Sale Grammar School

Mathematics – Year 9

KNOWLEDGE ORGANISER

To support your revision for the End of Year Assessment

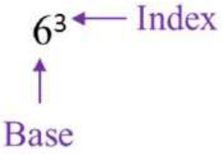
To effectively revise for a Maths assessment, you must finish, check, and correct many Maths questions.

- This document is to support your revision but remember the key with Maths revision is to **finish lots of questions**. Techniques like rewriting revision notes or copying from a revision guide, colour coding, and making posters can be enjoyable, but generally, they aren't the most effective use of revision time.
- Use your progress books and finish outstanding chapters or redo questions you struggled with.
- www.corbettmaths.com has lots of helpful videos and worksheets you can use as well.
- Use notes, and work through examples and questions in your book.
- Don't use your calculator unless the question specifically asks for it, you need to practise non-calculator skills as well. But checking answers with a calculator is very useful.
- If your struggle with anything, come to the support session during Tuesday lunchtime, in M51 or ask your teacher.
- The full-colour version can be found on www.smlmaths.com.

Year 9 Mathematics Knowledge Organiser – Unit 1: Powers and Roots

Keywords

Powers	The power of a number shows you how many times to use the number (base number) in a multiplication.
The base number	is the number that is being multiplied by Itself.
The power number	tells us how many times the base number is multiplied by itself.
Index notation	The notation in which a product such as $a \times a \times a \times a$ is recorded as a^4. In this example, the number 4 is the index (plural indices)



Irrational number cannot be written as a fraction.
The decimal form of an irrational number **does not terminate or recur**.

Examples of irrational numbers are $\pi = 3.14159…$

$\sqrt{2} = 1.414213…$

Rational number can be written as a fraction .
The decimal form of a rational number is either a **terminating** or a **recurring** decimal.

Examples of rational numbers are *17, -3 and 12.4 or*

$\frac{5}{4} = 1.25$ *(terminating decimal)*

$\frac{2}{3} = 0.\dot{6}$ *(recurring decimal)*

RECIPROCAL OR MULTIPLICATIVE INVERSE

The reciprocal of a **number** n is $\frac{1}{n}$

Example: the reciprocal of 5 is $\frac{1}{5}$

To find the reciprocal of **a fraction**, divide 1 by the fraction, (or 'flip' the fraction).

Example:

the reciprocal of $\frac{3}{4}$ *is* $1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}$

To calculate the reciprocal of a **decimal** d,

- use a calculation $1 \div d$, or
- convert the decimal into a fraction and find the reciprocal of the fraction.

Example: the reciprocal of 0.25 is

- $1 \div 0.25 = 4$ *or*
- $0.25 = \frac{25}{100} = \frac{1}{4}$, *reciprocal of* $\frac{1}{4}$ *is* $\frac{4}{1} = 4$

The product of a number and its reciprocal is always equal to 1.

Example: $5 \times \frac{1}{5} = \frac{5}{5} = 1$

$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

$0.25 \times 4 = 1$

MULTIPLICATION INDEX LAW

When expressions with the same base are multiplied, the indices are added.

$$\boxed{a^m \times a^n = a^{(m+n)}}$$

Examples: $7^5 \times 7^3 = 7^8$

$a^{12} \times a = a^{13}$

$4x^5 \times 2x^8 = 8x^{13}$

DIVISION INDEX LAW

When expressions with the same base are divided, the indices are subtracted.

$$\boxed{a^m \div a^n = a^{(m-n)} \quad \text{or} \quad \frac{a^m}{a^n} = a^{m-n}}$$

Examples: $15^7 \div 15^4 = 15^3$

$x^9 \div x^2 = x^7$

$20a^{11} \div 5a^3 = 4a^8$

BRACKETS INDEX LAWS

When raising a power to another power, multiply the powers together.

$$\boxed{(a^m)^n = a^{m \times n}}$$

Examples: $(y^2)^5 = y^{10}$

$(6^3)^4 = 6^{12}$

$(5x^6)^3 = 5^3 \times x^{18} = 125x^{18}$

NOTABLE POWERS

$$\boxed{\begin{gathered} a^1 = p \\ a^0 = 1 \end{gathered}}$$

Examples: $47^1 = 47$

$47^0 = 1$

$x^1 = x$

$x^0 = 1$

NEGATIVE POWERS

An expression with a negative index is the reciprocal of the expression with positive index

$$\boxed{a^{(-m)} = \frac{1}{a^m}}$$

Examples: $3^{(-2)} = \frac{1}{3^2} = \frac{1}{9}$

$\left(\frac{3}{8}\right)^{-1} = \frac{8}{3}$

$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

FRACTIONAL POWERS

Power $\frac{1}{2}$ means square root

Power $\frac{1}{3}$ means cube root

$$\boxed{\begin{gathered} a^{\frac{1}{2}} = \sqrt[2]{a} \\ a^{\frac{1}{3}} = \sqrt[3]{a} \end{gathered}}$$

The denominator of a fractional power acts as a 'root'.
The numerator of a fractional power acts as a normal power.

$$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}$$

Examples: $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

$\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\sqrt[2]{\frac{25}{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Year 9 Mathematics Knowledge Organiser – Unit 1: Powers and Roots

SURDS

Surds are expressions with irrational square roots.

LAWS OF SURDS

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$
$$\sqrt{a} \times \sqrt{a} = a$$
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$
$$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$$

SIMPLIFYING SURDS

- Write the number under the root sign as the product of two factors, one of which is the largest perfect square.

  Examples:

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5 \times \sqrt{3} = 5\sqrt{3}$$
$$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4 \times \sqrt{3} = 4\sqrt{3}$$

MULTIPLYING / DIVIDING SURDS

- Multiply / divide the numbers outside the square root sign together.
- Multiply / divide the numbers under the square root sign.
- Simplify the result.

  Examples:

$$\sqrt{8} \times \sqrt{3} = \sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$$
$$\begin{aligned} 4\sqrt{12} \times 3\sqrt{6} &= 4 \times 3 \times \sqrt{12} \times \sqrt{6} = 12\sqrt{72} \\ &= 12\sqrt{36 \times 2} = 12 \times \sqrt{36} \times \sqrt{2} \\ &= 12 \times 6 \times \sqrt{2} = 72\sqrt{2} \end{aligned}$$
$$\frac{\sqrt{200}}{\sqrt{10}} = \sqrt{\frac{200}{10}} = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

ADDING / SUBTRACTING SURDS

- The numbers inside the square root must be the same.
- Add/ subtract the numbers outside the square root, similarly to collecting like terms.

  Examples:

$$8\sqrt{5} - 2\sqrt{3} + 4\sqrt{5} - \sqrt{3} = 12\sqrt{5} - 3\sqrt{3}$$

STANDARD FORM

Standard form is a convenient way of writing very large, or very small numbers in form:

$A \times 10^n$

A is number between 1 and 10 (excluding 10) **$1 \le A < 10$**

10^n is a power of 10

^{n} is **positive** for big and **negative** for small numbers

Example:

Write 25400 in standard form:

A = 2.5400 *$2.54 \times 10000 = 25400$ ($10000 = 10^4$)*

$25400 = 2.54 \times 10^4$ *(power is positive)*

Write 0.0000342 in standard form:

A = 3.42 *$3.42 \times 0.00001 = 0.0000342$*

$0.0000342 = 3.42 \times 10^{-5}$ *(power is negative)*

Write 5.678×10^6 as ordinary number:

$5.678 \times 10^6 = 5.778 \times 1000000 = 5778000$

Write 5.678×10^{-6} as ordinary number:

$5.678 \times 10^{-6} = 5.778 \times 0.000001 = 0.000005778$

MULTIPLYING NUMBERS IN STANDARD FORM

- Multiply the numbers together.
- Multiply the powers of 10 together.
- Convert the result into standard form if needed.

Example: Calculate $(8 \times 10^4) \times (6 \times 10^2)$

1. Rearrange the calculation numbers first and then powers of 10 (multiplication is commutative):
   $(8 \times 10^4) \times (6 \times 10^2) = 8 \times 10^4 \times 6 \times 10^2 =$
   $8 \times 6 \times 10^4 \times 10^2$
2. Multiply numbers and use index laws to multiply powers of 10: *$8 \times 6 \times 10^4 \times 10^2 = 48 \times 10^{4+2} = 48 \times 10^6$*
3. Convert the answer to standard form if needed:
   $48 \times 10^6 = 4.8 \times 10^7$
   $\left[\begin{array}{l} 48 \div 10 = 4.8 \\ 10^6 \times 10 = 10^7 \end{array}\right]$

DIVIDING NUMBERS IN STANDARD FORM

- Divide the numbers.
- Divide the powers of 10.
- Convert the result into standard form if needed.

