A2 Practice Paper 0

1. $\int \cot 3x \, dx$ $\qquad \int \frac{f'(x)}{f(x)} \, dx = \ln f(x)$

$$\int \frac{\cos 3x}{\sin 3x} \, dx$$

$$= \frac{1}{3} \ln |\sin 3x| + c$$

2. Proof by contradiction:

Assumption: There is a greatest positive rational number, $\frac{a}{b}$.

$$\frac{a}{b} + 1 = \frac{a}{b} + \frac{b}{b} = \frac{a+b}{b} \Rightarrow \text{Rational}$$

This contradicts the assumption that $\frac{a}{b}$ is the greatest positive rational number so we can conclude that there is not a greatest positive rational number.

3. $\int_{\frac{\pi}{12}}^{\pi/8} \sin 4x (1 - \cos 4x)^3 \, dx$

$u = 1 - \cos 4x$

$\frac{du}{dx} = 4 \sin 4x$

$dx = \frac{du}{4 \sin 4x}$

$u = 1 - \cos 4x$

$x = \frac{\pi}{8}, \; u = 1$

$x = \frac{\pi}{12}, \; u = \frac{1}{2}$

$$\int_{\frac{1}{2}}^{1} \sin \cancel{4x} (u)^3 \frac{du}{4 \sin \cancel{4x}}$$

$$\frac{1}{4} \int_{\frac{1}{2}}^{1} u^3 \, du$$

$$\frac{1}{4} \left[\frac{u^4}{4} \right]_{\frac{1}{2}}^{1} = \frac{1}{4} \left[\left(\frac{1}{4} \right) - \left(\frac{1}{64} \right) \right] = \frac{15}{256}$$

4. $\dfrac{x^2-36}{x^2-11x+30} \times \dfrac{25-x^2}{Ax^2+Bx+C} \times \dfrac{6x^2+7x-3}{3x^2+17x-6} = \dfrac{x+5}{6-x}$

$\dfrac{(\cancel{x+6})(x-6)}{(x-5)(\cancel{x-6})} \times \dfrac{(5+x)(5-x)}{Ax^2+Bx+C} \times \dfrac{(\cancel{3x-1})(2x+3)}{(\cancel{3x-1})(\cancel{x+6})} = \dfrac{x+5}{6-x}$.

$(x+6)\dfrac{(5+x)(\cancel{-1})(\cancel{x-5})(2x+3)}{(\cancel{x-5})(Ax^2+Bx+C)} = \dfrac{x+5}{(\cancel{-1})(x-6)}$

$\dfrac{(5+x)(2x+3)}{(x-6)(2x+3)} = \dfrac{x+5}{x-6}$

$(x-6)(2x+3) = \underline{2x^2-9x-18}$

5. Proof by contradiction:

Assumption: If n is odd, n^2+1 is odd

$2n+1 \rightarrow$ odd

$(2n+1)^2+1$

$(2n+1)(2n+1)+1 = (4n^2+4n+1)+1$

$= 2(2n^2+2n+1) \rightarrow$ even

Therefore contradicts the assumption and if n is odd, n^2+1 is even.

6. a) $1 - 4x + 16x^2 - 64x^3 + \ldots$ convergent series

$|r| < 1$

$\left|\dfrac{-4x}{1}\right| < 1$

$|-4x| < 1$

$|x| < \frac{1}{4}$

b) $a = 1$

$r = -4x$

$S_\infty = \dfrac{a}{1-r}$

$4 = \dfrac{1}{1+4x}$

$4(1+4x) = 1$

$4 + 16x = 1$

$x = \frac{-3}{16}$

7. $(b-a)i - 2abcj + 2k = 10i - 96j + (7a+5b)k$

$b-a=10 \quad 2abc=96 \quad 2=7a+5b.$

$-a+b=10 \rightarrow \times 5$

$-5a+5b=50$
$-\;7a+5b=2$
$-12a = 48$
$\underline{a=-4}$

$b-a=10$
$b+4=10$
$\underline{b=6}$

$2abc=96$
$2\times -4\times 6\times c=96$
$-48c=96$
$\underline{c=-2}$

8. $g(x)=x^2-8x+7$

$y=x^2-8x+7$

$x=y^2-8y+7$

$x=(y-4)^2-4^2+7$

$x=(y-4)^2-9$

$x+9=(y-4)^2$

$\sqrt{x+9}+4=y$

$g^{-1}(x)=4+\sqrt{x+9}$

Range is $y>4$
Domain is $x>9$

9. $\dfrac{9x^2+25x+16}{9x^2-16} = A + \dfrac{B}{3x-4} + \dfrac{C}{3x+4}$

$$\begin{array}{r|l} & 1 \\ 9x^2+0x-16 & 9x^2+25x+16 \\ & -\;9x^2+0x-16 \\ \hline & 25x+32 \end{array}$$

