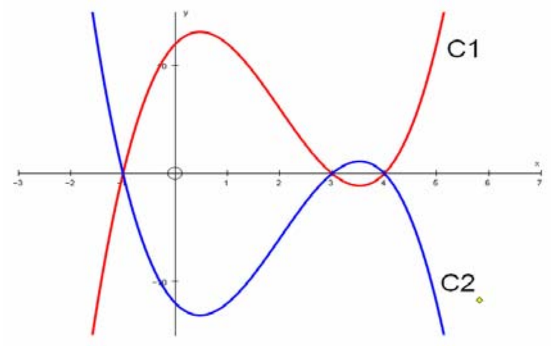
Mark Scheme 4721
June 2006

1	(i)	$\frac{21-3}{4-1}=\frac{18}{3}=6$	M1 A1	 2	Uses $\frac{y_2-y_1}{x_2-x_1}$ 6 (not left as $\frac{18}{3}$)
	(ii)	$\frac{dy}{dx}=2x+1$ $2\times3+1=7$	B1 B1	 2	
2	(i)	$27^{-\frac{2}{3}}=\frac{1}{27^{\frac{2}{3}}}=\frac{1}{9}$	M1 A1	 2	$\frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}}$ = 9 or 3^{-2} soi $\frac{1}{9}$
	(ii)	$5\sqrt{5}=5^{\frac{3}{2}}$	B1	1	
	(iii)	$\frac{1-\sqrt{5}}{3+\sqrt{5}}=\frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ $=\frac{8-4\sqrt{5}}{4}$ $=2-\sqrt{5}$	M1 B1 A1	 3	Multiply numerator and denominator by conjugate $\left(\sqrt{5}\right)^2=5$ soi $2-\sqrt{5}$
3	(i)	$2x^2+12x+13=2\left(x^2+6x\right)+13$ $=2[\left(x+3\right)^2-9]+13$ $=2\left(x+3\right)^2-5$	B1 B1 M1 A1	 4	*a* = 2 *b* = 3 $13-2b^2$ or 13 – b^2 or $\frac{13}{2}$ - b^2 (their *b*) *c*= –5
	(ii)	$2\left(x+3\right)^2-5=0$ $\left(x+3\right)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$	M1 A1 A1	 3	Uses correct quadratic formula or completing square method *x* = $\frac{-12\pm\sqrt{40}}{4}$ or $\left(x+3\right)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$ or -3 $\pm\frac{1}{2}\sqrt{10}$

4	(i)	$(x-4)(x-3)(x+1)$	B1		$x^2-7x+12$ or x^2-2x-3 or x^2-3x-4 seen
		$\equiv (x^2-7x+12)(x+1)$ $\equiv x^3+x^2-7x^2-7x+12x+12$	M1		Attempt to multiply a quadratic by a linear factor or attempt to list an 8 term expansion of all 3 brackets
		$\equiv x^3-6x^2+5x+12$	A1	3	$x^3-6x^2+5x+12$ **(AG)** obtained (no wrong working seen)
	(ii) (iii)		B1		+ve cubic with 3 roots (not 3 line segments)
			B1		(0, 12) labelled or indicated on *y*-axis
			B1	3	(– 1, 0), (3,0), (4, 0) labelled or indicated on *x*-axis
			M1		Reflect *their* (ii) in either *x*- or *y*-axis
			A1√	2	Reflect *their* (ii) in *x*-axis
5	(i)	$1<4x-9<5$	M1		2 equations or inequalities both dealing with all 3 terms
		$10<4x<14$ $2.5<x<3.5$	A1		2.5 and 3.5 seen oe
			A1	3	2.5 < *x* < 3.5 (or '*x* > 2.5 <u>and</u> *x* < 3.5')
	(ii)	$y^2\ge 4y+5$	B1		$y^2-4y-5=0$ soi
		$y^2-4y-5\ge 0$	M1		Correct method to solve quadratic
		$(y-5)(y+1)\ge 0$	A1		-1, 5 (**SR** If **both** values obtained from trial and improvement, award **B3**)
		$y\le -1,\ y\ge 5$	M1		Correct method to solve inequality
			A1	5	$y\le -1,\ y\ge 5$