Example: Calculate $(4.5 \times 10^9) \div (9 \times 10^4)$

1. Rearrange calculation into a fraction:
   $$\frac{4.5 \times 10^9}{9 \times 10^4} = \frac{4.5}{9} \times \frac{10^9}{10^4}$$
2. Divide the numbers and use index laws to divide powers of 10:
   $$\frac{4.5}{9} \times \frac{10^9}{10^4} = 0.5 \times 10^{9-4} = 0.5 \times 10^5$$
3. Convert the answer to standard form if needed:
   $0.5 \times 10^5 = 5 \times 10^4$
   $\left[\begin{array}{l} 0.5 \times 10 = 5 \\ 10^5 \div 10 = 10^4 \end{array}\right]$

ADDING AND SUBTRACTING NUMBERS IN STANDARD FORM

Example: Calculate $(3.56 \times 10^5) + (2 \times 10^4)$

1. Convert numbers into numbers with the same power of 10:
   $(3.56 \times 10^5) + (2 \times 10^4) = (35.6 \times 10^4) + (2 \times 10^4)$
2. Add / subtract the numbers, keep power of 10 the same:
   $(35.6 \times 10^4) + (2 \times 10^4) = 37.6 \times 10^4$
3. Convert the answer to standard form if needed:
   $37.5 \times 10^4 = 3.75 \times 10^5$
   $\left[\begin{array}{l} 37.5 \div 10 = 3.75 \\ 10^4 \times 10 = 10^5 \end{array}\right]$

<u>Different way:</u>

1. Convert numbers to ordinary numbers:
   $(3.56 \times 10^5) + (2 \times 10^4) = 356000 + 20000$
2. Add / subtract them and convert back to stand.form:
   $356000 + 20000 = 376000 = 3.75 \times 10^5$

Year 9 Mathematics Knowledge Organiser – Unit 2: Quadratics

Keywords

Expanding brackets means removing the brackets.
Factorising means putting brackets back into expressions.
Factors of a number are the numbers that divide the original number without a remainder.
Writing a number as a product of factors is called a **factorisation** of the number.
The **Highest Common Factor (HCF)** is the largest common factor (the factor that two or more numbers have in common).

EXPANDING SINGLE BRACKETS

- Multiply everything in the brackets by a number or variable in front of the bracket.

Examples: Expand

$4(a + 6) = 4a + 24$

$-2(b - 4) = -2b + 8$

$2d(3d - e) = 6d^2 \text{-} 2de$

	a	+6
4	4a	+24

	b	-4
-2	-2b	+8

Expanding collection of single brackets

- Expand each bracket.
- Collect like terms.

Example: Expand and simplify

$2(3a^2 + 4a - 1) + 3(4a + 2) =$
$6a^2 + 8a - 2 + 12a + 6 =$
$6a^2 + 20a + 4$

FACTORISING

- Find the HCF of the terms in the brackets.
- Put the HCF in front of the brackets.
- Divide all terms by the HCF.
- Check your answers by expanding brackets.

Examples: Factorise

$4x + 12 = 4(x + 3)$
$7x^2 + 3x = x(7x + 3)$
$8x^2 + 16x = 8x(x + 2)$

EXPANDING DOUBLE BRACKETS (BINOMIALS)

- Multiply everything in the FIRST bracket by everything in the SECOND bracket.
- Collect like terms.

Examples: Expand and simplify

$(x - 5)(x + 2) =$
$x^2 + 2x - 5x - 10 =$
$x^2 - 3x - 10$

	x	-5
x	x^2	$-5x$
$+2$	$2x$	-10

$(2x - 5)(x - 2) = 2x^2 - 4x - 5x + 10 =$
$2x^2 - 9x + 10$

Expanding squared brackets

- Write squared brackets as two identical brackets multiplied together.
- Expand the brackets.
- Collect like terms.

Example: Expand and simplify

$(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 =$
$x^2 + 6x + 9$

$(x - 3)^2 = (x - 3)(x - 3) = x^2 - 3x - 3x + 9 =$
$x^2 - 6x + 9$

THE DIFFERENCE BETWEEN TWO SQUARES

$$A^2 - B^2 = (A + B)(A - B)$$

Example: Expand and simplify

$x^2 - 16 = (x + 4)(x - 4)$
$x^2 - 9 = (x + 3)(x - 3)$

FACTORISING QUADRATIC EXPRESSIONS

Quadratic expression is an expression where the highest power of the variable is power 2. The standard form of the **quadratic expression** is

$$\mathbf{ax^2 + bx + c}$$
where a, b, c are numbers, x is variable, $a \neq 0$

Factorising quadratic expressions means breaking quadratics into two brackets.

Example: Factorise $x^2 + 7x + 12$

- *Write down two brackets* $(x \quad)(x \quad)$
- *Find two numbers that multiply to c and add to b. Add them into the brackets*

$c = +12 \qquad 3 \times 4 = 12$
$b = +7 \qquad 3 + 4 = 7$
$x^2 + 7x + 12 = (x + 3)(x + 4)$

- *Check your answers by expanding*

$(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

Example: Factorise $x^2 + 3x - 4$

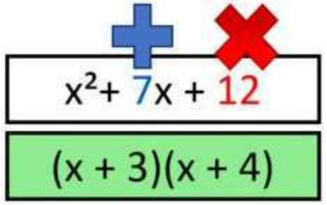
$c = -4 \qquad (-1) \times 4 = -4$
$b = +3 \qquad (-1) + 4 = +3$
$x^2 + 3x - 4 = (x - 1)(x + 4)$

Example: Factorise $x^2 - 12x + 35$

$c = +35 \qquad (-5) \times (-7) = +35$
$b = -12 \qquad (-5) + (-7) = -12$
$x^2 - 12x + 35 = (x - 5)(x - 7)$

Example: Factorise $x^2 - x - 2$

$c = -2 \qquad (-2) \times 1 = -2$
$b = -1 \qquad (-2) + 1 = -1$
$x^2 - x - 2 = (x - 2)(x + 1)$



Year 9 Mathematics Knowledge Organiser – Unit 2: Quadratics

SOLVING QUADRATIC EQUATIONS

A quadratic equation is an equation where the highest power of the variable is power 2. The standard format of the **quadratic equation** is

$$\boldsymbol{ax^2 + bx + c = 0}$$

where a, b, c are numbers, x is variable, $a \neq 0$

Solving quadratic equations means finding values of the variable that fulfil equality. There can be no solution, one solution, but most usually two solutions.

Example: Solve $x^2 - 6x = -8$

- *Rearrange the equation into standard form*
  $x^2 - 6x + 8 = 0$
- *Factorise quadratic expression*
  $x^2 - 6x + 8 = 0$
  $(x - 4)(x - 2) = 0$
- *Solve the equation by putting each of the brackets equal to 0*
  $x – 4 = 0$ or $x – 2 = 0$
  $x = 4$ $x = 2$
- *Check your answers by substitution*
  substitute $x = 4$:
  $4^2 - 6×4 + 8 = 16 – 24 + 8 = 0$
  substitute $x = 2$:
  $2^2 - 6×2 + 8 = 4 – 12 + 8 = 0$

Example: Solve $x^2 + 2x - 7 = 8$

$x^2 + 2x - 15 = 0$
$(x + 5)(x – 3) = 0$
$x + 5 = 0$ or $x – 3 = 0$
$\boldsymbol{x = -5}$ $\boldsymbol{x = 3}$
Check: $(-5)^2 + 2×(-5) - 15 =$
$= 25 - 10 - 15 = 0$
Check: $3^2 + 2×3 - 15 =$
$= 9 + 6 - 15 = 0$

SEQUENCE	A list of numbers or shapes that follow some pattern.
TERM	A number or shape in the sequence.
POSITION of the term	in the sequence describes where each term is located, for example 1st , 2nd …position.
TERM-TO-TERM RULE	gives a rule for finding each term of a sequence from the previous term. *Example: 1st term is 7, the rule is 'add 9'*
N-TH RULE	A rule calculates the term that is at the n-th position of the sequence. Also known as the '**POSITION-TO-TERM**' rule. n refers to the position of a term in a sequence. *Example: nth term is* $3n - 1$*, the 100th term is 3×100 - 1=299*
LINEAR SEQUENCE	A number pattern which increases (or decreases) by the same amount each time.
QUADRATIC SEQUENCE	A sequence of numbers where the **second difference** is constant.
COMMON DIFFERENCE	The constant rate at which a sequence increases or decreases.

FINDING THE N-TH TERM OF A LINEAR SEQUENCE

N-th term of linear sequence is written as $an + b$ where

n is a position of the term
a is the common difference between terms
b is a particular number

- Find the common difference.
- Multiply the common difference by n.
- Substitute n = 1, 2, 3… to find out the difference between created and original sequence.
- Add the difference to the rule.
- Substitute values in to check your nth term.

Example: Find the nth term of: 3, 7, 11, 15…

The common difference is +4.

Start to create your rule with $\boldsymbol{4n}$*.*

n	1	2	3
sequence	3	7	11
$4n$	4	8	12
difference	-1	-1	-1

The difference between the original and the new sequence is -1, the n-th term is $\boldsymbol{4n - 1}$*.*

Check: $n=1$, $4n-1 = 3$,
$n = 2$, $4n-1 = 7$

FINDING THE N-TH TERM OF A QUADRATIC SEQUENCE

N-th term of the quadratic sequence is written as $an^2 + bn + c$

where n is a position of the term
a is a half of the second difference
b and c are a particular numbers

- Find the first and second differences.
- Halve the second difference and multiply this by n^2.
- Substitute n = 1,2,3,4… into your expression so far.
- Subtract this set of numbers from the corresponding terms in the sequence from the question.
- Find the nth term of this set of numbers.
- Combine the nth terms to find the overall nth term of the quadratic sequence.
- Substitute values in to check your nth term.

Example: Find the nth term of: 4, 7, 14, 25, 40..

the first differences +3 +7 +11 +15
the second difference +4 +4 +4

Second difference is +4, so the nth term starts with $\boldsymbol{2n^2}$*.*

n	1	2	3	4	5
sequence	4	7	14	25	40
$2n^2$	2	8	18	32	50
difference	2	-1	-4	-7	-10

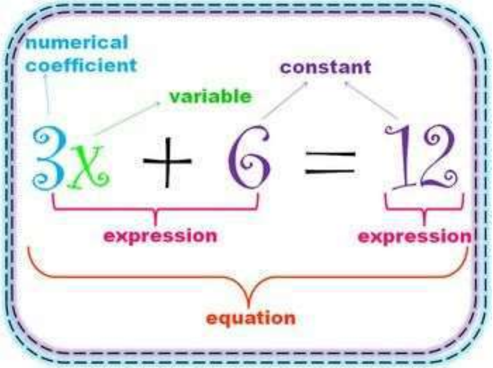
N-th term of the differences $-3n + 5$.