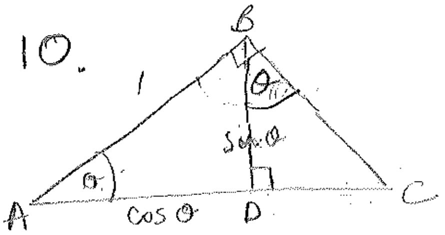
$25x+32 = B(3x+4)+C(3x-4)$

$x=\frac{4}{3},\; 25\left(\frac{4}{3}\right)+32 = B\left(3\left(\frac{4}{3}\right)+4\right)$

$\frac{196}{3}=8B \quad B=\frac{49}{6}$

$25x = 3Bx+3Cx$
$25 = 3\left(\frac{49}{6}\right)+3C$
$\frac{1}{2}=3C$
$C=\frac{1}{6}.$

10.



$\cos\theta = \frac{AD}{1}$

$\cos\theta = AD$

$\angle DBC = \theta \rightarrow$ Similar triangles

$\sin\theta = \frac{BD}{1}$

$\sin\theta = BD$

$\tan\theta = \frac{DC}{\sin\theta}$

$\frac{\sin\theta}{\cos\theta} = \frac{DC}{\sin\theta}$

$\frac{\sin^2\theta}{\cos\theta} = DC$

$\cos\theta = \frac{\sin\theta}{BC}$

$BC\cos\theta = \sin\theta$

$BC = \frac{\sin\theta}{\cos\theta}$

$BC = \tan\theta$

$AB^2 + BC^2 = AC^2$

$1^2 + \tan^2\theta = \left(\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta}\right)^2$

$1 + \tan^2\theta = \left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right)^2$

$1 + \tan^2\theta = \frac{1^2}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

11. a) $x = 7\sin t - 4$ $\quad y = 7\cos t + 3$

$\frac{x+4}{7} = \sin t$ $\quad \frac{y-3}{7} = \cos t$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x+4}{7}\right)^2 + \left(\frac{y-3}{7}\right)^2 = 1$

$(x+4)^2 + (y-3)^2 = 49$

b) Centre $(-4, 3)$

Radius $= 7$

$t = -\frac{\pi}{2}$, $x = -11$, $y = 3$ $\quad t = 0$, $x = -4$, $y = 10$

$t = \frac{\pi}{3}$, $x = 2.06$, $y = 6.5$

$-\frac{\pi}{2}$	$\frac{\pi}{3}$

c) $C = \pi d$

$C = 2\pi r$

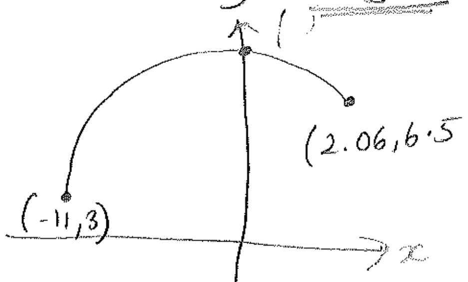
$C = 2 \times \pi \times 7$

$C = 14\pi$

$\frac{5}{12}$ of circumference

Arc length $= \frac{5}{12} \times 14\pi$

$= \frac{35}{6}\pi$



12. a) $x = \cos 2t \quad y = \sin t$

$$\frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt}$$

$$\frac{dx}{dt} = -2\sin 2t \quad \frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = \frac{1}{-4\sin t\cos t}$$

$$\frac{dy}{dx} = \frac{1}{-4\sin t\cos t} \times \cos t = -\frac{1}{4}\operatorname{cosec} t$$

b) $\frac{dy}{dx} = -\frac{1}{4}\operatorname{cosec} t$

$t = \frac{-5\pi}{6}, \quad \frac{dy}{dx} = \frac{-1}{4\sin\left(\frac{-5\pi}{6}\right)} = \frac{1}{2}$ Gradient of normal $= -2$

$t = \frac{-5\pi}{6}, \quad x = \frac{1}{2}, \quad y = -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y + \frac{1}{2} = -2\left(x - \frac{1}{2}\right)$$

