\, 1.29\right)\left(awrt\, 1.58\right)\left(\sin 15 \text{ or } awrt\, 0.259\right)$.	**M1**
	Finds the correct answer to 2 decimal places. 0.26	**A1**
	NOTE: Exact value of area is $\frac{1}{200}\left(24 + 11\sqrt{6}\right)\left(\sqrt{6} - \sqrt{2}\right)$. If 0.26 not given, award M1M1A0 if exact value seen.	**(3 marks)** **Total** **10 marks**

Question	Scheme	Marks
11a	Makes an attempt to find $\int(10-6x)\mathrm{d}x$ Raising x powers by 1 would constitute an attempt.	**M1**
	Shows a fully correct integral with limits. $\left[10x-3x^2\right]_a^{2a}=1$	**A1**
	Makes an attempt to substitute the limits into their expression. For example, $\left(10(2a)-3(2a)^2\right)-\left(10(a)-3(a)^2\right)$ or $\left(20a-12a^2\right)-\left(10a-3a^2\right)$ is seen.	**M1ft**
	Rearranges to a 3-term quadratic equation (with = 0). $9a^2-10a+1=0$	**M1ft**
	Correctly factorises the LHS: $(9a-1)(a-1)=0$ or uses a valid method for solving a quadratic equation (can be implied by correct answers).	**M1ft**
	States the two fully correct answers $a=\frac{1}{9}$ or $a = 1$ For the first solution accept awrt 0.111	**A1**
		(6 marks)

11b

Figure 1



Question	Scheme	Marks
11b	Straight line sloping downwards with positive x and y intercepts. Ignore portions of graph outside $0 \leqslant x \leqslant 2$	**M1**
	Fully correct sketch with points (0, 10), and ($\frac{5}{3}$, 0) labelled. Ignore portions of graph outside $0 \leqslant x \leqslant 2$	**A1**
		(2 marks)
11c	Statements to the effect that the (definite) integral will only equal the area (1) if the function is above the x-axis (between the limits) AND when $a = 1$, $2a = 2$, so part of the area will be above the x-axis and part will be below the x-axis.	**B1**
	Greater than 1.	**B1**
		(2 marks)
		Total 10 marks

12a	States that the perimeter of the track is $2\pi r + 2x = 300$ The choice of the variable x is not important, but there should be a variable other than r.	**M1**
	Correctly solves for x. Award method mark if this is seen in a subsequent step. $x = \frac{300 - 2\pi r}{2} = 150 - \pi r$	**A1**
	States that the area of the shape is $A = \pi r^2 + 2rx$	**B1**
	Attempts to simplify this by substituting their expression for x. $A = \pi r^2 + 2r(150 - \pi r)$ $A = \pi r^2 + 300r - 2\pi r^2$	**M1**
	States that the area is $A = 300r - \pi r^2$ *	**A1***
		(5 marks)
12b	Attempts to differentiate A with respect to r	**M1**
	Finds $\frac{dA}{dr} = 300 - 2\pi r$	**A1**
	Shows or implies that a maximum value will occur when $300 - 2\pi r = 0$	**M1**
	Solves the equation for r, stating $r = \frac{150}{\pi}$	**A1**
	Attempts to substitute for r in $A = 300r - \pi r^2$, for example writing $A = 300\left(\frac{150}{\pi}\right) - \pi\left(\frac{150}{\pi}\right)^2$	**M1**
	Solves for A, stating $A = \frac{22\,500}{\pi}$	**A1**
		(6 marks)
		Total 11 marks

NOTES: 12b: Ignore any attempts at deriving second derivative and related calculations.

13a	Uses the equation of a straight line in the form $\log_4 V = mt + c$ or $\log_4 V - k = m(t - t_0)$ o.e.	**M1**
	Makes correct substitution. $\log_4 V = -\frac{1}{10}t + \log_4 40000$ o.e.	**A1**
		(2 marks)
13b	Either correctly rearranges their equation by exponentiation For example, $V = 4^{-\frac{1}{10}t + \log_4 40000}$ or takes the log of both sides of the equation $V = ab^t$. For example, $\log_4 V = \log_4(ab^t)$.	**M1**
	Completes rearrangement so that both equations are in directly comparable form $V = 40000 \times \left(4^{-\frac{1}{10}}\right)^t$ and $V = ab^t$ or $\log_4 V = -\frac{1}{10}t + \log_4 40000$ and $\log_4 V = \log_4 a + t\log_4 b$.	**M1**
	States that a = 40 000	**A1**
	States that $b = 4^{-\frac{1}{10}}$	**A1**
	NOTE: 2nd M mark can be implied by correct values of *a* and *b*.	**(4 marks)**
13c	*a* is the initial value of the car o.e.	**B1**
	b is the annual proportional decrease in the value of the car o.e. (allow if explained in figures using their *b*. For example, (since *b* is ≈0.87) the car loses 13% of its value each year.)	**B1**
	NOTE: Accept answers that are the equivalent mathematically. For example, for *b*. the value of the car in 87% of the value the previous year.	**(2 marks)**
13d	Substitutes 7 into their formula from part b. Correct answer is £15 157, accept awrt £15 000	**B1ft**
		(1 mark)
13e	Uses $10000 = ab^t$ with their values of *a* and *b* or writes $\log_4 10000 = -\frac{1}{10}t + \log_4 40000$ (could be inequality).	**M1**
	Solves to find *t* = 10 years.	**A1ft**
		(2 marks)

13f	Acceptable answers include. The model is not necessarily valid for larger values of t. Value of the car is not necessarily just related to age. Mileage (or other factors) will affect the value of the car.	**B1**
		(1 mark)
		Total 12 marks