The overall n-th term is $\boldsymbol{2n^2 - 3n + 5}$.

Year 9 Mathematics Knowledge Organiser – Unit 3: Inequalities, equations and formulae

Keywords

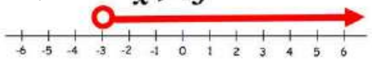
A **variable** is a letter or symbol that represents an unknown value.
When variables are used with other numbers, parentheses, or operations, they create an **algebraic expression**.
The equation is an algebraic expression with an equal sign, which can be solved (the value of the variable is found).
A **coefficient** is a number multiplied by the variable in an algebraic expression.
A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division, including + or – symbol in front of it.
A **constant** is a number that cannot change its value.
Identity is an equation that is true no matter what values of variables are chosen. (symbol ≡)
A **formula** is where one variable is equal to an expression in a different variable.



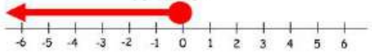
MARKING INEQUALITIES ON THE NUMBER LINE

Examples:

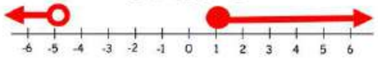
$x > -3$



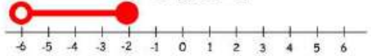
$x \leq 0$



$-5 > x \geq 1$



$-6 < x \leq -2$



>	means “greater than”,
≥	means “greater than or equal to”,
<	means “less than”,
≤	means “less than or equal to”.

SOLVING INEQUALITIES

The aim is to have variable on its own on the left of the inequality sign.
Solving inequalities is similar to solving equations, using inverse operations.

The **direction of inequality** stays the same and is not affected by

- adding (or subtracting) a number from both sides,
- multiplying (or dividing) both sides by a positive number,
- simplifying a side.

Example: Solve $3x \leq 12$ *// divide by 3*
$x \leq 4$

$2x - 1 > 3$ *// add 1*
$2x > 4$ *// divide by 2*
$x > 2$

! Multiplication or division by a negative number reverses the inequality.

Example: Solve $-5x > 10$ *// divide by -5*
$x < -2$

OR $-5x > 10$ *// add 5x*
$0 > 10 + 5x$ *// subtract 10*
$-10 > 5x$ *// divide by 5*
$-2 > x$

SOLVING THREE PART INEQUALITIES

To solve three part inequalities,

- apply inverse operations to all sides, or
- break inequalities into two separate ones, solve them and combine them back together.

Example: Solve $2 < 2x < -6$
(divide all three parts by 2)
$1 < x < -3$

or $2 < 2x < -6$
$2 < 2x$ $2x < -6$
(divide by 2)
$1 < x$ $x < -3$
$1 < x < -3$

Example: Solve $5 \leq 3x + 2 \leq 14$
(subtract 2 from all three parts)
$3 \leq 3x \leq 12$
(divide by 3)
$1 \leq x \leq 4$

or $5 \leq 3x + 2 \leq 14$
$5 \leq 3x + 2$ $3x + 2 \leq 14$
(subtract 2)
$3 \leq 3x$ $3x \leq 12$
(divide by 3)
$1 \leq x$ $x \leq 4$
$1 \leq x \leq 4$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

- collect all the **variables** onto one side of the equation and all numbers onto the other side.
- start by moving the unknown with the smallest **coefficient** in the equation.

Example: Solve $6x - 5 = 27 - 2x$ *//add 2x to both sides*
$8x - 5 = 27$ *//add 5 to both sides*
$8x = 32$ *//divide by 8*
$x = 4$

Year 9 Mathematics Knowledge Organiser – Unit 3: Inequalities, equations and formulae

SOLVING EQUATIONS WITH FRACTIONS

- Find the least common denominator of all the fractions in the equation.
- Multiply both sides of the equation by that least common denominator. This clears the fractions.
- Isolate the variable terms on one side and the constant terms on the other side.
- Simplify both sides.
- Solve the equation

Example: Solve $\frac{2x}{5} - 4 = 6$ //multiply both sides by 5

$2x - 20 = 30$ //solve 2-step equation

$x = 25$

$1 + \frac{x}{2} = \frac{x}{3} + 2$ //LCD = 6, multiply by 6

$6 + 3x = 2x + 12$ //subtract $2x$ from both sides

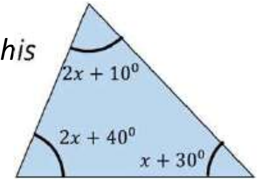
$6 + x = 12$ //subtract 6

$x = 6$

FORMING AND SOLVING EQUATIONS

Example:

Write an equation for the sum of the angles in this triangle:



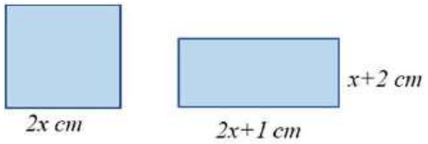
$(2x + 10) + (2x + 40) + (x + 30) = 180$

$5x + 80 = 180$

Solving this equation, finds the size of x and consequently the sizes of angles.

Example:

The perimeters of the square and rectangle are the same, write the equation:



$4(2x) = 2(2x + 1) + 2(x + 2)$

$8x = 4x + 2 + 2x + 4$

$8x = 6x + 6$

Solving this equation, finds the size of x and consequently the perimeters of the shapes.

$x = 3$

CHANGING THE SUBJECT OF THE FORMULA

The subject of a formula is the variable that is being worked out. It can be recognised as the letter on its own on one side of the equals sign.

To change the subject of a formula, rearrange the formula so that it has a different subject. The method is exactly the same as solving an equation.

Examples: Make x the subject of the formula

$4t = x - 3p$ //add $3p$ to both sides

$4t + 3p = x$

$x = 4t + 3p$

$ax = y + z$ //divide both sides by a

$x = \frac{y+z}{a}$

$ax - y = 2y$ //add y to both sides

$ax = 3y$ //divide both sides by a

$x = \frac{3y}{a}$

$x + y = xy$ //collect x on one side

$x + y - xy = 0$

$x - xy = -y$ //factorise

$x(1 - y) = -y$ //divide both sides by $1-y$

$x = -\frac{y}{1-y}$

$\sqrt{x} - 3 = y$ //add 3 to both sides

$\sqrt{x} = y + 3$ //square both sides

$x = (y + 3)^2$

$\sqrt{x - 3} = y$ //square both sides

$x - 3 = y^2$ //add 3 to both sides

$x = y^2 + 3$

$x^2 - 4 = a^2$ //add 4 to both sides

$x^2 = a^2 + 4$ //square root both sides

$x = \pm\sqrt{a^2 + 4}$

ALGEBRAIC FRACTIONS

The fractions, where numerator and/or denominator are algebraic expressions.

Simplifying Algebraic Fractions

- factorise the numerator and denominator and cancel common factors.

Example: Simplify $\frac{10xy}{12xy^2}$

$= \frac{2 \times 5 \times x \times y}{2 \times 6 \times x \times y \times y} = \frac{5}{6y}$

Adding/ Subtracting Algebraic Fractions

- for $\frac{a}{b} \pm \frac{c}{d}$, the common denominator is bd

$$\boxed{\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}}$$

Example: $\frac{1}{x} + \frac{x}{2y} = \frac{1 \times 2y}{2xy} + \frac{x \times x}{2xy} = \frac{2y+x^2}{2xy}$

Multiplying Algebraic Fractions

- multiply the numerators together and the denominators together.

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}}$$

Example: $\frac{x}{3} \times \frac{x+2}{x-2} = \frac{x(x+2)}{3(x-2)} = \frac{x^2+2x}{3x-6}$

Dividing Algebraic Fractions

- multiply the first fraction by the reciprocal of the second fraction.

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}}$$

Example: $\frac{x}{3} \div \frac{2x}{7} = \frac{x}{3} \times \frac{7}{2x} = \frac{7x}{6x} = \frac{7}{6}$

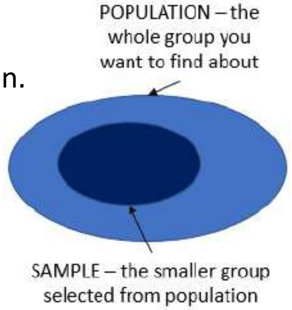
$\frac{y}{x} = b$ // multiply by x

$y = bx$ // divide by b

$x = \frac{y}{b}$

Year 9 Mathematics Knowledge Organiser – Unit 4: Collecting and analysing data

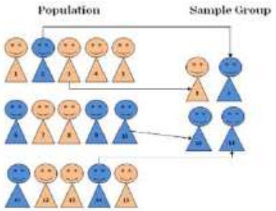
SAMPLING



A **census** surveys the whole population.
The population is everyone who can be questioned.
A **sample** involves just part of the population, it should **not be biased**.

To avoid bias a sample should be:
- representative (represents the whole population)
- selected by a random process (every member of the population has an <u>equal chance to be selected</u>)
- big enough

SIMPLE SAMPLING



- number the population
- choose random numbers to create the sample

Example: there are 300 frogs in a pond. In a sample of 10 frogs 3 are blue, what is an estimate for the number of blue frogs in the pond?