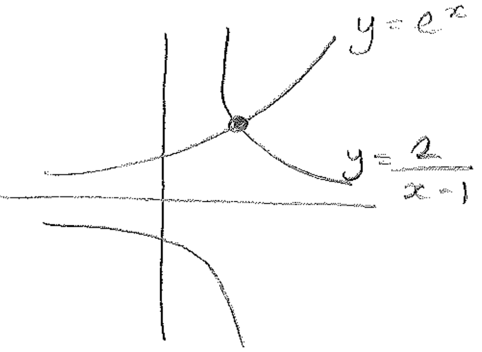
$$2y + 1 = -4x + 2$$

$$2y = -4x + 1$$

$$y = -2x + \frac{1}{2}$$

13. a) $g(x) = \frac{2}{x-1} - e^x$

$$\frac{2}{x-1} = e^x$$



Intersect at one point so $g(x) = 0$ has one root

b) $\frac{2}{x-1} - e^x = 0$

$$\frac{2}{x-1} = e^x$$

$$2 = e^x(x-1)$$

$$2 = xe^x - e^x$$

$$\frac{2 + e^x}{e^x} = x$$

$$2e^{-x} + 1 = x$$

c) $x_1 = 1.4463$

$x_2 = 1.4709$

$x_3 = 1.4594$

$x_4 = 1.4647$

d) $g'(x) = -2(x-1)^{-2} - e^x$

$g(1.5) = -0.4816$

$g'(1.5) = -12.4816$

$x_1 = 1.461$

14. a) $\dfrac{1+x}{\sqrt{1-2x}} = (1+x)(1-2x)^{-\frac{1}{2}}$

$$(1-2x)^{-\frac{1}{2}} = 1 + \left(\tfrac{-1}{2}\right)(-2x) + \frac{\left(\tfrac{-1}{2}\right)\left(\tfrac{-3}{2}\right)(-2x)^2}{1\times 2} + \ldots$$

$$= 1 + x + \frac{3}{2}x^2$$

$$(1+x)\left(1+x+\tfrac{3}{2}x^2\right) = 1 + x + \tfrac{3}{2}x^2 + \ldots$$

$$+ \quad + x + x^2 + \ldots$$

$$\underline{= 1 + 2x + \tfrac{5}{2}x^2 \ldots}$$

b) $|-2x| < 1$

$|x| < \frac{1}{2}$

c) $x = \dfrac{1}{100}$, $\dfrac{1+\frac{1}{100}}{\sqrt{1-2\left(\frac{1}{100}\right)}} = \dfrac{101\sqrt{2}}{140}$

d) $x = \dfrac{1}{100}$, $1 + 2\left(\dfrac{1}{100}\right) + \dfrac{5}{2}\left(\dfrac{1}{100}\right)^2 \approx 1.02025$

$1.02025 = \dfrac{101\sqrt{2}}{140}$

$\underline{\sqrt{2} = 1.41421}$

15. a) $y = 2.2323\ldots$

b) $A = \dfrac{0.5}{2}\left(0 + 2(0.12103 + 0.86603 + 2.23235) + 0\right)$

$\underline{= 1.610 \ (4sf)}$

c) $\displaystyle\int_0^2 \left(\tfrac{1}{2}x^3\sqrt{4-x^2}\right)dx$

$u = 4 - x^2$

$\dfrac{du}{dx} = -2x$

$\dfrac{du}{2x} = dx$

$x = 2, u = 0$

$x = 0, u = 4$

$\displaystyle\int_4^0 \tfrac{1}{2}x^3 u^{\frac{1}{2}} \frac{du}{-2x}$

$\displaystyle -\frac{1}{4}\int_4^0 x^2 u^{\frac{1}{2}} du$

$\displaystyle -\frac{1}{4}\int_4^0 (4-u) u^{\frac{1}{2}} du$

$\displaystyle -\frac{1}{4}\int_4^0 \left(4u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$

$= -\dfrac{1}{4}\left[\dfrac{8}{3}u^{\frac{3}{2}} - \dfrac{2}{5}u^{3/2}\right]_4^0$

$= -\dfrac{1}{4}\left[\left(\dfrac{8}{3}0^{3/2} - \dfrac{2}{5}0^{3/2}\right) - \left(\dfrac{8}{3}(4)^{3/2} - \dfrac{2}{5}(4)^{3/2}\right)\right]$

$= \dfrac{32}{15}$

d) Use more strips to improve the accuracy of the answer.