6	(i)	$x^4-10x^2+25=0$ Let $y=x^2$ $y^2-10y+25=0$ $(y-5)^2=0$ $y=5$ $x^2=5$ $x=\pm\sqrt{5}$	*M1 dep*M1 A1 A1	 4	Use a substitution to obtain a quadratic or $(x^2-5)(x^2-5)=0$ Correct method to solve a quadratic 5 (not x = 5 with no subsequent working) $x=\pm\sqrt{5}$
	(ii)	$y=\frac{2x^5}{5}-\frac{20x^3}{3}+50x+3$ $\frac{dy}{dx}=2x^4-20x^2+50$	B1 B1	 2	$2x^4$ or $-20x^2$ oe seen $2x^4-20x^2+50$ (integers required)
	(iii)	$2x^4-20x^2+50=0$ $x^4-10x^2+25=0$ which has 2 roots	M1 A1	 2	*their* $\frac{dy}{dx}=0$ seen (or implied by correct answer) 2 stationary points **www in any part**
7	(i)	$y=x^2-5x+4$ $y=x-1$ $x^2-5x+4=x-1$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1 \quad x=5$ $y=0 \quad y=4$	M1 M1 A1 A1	 4	Substitute to find an equation in x (or y) Correct method to solve quadratic x = 1, 5 y = 0, 4 (**N.B.** This final A1 may be awarded in part (ii) if y coordinates only seen in part (ii)) **SR** one correct (x,y) pair **www** **B1**
	(ii)	2 points of intersection	B1	1	
	(iii)	EITHER $x^2-5x+4=x+c$ has 1 solution $x^2-6x+(4-c)=0$ $b^2-4ac=0$ $36-4(4-c)=0$ $c=-5$ OR $\frac{dy}{dx}=1=2x-5$ $x=3 \quad y=-2$ $-2=3+c$ $c=-5$	M1 M1 A1 A1 M1 A1 A1 A1	 4 4	$x^2-5x+4=x+c$ has 1 soln seen or implied Discriminant = 0 or $(x-a)^2=0$ soi 36 – 4(4 – c) = 0 or 9 = 4 – c c = – 5 Algebraic expression for gradient of curve = non-zero gradient of line used $2x$ – 5 = 1 x = 3 c = –5 **SR** c = -5 without any working **B1**

8	(i)	Height of box = $\frac{8}{x^2}$	*B1		Area of 1 vertical face $= \frac{8}{x^2} \times x$ $= \frac{8}{x}$
		4 vertical faces $= 4 \times \frac{8}{x}$ $= \frac{32}{x}$	*B1		
		Total surface area = $x^2 + x^2 + \frac{32}{x}$	B1 dep on both **		Correct final expression
		$A = 2x^2 + \frac{32}{x}$		3	
	(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	B1 B1 B1	3	$4x$ kx^{-2} $-32x^{-2}$
	(iii)	$4x - \frac{32}{x^2} = 0$	M1		$\frac{dA}{dx} = 0$ soi
		$4x^3 = 32$ $x = 2$	A1		$x = 2$
			M1 A1	4	Check for minimum Correctly justified
					SR If $x = 2$ stated **www** but with no evidence of differentiated expression(s) having been used in part (iii) **B1**

9	(i)	$\left(\frac{4+10}{2},\frac{-2+6}{2}\right)$ (7, 2)	M1 A1	 2	Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (7, 2) (integers required)
	(ii)	$\sqrt{(7-4)^2+(2--2)^2}$ $=\sqrt{3^2+4^2}$ $=5$	M1 A1	 2	Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ 5
	(iii)	$(x-7)^2+(y-2)^2=25$	B1√ B1√ B1	 3	$(x-7)^2$ and $(y-2)^2$ used (*their* centre) $r^2 = 25$ used (*their* r^2) $(x-7)^2+(y-2)^2=25$ cao <u>Expanded form:</u> -14x and -4y used B1√ $r = \sqrt{g^2+f^2-c}$ used B1√ $x^2+y^2-14x-4y+28=0$ B1 cao <u>By using ends of diameter:</u> (x - 4)(x - 10) + (y + 2)(y – 6) = 0 Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1
	(iv)	Gradient of AB = $\frac{6--2}{10-4}=\frac{4}{3}$ Gradient of tangent = $-\frac{3}{4}$ $y--2=-\frac{3}{4}(x-4)$ $3x+4y=4$	B1 B1√ M1 A1 A1	 5	oe Correct equation of straight line through A, any non-zero gradient a ,b, c need not be integers