$\frac{3}{10}$ *in the sample are blue, so* $\frac{90}{300}$ *in population are blue*

$\times 30$

STRATIFIED SAMPLING

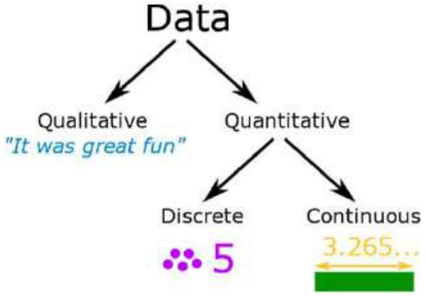
- divide population into sub-groups called **strata**, based on relevant characteristics
- count the population are in each stratum
- use random sampling proportionally

Example:

Year group	*7*	*8*	*9*	*10*	*11*
# of students	*190*	*145*	*145*	*140*	*130*

A stratified sample of 60 students is used in the survey. Calculate the number of Year 11 students in the sample.

1. *Total number of students = 750*
2. *Proportion of Year 11 in the total =* $\frac{130}{750}$
3. *Number of Year 11 in the sample =* $60 \times \frac{130}{750} = 10.4$
4. *10.4 ≈ 10. Number of Year 11 in the sample is 10.*



Keywords

Discrete data is counted, it can only take certain values.

Example: the number of students in a class

Continuous data is measured, it can take any value (within a range).

Example: a person's height

Raw data is collected, unprocessed data

Primary data is data that you collect yourself.

Secondary data is data collected by someone else.

The **frequency** of a data is the number of times the data occurs.

QUESTIONNAIRES

Types of questions in questionnaire:
- **Open questions** have no suggested answers.
- **Closed questions** have a set of answers to choose from.

Questionnaires should include
- short questions,
- words that are easily understood,
- non-biased or no 'leading' questions,
- option boxes for answers where possible.

Option boxes should
- cover **every possible answer** (using 'other' if necessary),
- be **easily understood**,
- **not overlap**.

Questionnaires must not be biased and should be tested before being used (a pilot survey).

Example: List two things that are wrong with this question in the questionnaire:
"How many texts have you sent on your mobile phone?
☐ 0 - 10
☐ 10 - 20
☐ 20 or more"

Mistakes: 1. Overlapping regions.
2. No time frame.

Corrected question:
"How many texts have you sent on your mobile phone last week? ☐ 0 - 9
☐ 10 - 19
☐ 20 or more"

FREQUENCY POLYGONS

A frequency polygon is a graph constructed by using **straight lines** joining the **midpoints of each interval**. The heights of the points represent the **frequencies**.

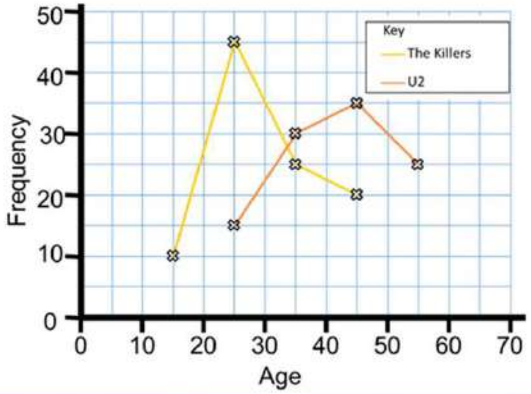
Example: The table shows the ages of the first 100 people attending concert of U2 and The Killers. Draw a frequency polygon.

U2

Age	Freq.	Mid Point
$20 < a \leq 30$	15	25
$30 < a \leq 40$	30	35
$40 < a \leq 50$	35	45
$50 < a \leq 60$	20	55

The Killers

Age	Freq.	Mid Point
$10 < a \leq 20$	10	15
$20 < a \leq 30$	45	25
$30 < a \leq 40$	25	35
$40 < a \leq 50$	20	45



Year 9 Mathematics Knowledge Organiser – Unit 4: Collecting and analysing data

Keywords

MEAN	means the fair share (**total of values** ÷ **number of values**).	AVERAGES
MEDIAN	is the **middle value** when the values are put in order.	AVERAGES
MODE	is the **most common** value.	AVERAGES
RANGE	is the **difference between the biggest and smallest** values.	
Frequency	is the number of times an event happens.	

Frequency table is a table for a set of observations showing how frequently each event occurs.

Grouped data is data grouped into non-overlapping classes or intervals.

Class is an interval for grouping data.

AVERAGES FROM FREQUENCY TABLE

Example: A team plays 20 games, the coach records the number of goals they score in each game in a frequency table. Find averages and range.

Number of Goals	Frequency	Total number of goals
0	5	0 × 5 = 0
1	6	1 × 6 = 6
2	4	2 × 4 = 8
3	3	3 × 3 = 9
4	2	4 × 2 = 8
Total	20	31

Mode: the most common number *of goals is 1 (6 times in the table)*

Mode = 1

Range: *the highest value is 4 goals and the lowest 0 goals.*

Range = 4 – 0 = 4

Mean:

1. Create the third column and multiply (**value × frequency**) to find the total number of values (goals)
2. Find total of frequencies and total of the 3rd column.
3. Divide $\frac{\boldsymbol{total\ number\ of\ values\ (goals)}}{\boldsymbol{total\ frequencies}} = \frac{31}{20} = \mathbf{1.55}$

Median:

1. **Position of the median =** $\frac{\boldsymbol{total\ frequency+1}}{\mathbf{2}}$

$= \frac{20+1}{2} = 10.5^{th}$ *position*

2. *There are 5 '0 goals' + 6 '1 goals', which makes 11 values. The median is 10.5th value, **median = 1**. (imagine values in a list: 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2…)*

10.5th value

AVERAGES FROM THE GROUPED DATA

Example: Find the estimate of mean, median and modal classes from the table below:

POCKET MONEY (£)	FREQUENCY (F)	MIDPOINT (X)	F × X = FX
0 < P ≤ 1	2	0.5	2 × 0.5 = 1
1 < P ≤ 2	5	1.5	5 × 1.5 = 7.5
2 < P ≤ 3	5	2.5	5 × 2.5 = 12.5
3 < P ≤ 4	9	3.5	9 × 3.5 = 31.5
4 < P ≤ 5	15	4.5	15 × 4.5 = 67.5
TOTAL	36	TOTAL	120

Modal class: ***4 < P ≤ 5*** *(the most common class, 15 times)*

Range: *£5 – £0 =* ***£5***

Mean:

1. Create 3rd column (**midpoint** of the classes)
2. Create 4th column (**midpoint** × **frequency**)
3. Find total of frequencies and total of the 4th column.
4. Divide $\frac{\boldsymbol{total\ number\ of\ values\ (£)}}{\boldsymbol{total\ frequencies}} = \frac{120}{36} =$ ***£3.33***

Median class:

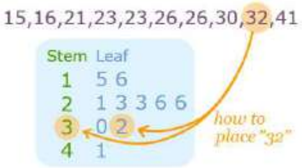
1. Position of the median = $\frac{\boldsymbol{total\ frequency+1}}{\mathbf{2}} = \frac{36+1}{2} =$ *18.5th position*

POCKET MONEY (£)	FREQUENCY (F)	
0 < P ≤ 1	2	2
1 < P ≤ 2	5	2+5=7
2 < P ≤ 3	5	2+5+5=12
3 < P ≤ 4	9	2+5+5+9=21
4 < P ≤ 5	15	2+5+5+9+15=36

2. *Median is 18.5th value, that is in **median class 3 < P ≤4***

STEM AND LEAF DIAGRAM

- data value is split into a "**leaf**" (usually the last digit) and a "**stem**" (the leading digit(s)).
- allows the visualisation of the distribution of data.



Example: Create stem and leaf diagram from this set of data: 133, 107, 113, 94, 97, 94, 109, 107, 113, 132, 99

1. create 'STEM' part of the diagram (there should not be any number missing between the smallest and the biggest value – *that is why there is a row with stem 12 without any leaves*)
2. one by one, put in the unit's figures in the proper row, these are the 'LEAVES'.

not ordered diagram

Stem	Leaf
9	4 7 4 9
10	7 9 7
11	3 3
12	
13	3 2

3. rewrite the diagram so that the leaves are in order.
4. add **KEY**

correct ordered diagram

Stem	Leaf
9	4 4 7 9
10	6 7 9
11	3 3
12	
13	2 3

Key: 10│3 means 103

Year 9 Mathematics Knowledge Organiser – Unit 4: Collecting and analysing data

AVERAGES FROM STEM-AND-LEAF

Example: Find mean, median, mode and range from this stem and leaf diagram.

5	1	4		
6	2	5	8	
7	0	2	2	2
8	2	4	5	
9	1	3	3	9

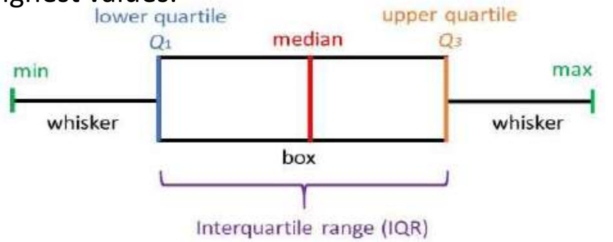
Key: 7|3 means 73%

1. Mode = ***72%*** (the most common result)
2. Range = *99 – 51 =* ***48%*** (maximum - minimum)
3. Median:
   - for the total frequency, count the number of values in the diagram: *16*
   - the position of the median = $\frac{\boldsymbol{total\ frequency+1}}{\mathbf{2}} = \frac{16+1}{2} = 8.5^{th}$ *position*
   - *find the 8th value (72%) and 9th value (72%).*
   - *the value in between them is* ***median = 72%.***

   (! The most common mistake is reading the value incorrectly: *incorrect answers would be 2)*

BOX AND WHISKERS PLOTS

Box and whiskers plot is a diagram, showing **quartiles** in a box, with lines extending to the lowest and the highest values.



Median (Q2) is the middle value.
Lower quartile (Q1) is the middle value of the bottom half.
Upper quartile (Q3) is the middle value of the upper half.
The **interquartile range (IQR)** is the difference between the upper quartile and lower quartile **IQR = Q3 – Q1**.

Finding lower and upper quartiles:

- **Even** number of items in the list: find median of bottom and upper half.

  Example: 2, 8, 9, 11, 13, 56

  Position of median = $\frac{6+1}{2} = \frac{7}{2} = 3.5^{th}$ *value.*

  Median (Q2) = 10.

  LQ = median of the bottom half (numbers 2, 8, 9).

  Position of LQ = $\frac{3+1}{2} = \frac{4}{2} = 2^{nd}$ *value.* ***Q1 = 8.***

  UQ = median of the top half (numbers 11, 13, 56).

  Position of UQ = $\frac{3+1}{2} = \frac{4}{2} = 2^{nd}$ *value.* ***Q3 = 13.***

- **Odd** number of items: throw away middle item, find median of remaining bottom half and upper half.

  Example: 2, 8, 9, 11, 13

  Position of median = $\frac{5+1}{2} = \frac{6}{2} = 3^{th}$ *value.*

  Median (Q2) = 9.

  LQ = median of the bottom half (numbers 2, 8).

  Position of LQ = $\frac{2+1}{2} = \frac{3}{2} = 1.5^{th}$ *value.* ***Q1 = 5.***

  UQ = median of the top half (numbers 11, 13).

  Position of UQ = $\frac{3+1}{2} = \frac{4}{2} = 2^{nd}$ *value.* ***Q3 = 25.***

Constructing Box and whiskers plots:

Example: Construct a box plot for number of customers in the shop each hour:

2, 3, 5, 7, 7, 9, 9, 9, 9, 13, 14, 19

Minimum value = 2.

Maximum value = 19.

Position of median = $\frac{12+1}{2} = \frac{13}{2} = 6.5^{th}$ *value.*

Median Q2 = 9.

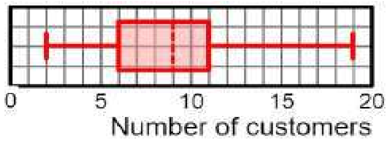
Q1 = median of the bottom half.

Q3 = median of the top half .

Position of Q1 and Q3 = $\frac{6+1}{2} = \frac{7}{2} = 3.5^{th}$ *value.*

Q1 = 6.

Q3 = 11.



CUMULATIVE FREQUENCY DIAGRAM

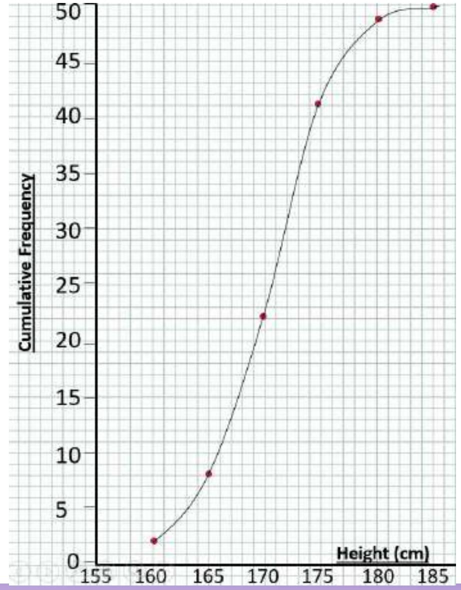
Cumulative frequency is a running total.
A **cumulative frequency diagram** is a curve that illustrates the trend of the data.

To plot the cumulative frequency diagram:

- calculate the **cumulative frequency**, the sum of all the frequencies up to and including that value.
- plot the cumulative frequency against the upper interval value.
- join up the points with a **smooth curve.**

Example: Plot cumulative frequency diagram

Height (cm)	*Freq.*	*Cum. Freq.*
155 ≤ h <160	*2*	*2*
160 ≤ h <165	*6*	*2+ 6 = 8*
165 ≤ h <170	*14*	*2 + 6 + 14 = 22*
170 ≤ h <175	*19*	*2 + 6 + 14 + 19 = 41*
175 ≤ h <180	*8*	*2 + …+ 19 + 8 = 49*
180 ≤ h <185	*1*	*2 + .. + 19 + 8 + 1 = 50*
Total =	*50*	



Year 9 Mathematics Knowledge Organiser – Unit 4: Collecting and analysing data

BOX PLOTS AND CUMULATIVE FREQUENCY GRAPHS

Lower Quartile (Q1): 25% of the data is less than the lower quartile.
Median (Q2): 50% of the data is less than the median.
Upper Quartile (Q3): 75% of the data is less than the upper quartile.
Interquartile Range (IQR): represents the middle 50% of the data.

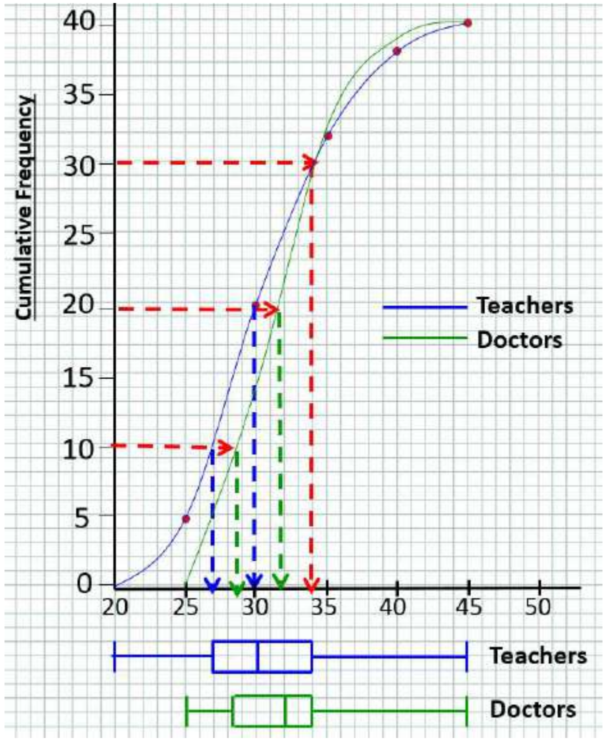
Example: The table below shows the ages that men from two professions spotted their first grey hair.

	Teachers		*Doctors*	
Age y years	*Freq.*	*CF*	*Freq.*	*CF*
20 < y ≤ 25	*5*	*5*	*0*	*0*
25 < y ≤ 30	*15*	*20*	*14*	*14*
30 < y ≤ 35	*12*	*32*	*19*	*33*
35 < y ≤ 40	*6*	*38*	*6*	*39*
40 < y ≤ 45	*2*	*40*	*1*	*40*

Median = 40 ÷ 2 = 20th value
Median (T) = 30 years
Median (D) = 32 years

LQ = ¼ × 40 = 10th value
LQ (T) = 27 years
LQ (D) = 29 years

UQ = ¾ × 40 = 30th value
UQ (T & D) = 34 years



Interpreting box plots:

- *On average, teachers go grey at a younger age as their median is lower.*
- *However, doctors go grey at a more similar age as their range and interquartile range is smaller.*

HISTOGRAMS

Histograms allow us to display **continuous** data grouped into intervals. They reflect the ‘concentration’ of things within each range of values.
Bars can be unequal in width and there are no spaces in between the bars..

Histograms show **frequency density** on the y-axis, not frequency.

Working out the frequency density:

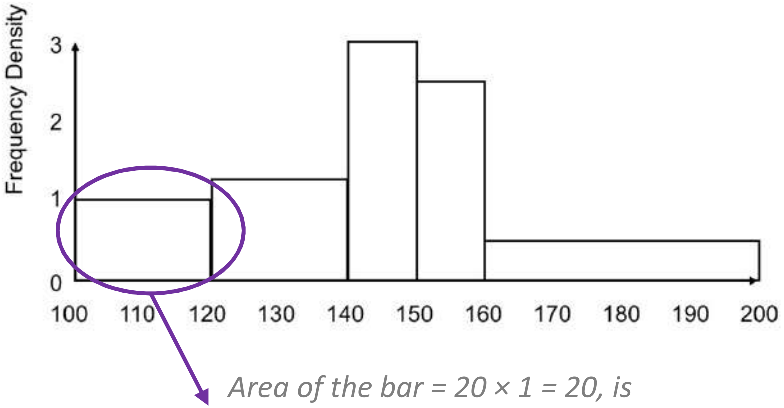
$$\textbf{Frequency Density} = \frac{frequency}{class\ width}$$

The area of a bar is equal to the frequency of that class interval.

Frequency = Freq Density × Class Width

Example: Complete the histogram

Height (cm)	Frequency	Frequency Density= $\frac{frequency}{class\ width}$
100 < x ≤ 120	20	20 ÷ 20 = **1**
120 < x ≤ 140	25	25 ÷ 20 = **1.25**
140 < x ≤ 150	30	30 ÷ 10 = **3**
150 < x ≤ 160	25	25 ÷ 10 = **2.5**
160 < x ≤ 200	20	20 ÷ 40 = **0.5**



Area of the bar = 20 × 1 = 20, is equal to the frequency

Year 9 Mathematics Knowledge Organiser – Unit 5: Multiplicative reasoning

Keywords

Proportion is used to show how quantities and amounts are related to each other.

$\propto$ is the symbol for proportion.

DIRECT PROPORTION

Two quantities x and y are said to be in **direct proportion** if they increase or decrease at the same rate. That is, if the ratio between the two quantities $\frac{y}{x}$ is always the same ($\frac{y}{x} = \boldsymbol{k}$, where k is the constant of proportionality).

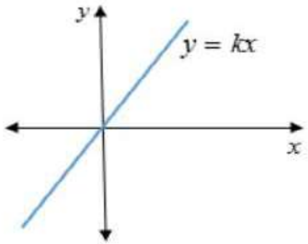
If y is directly proportional to x, this can be written as

$$\boxed{y \propto x}$$

An equation representing direct proportion, where k is the constant of proportionality is

$$\boxed{y = kx}$$

Graph of linear direct proportion is a **straight line** running through an origin (0, 0).



Using multiplicative tables

Example: In the following table A is directly proportional to B. Find the equation connecting A and B. Hence complete the table.

A	1	3		10
B	4	12	16	

$\frac{B}{A} = k$ $\quad \frac{4}{1} = 4$ $\quad \frac{12}{3} = 4$

Constant of proportionality $k = 4$, therefore equation connecting A and B is $\boldsymbol{B = 4A}$ *or* $\boldsymbol{A = \frac{1}{4}B}$

×4

A	1	3	4	10
B	4	12	16	40

÷4 or $\times \frac{1}{4}$

$\frac{16}{4} = 4$

$10 \times 4 = 40$

Using proportionality formulae

1. Start with a general equation using a constant of proportionality $y = kx$.
2. Solve the equation to find k using the pair of values in the question.
3. Rewrite the equation using the value of k you have just found.
4. Substitute the other given value from the question into the equation to find the missing value.

Example: S is directly proportional to T.
When $S = 30$, $T = 5$.
Write an equation linking S and T.
Hence a) Find the value of S when $T = 7$.
b) Find the value of T when $S = 60$

$S \propto T$
$S = kT$
$30 = k \times 5$
$k = \frac{30}{5} = 6$
$\boldsymbol{S = 6T}$

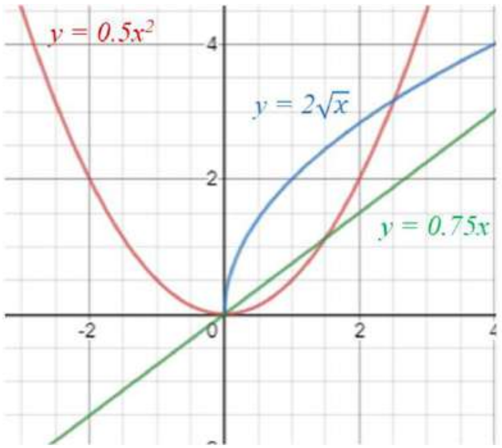
a) $S = 6T$
$S = 6 \times 7 = 42$

b) $S = 6T$
$60 = 6T$
$T = \frac{60}{6} = 10$

NON LINEAR DIRECT PROPORTION

$$\boxed{y = kx^n}$$

Graph of non-linear direct proportion is not a straight line but runs through an origin (0, 0).



Example: P is directly proportional to square of Q.
When $P = 8$, $Q = 4$.
Find the equation connecting P and Q.
Hence find a) P when $Q = 7$.
b) Q when $P = 84.5$.

$P \propto Q^2$
$P = kQ^2$
$8 = k \times 4^2$
$8 = 16k$
$k = \frac{8}{16} = 0.5$
$\boldsymbol{P = 0.5Q^2}$

a) $P = 0.5Q^2$
$P = 0.5 \times 7^2 = 24.5$

b) $P = 0.5Q^2$
$84.5 = 0.5Q^2$
$Q^2 = \frac{84.5}{0.5} = 169$
$Q = \sqrt{169} = 13$

Year 9 Mathematics Knowledge Organiser – Unit 5: Multiplicative reasoning

INVERSE PROPORTION

If two quantities x and y are **inversely proportional**, as one increases, the other decreases by the same rate. When you multiply the variables together $x \times y$ you get a constant value ($\boldsymbol{x \times y = k}$, where k is the constant of proportionality).

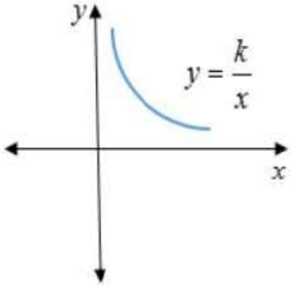
If y is inversely proportional to x, this can be written as

$$\boxed{y \propto \frac{1}{x} \qquad x \neq 0}$$

An equation of the inverse proportion, where k is the constant of proportionality is

$$\boxed{y = \frac{k}{x} \qquad x \neq 0}$$

Graph of inverse proportion never crosses x and y axis.
$x \neq 0,\ y \neq 0$



Using multiplicative tables

Example: In the following table A is inversely proportional to B. Find the equation connecting A and B. Hence complete the table.

A	5	8	
B	10		4

$A \times B = k$ $\quad$ $5 \times 10 = 50$

Constant of proportionality $k = 50$, therefore equation connecting B and A is $\boldsymbol{B = \frac{50}{A}}$ or $\boldsymbol{A = \frac{50}{B}}$

A	5	8	12.5
B	10	6.25	4

$50 \div 8 = 6.25$
$50 \div 4 = 12.5$

Using proportionality formulae

1. Start with general equation using a constant of proportionality $y = \frac{k}{x}$.
2. Solve the equation to find k using the pair of values in the question.
3. Rewrite the equation using the value of k you have just found.
4. Substitute the other given value from the question into the equation to find the missing value.

Example: P is inversely proportional to Q.
When $P = 2$, $Q = 8$.
Find an equation linking P and Q.
a) Find the value of P when $Q = 10$
b) Find the value of Q when $P = 16$

$P \propto \frac{1}{Q}$
$P = \frac{k}{Q}$
$2 = \frac{k}{8}$
$k = 2 \times 8 = 16$
$\boldsymbol{P = \frac{16}{Q}}$

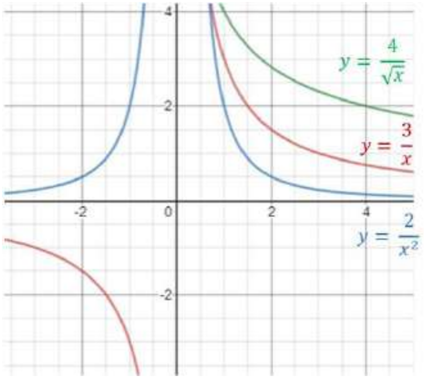
a) $P = \frac{16}{Q}$
$P = \frac{16}{10} = 1.6$

b) $P = \frac{16}{Q}$
$16 = \frac{16}{Q}$
$16Q = 16$
$Q = \frac{16}{16} = 1$

NON LINEAR INVERSE PROPORTION

$$\boxed{y = \frac{k}{x^n} \qquad x \neq 0}$$

Inverse proportion graphs will never start at the origin (0, 0).



Example: y is inversely proportional to the square root of x.
When $x = 9$, $y = 2$.
Find an equation connecting x and y.
a) Find the value of y when $x = 16$
b) Find the value of x when $y = 9$

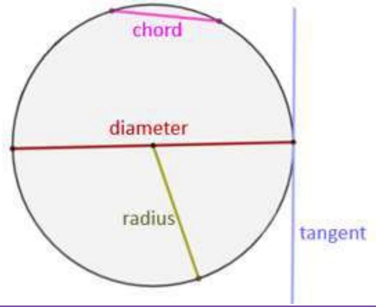
$y \propto \frac{1}{\sqrt{x}}$
$y = \frac{k}{\sqrt{x}}$
$2 = \frac{k}{9}$
$k = 2 \times 9 = 18$
$\boldsymbol{y = \frac{18}{\sqrt{x}}}$

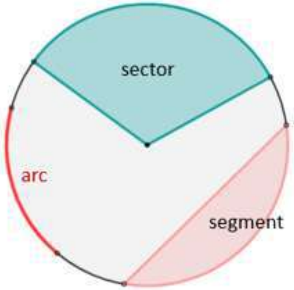
a) $y = \frac{18}{\sqrt{x}}$
$y = \frac{18}{\sqrt{16}} = \frac{18}{4} = 4.5$

b) $y = \frac{18}{\sqrt{x}}$
$9 = \frac{18}{\sqrt{x}}$
$\sqrt{x} = \frac{18}{9} = 2$
$x = 4$

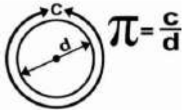
Year 9 Mathematics Knowledge Organiser – Unit 5: Multiplicative reasoning

Keywords





Radius	the distance from the centre of a circle to the edge.
Diameter	the total distance across the width of a circle through the centre.
Circumference	the total distance around the outside of a circle.
Arc	a part of the circumference of a circle.
Sector	the region of a circle enclosed by two radii and their intercepted arc.
Number π ('pi')	Pi is a mathematical constant, it equals the circumference of a circle divided by its diameter. **$\pi \approx 3.14$**



CIRCUMFERENCE OF A CIRCLE

$$\boxed{C = \pi d}$$

which means '*pi* × *diameter*'.

Example: Find circumference of the circle with diameter 10 cm.

$C = \pi \times 10 = 31.4$ cm

Example: Find circumference of the circle with radius 10 cm.

diameter = $10 \times 2 = 20$ cm *$C = \pi \times 20 = 62.8$ cm*

Example: Find radius of the circle with circumference 12.6 cm.

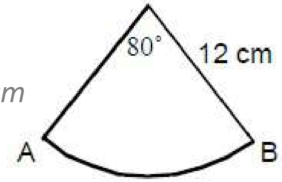
diameter = $\frac{C}{\pi} = \frac{12.6}{\pi} = 4$ cm *Radius = half of diameter = 2 cm*

LENGTH OF AN ARC

$$\boxed{\text{Arc length} = \frac{\text{angle}}{360} \times \pi \times d}$$

The arc length is **fraction** of the total circumference.

Example: Find the length of the arc AB.



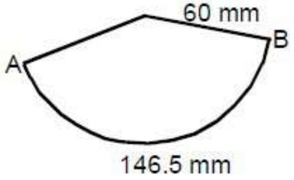
Radius = 12 cm
Diameter = 24 cm

Circumference of the whole circle is $C = \pi \times 24 = 24\pi = 75.4$ cm

Arc is $\frac{80}{360}$ of circumference.

Arc = $\frac{80}{360} \times \pi \times 24 = 16\pi = 16.8$ cm

Example: The length of the arc AB is 146.5 cm, what is the angle?



Radius = 60 mm
Diameter = 120 mm

Circumference of the whole circle is $C = \pi \times 120 = 377$ mm

Arc is $\frac{146.5}{377}$ of circumference.

Angle = $\frac{146.5}{377} \times 360° = 140°$

AREA OF A CIRCLE

$$\boxed{A = \pi r^2}$$

which means '*pi* × *radius squared*'.

Example: Find area of the circle with diameter 10 cm.

Radius = $10 \div 2 = 5$ cm *$A = \pi \times 5^2 = 78.5$ cm^2*

Example: Find circumference of the circle with radius 10 cm.

$A = \pi \times 10^2 = 314$ cm^2

Example: Find diameter of the circle with area 50.3 cm^2.

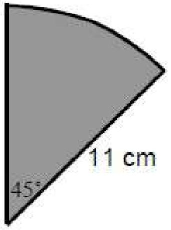
radius = $\sqrt{\frac{A}{\pi}} = \sqrt{\frac{50.3}{\pi}} = 4$ cm *diameter = 2 × radius = 8 cm*

AREA OF A SECTOR

$$\boxed{\text{Sector area} = \frac{\text{angle}}{360} \times \pi \times r^2}$$

The area of a sector is **fraction** of the total area.

Example: Find the area of the sector.



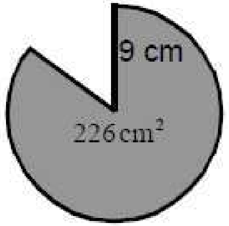
Radius = 11 cm

Area of the whole circle is $A = \pi \times 11^2 = 121\pi = 380.13$ cm^2

Area the of sector is $\frac{45}{360}$ of the area of the whole circle.

Area of sector = $\frac{45}{360} \times \pi \times 11^2 = \frac{1}{8} \times 380.13 = 47.52$ cm^2

Example: The area of the sector is 226cm^2, what is the angle?



Radius = 9 cm

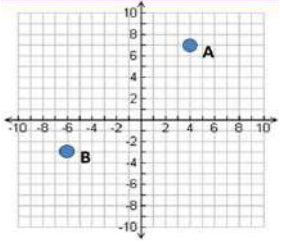
Area of the whole circle is $A = \pi \times 9^2 = 81\pi = 254.47$ cm^2

Area the of sector is $\frac{226}{254.47}$ of the area of the whole circle.

Angle = $\frac{226}{254.47} \times 360° = \frac{226}{254.47} \times 360° = 320°$

Year 9 Mathematics Knowledge Organiser – Unit 6: Non linear graphs

COORDINATES A set of values that describe an exact position of a point on a coordinate plane.
(x,y) the x-value or x-coordinate (horizontally) and y-value or y-coordinate (vertically).
Examples: point A (4, 7), point B (-6, -3)



QUADRATIC GRAPH

Quadratic expression is an expression where the highest power of the variable is **power 2**.
Standard form of the **quadratic EXPRESSION** is

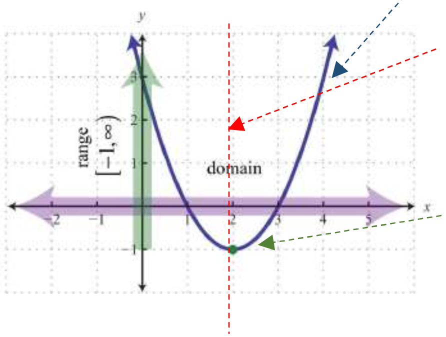
$$ax^2 + bx + c$$
where a, b, c are numbers, x is variable, $a \neq 0$

Standard form of the **quadratic EQUATION** is

$$ax^2 + bx + c = 0$$
where a, b, c are numbers, x is variable, $a \neq 0$

Standard form of the **quadratic FUNCTION** is

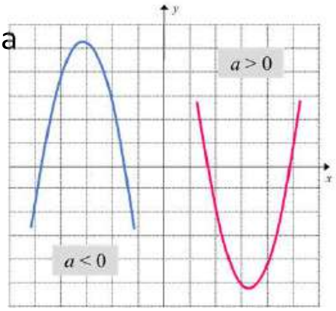
$$y = ax^2 + bx + c \quad \text{or}$$
$$f(x) = ax^2 + bx + c$$
where a, b, c are numbers, x, y are variables, $a \neq 0$



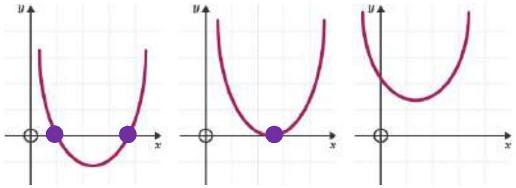
Parabola - The curve produced by a quadratic function.

Axis of symmetry - The line of symmetry of a parabola that divides a parabola into two equal halves that are reflections of each other about the line of symmetry.

Vertex – The lowest point (or the highest point, if the parabola is upside-down) of the parabola. This is the point, where the **parabola** changes direction.



Roots - Roots are also called x-intercepts, which means values of x that satisfy $ax^2 + bx + c = 0$. Parabola can have one, two or no roots.



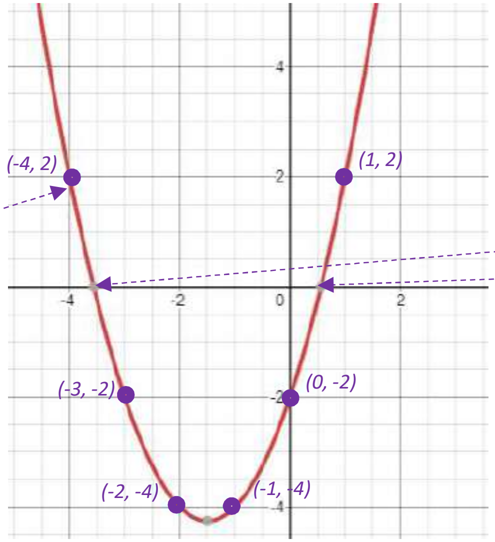
Example: Plot the graph of $y = x^2 + 3x - 2$

x	*-4*	*-3*	*-2*	*-1*	*0*	*1*	*2*
y	*2*	*-2*	*-4*	*-4*	*-2*	*2*	*8*

Substitute the values of x and find the values of y.
When $x = -4$:
$y = (-4)^2 + 3\times(-4) - 2 = 16 - 12 - 2 = 2$

Each set of x and y values gives coordinates of one point on the graph.
Points are (x, y)
(-4, 2)
(-3, -2) *(-2, -4)*
(-1, -4) *(0, -2)*
(1, 2) *(2, 8)*

After the points are plotted, they need to be connected with a smooth curve.



Example: Solve quadratic equation $x^2 + 3x - 2 = 0$ graphically.

1. *Plot the graph $y = x^2 + 3x - 2$.*
2. *Find **roots** (x–intercepts), the points on the graph, where $y = 0$.*
3. *Read the values of x on the x-axis.*
4. *Estimated solutions of the quadratic equation are: $x = -3.6$ and $x = 0.6$.*

(The exact solutions which we would find by solving quadratic equations algebraically are -3.562 and 0.562, so the estimation was very close.)

Year 9 Mathematics Knowledge Organiser – Unit 6: Non linear graphs

CUBIC GRAPH

Cubic expression is an expression where the highest power of the variable is **power 3**.
Standard form of the **cubic EXPRESSION** is

$$ax^3 + bx^2 + cx + d$$
where a, b, c are numbers, x is variable, $a \neq 0$

Standard form of the **cubic EQUATION** is

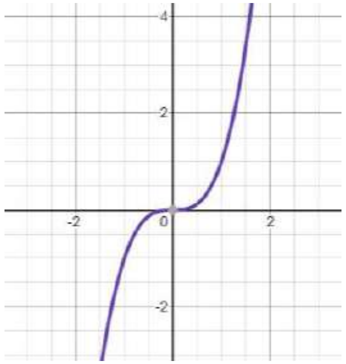
$$ax^3 + bx^2 + cx + d = 0$$
where a, b, c are numbers, x is variable, $a \neq 0$

Standard form of the **cubic FUNCTION** is

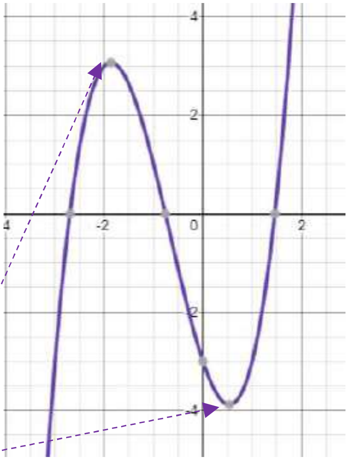
$$y = ax^3 + bx^2 + cx + d \quad \text{or}$$
$$f(x) = ax^3 + bx^2 + cx + d$$
where a, b, c are numbers, x, y are variables, $a \neq 0$

Examples of the graphs of the cubic functions:

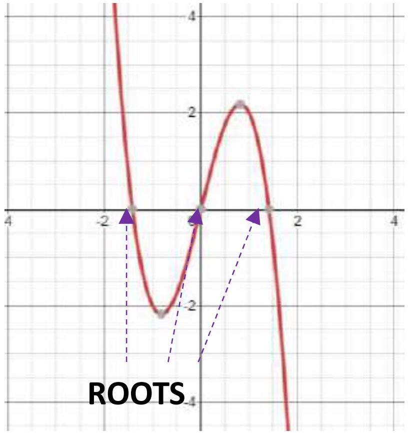
$y = x^3$



$y = x^3 + 2x^2 - 3x - 3$



$y = -2x^3 + 4x^2$



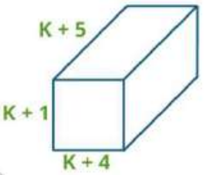
Keywords **TURNING POINT** on a graph is the point where the graph shifts from a positive gradient to a negative gradient, or vice versa. It is a **local maximum or minimum.**

Example: A cuboid has dimensions $k + 1$, $k + 5$ and $k + 4$ as shown.

a) *Write an expression representing the volume in m^3 in terms of k in m.*

b) *Plot a graph representing the volume in m^3 in terms of k in m.*

c) *Explain the features of the graph.*



a) Volume of the cuboid :

$$(k + 5)(k + 1)(k + 4) =$$
$$(k + 5)(k^2 + 4k + k + 4) =$$
$$(k + 5)(k^2 + 5k + 4) =$$
$$k^3 + 5k^2 + 4k + 5k^2 + 25k + 20 =$$
$$k^3 + 10k^2 + 29k + 20$$

b) Plotting the graph $y = x^3 + 10x^2 + 29x + 20$

x	-6	-5	-4	-3	-2	-1	0	1
y	-10	0	0	-4	-6	0	20	60

Substitute values of x into function and find the values of y.

When $x = -6$:

$y = (-6)^3 + 10\times(-6)^2 + 29\times(-6) + 20 =$

$= -216 + 360 - 174 + 20 = -10$

Each set of x and y values gives coordinates of one point on the graph.

Points are (x, y): (-6, -10), (-5, 0), (-4, 0), (-3, -4), (-2, -6), (-1, 0), (0, 20), (1, 60)

c) Possible comments on the graph:

- *x axis represents values k*
- *y axis represents the volume*
- *when $k = 0$, the volume of the cuboid is $20m^3$*
- *when $k = 1$, the volume of the cuboid is $60m^3$*
- *when $k = -5$ or -4 or -1, the volume of the cuboid is 0 because at least one dimension of the cuboid is equal to 0, for example if $k = -4$, width $(k + 4) = 0$*
- *volume of the cuboid cannot be negative, therefore possible values of k are the values which give the value of volume (y values) greater than 0:*

  $-5 < k < -4$ and $k > -1$

Example: Solve cubic equation $x^3 + 10x^2 + 29x + 20 = 0$ graphically.

1. *Plot the graph of $y = x^3 + 10x^2 + 29x + 20$.*
2. *Find **roots** (x – intercepts), the points on the graph, where $y = 0$.*
3. *Read the values of x on the x axis.*
4. *Solutions of the cubic equation are: $x = -5$, $x = -4$ and $x = -1$.*

Year 9 Mathematics Knowledge Organiser – Unit 6: Non linear graphs

RECIPROCAL is the multiplicative inverse of any non-zero number. Division by zero is not defined and zero has no reciprocal.

The reciprocal of a **number n** is $\frac{1}{n}$

Example: the reciprocal of 5 is $\frac{1}{5}$

To find the reciprocal of **a fraction**, flip the fraction

Example: the reciprocal of $\frac{3}{4}$ *is* $\frac{4}{3}$

Any number multiplied by its reciprocal is always equal to 1.

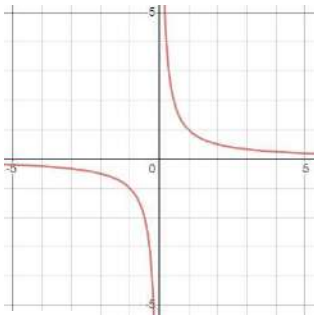
RECIPROCAL GRAPH

Reciprocal function of a function $f(x)$ is $\frac{1}{f(x)}$

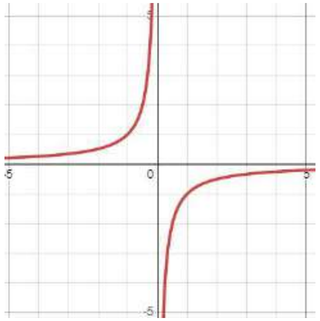
Example: The reciprocal of $f(x) = x + 1$ *is* $\frac{1}{x+1}$

Examples of the graphs of the reciprocal functions:

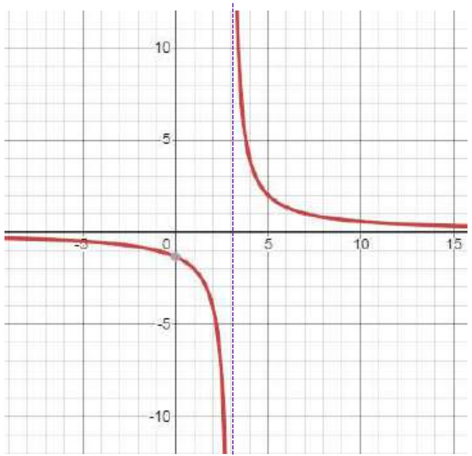
$y = \frac{1}{x}$



$y = -\frac{1}{x}$



$y = \frac{4}{x-3}$



Division by zero is not defined therefore the denominator cannot be equal to 0.

For function $y = \frac{4}{x-3}$ that means that $x - 3 \neq 0$

$x \neq 3$

ASYMPTOTE is a line that a curve approaches but does not touch or cross as it heads towards infinity.

Example: the graph of $y = \frac{4}{x-3}$ has an asymptote at $x = 3$.

Example: Plot the graph of $y = 3 - \frac{1}{x+1}$

x	-5	-4	-3	-2	-1	0	1	2	3
y	3.25	3.33	3.5	4	undefined	2	2.5	2.67	2.75

Each set of x and y values gives coordinates of one point on the graph.

Points are (x, y)

(-5, 3.25)

(-4, 3.33)

(-3, 3.5)

(-2, 4)

when x = -1, the value is undefined,

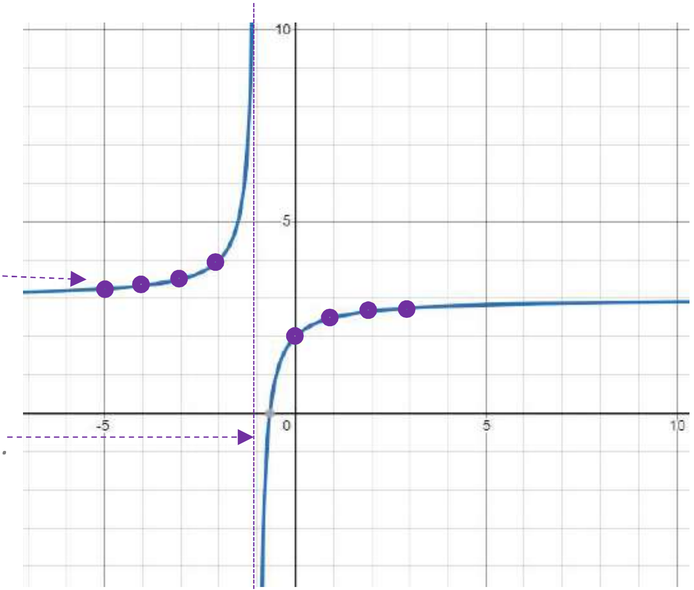
x + 1 cannot be equal 0. Therefore x = -1 is the asymptote of the graph.

(0, 2)

(1, 2.5)

(2, 2.67)

(3, 2.75)



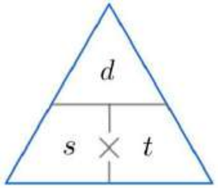
Year 9 Mathematics Knowledge Organiser – Unit 7: Accuracy and Measures

Keywords

SPEED	describes how fast something is moving.
DENSITY	is the compactness of a substance.
PRESSURE	is how much something is pushing on something else.
WEIGHT	is the force with which a body is attracted towards the earth's centre.
MASS	differs from the weight. Whereas, under certain conditions, a body can become weightless, **mass is constant**.

SPEED

is distance travelled per unit time.



$$\boxed{speed = \frac{distance}{time}}$$

$distance = speed \times time$

$time = \frac{distance}{speed}$

The main units of speed are:

- Kilometres per hour (**km/h**)
- Miles per hour (**mph**)
- Metres per second (**m/s**)

Example: I travel from Birmingham to London by train, a distance of 180 kilometres. It takes 2 hours, 15 minutes. What is the train's average speed?

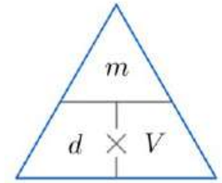
2 hours and 15 minutes = $2\frac{15}{60}$ hours = 2.25 hours

$speed = \frac{distance}{time} = \frac{180}{2.25} = 80$ km/h

Example: A tennis ball travels at 40 m/s. How far will it travel in 1½ seconds? Distance = 40 × 1.5 = 60 m

DENSITY

is mass per unit volume.



$$\boxed{density = \frac{mass}{volume}}$$

$mass = density \times volume$

$volume = \frac{mass}{density}$

The main units of density are:

- Kilograms per cubic metre (**kg/m³**)
- Gram per cubic centimetre (**g/cm³**)

Example: A gas has a mass of 20kg. It has a volume of 4m³. What is the density of the gas?

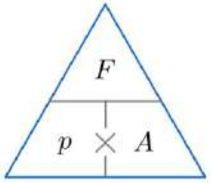
$density = \frac{mass}{volume} = \frac{20}{4} = 5\ kg/m^3$

Example: The density of air is 1.225kg/m³. A weather balloon holds 10kg of air.
To the nearest m³, what is the volume of the balloon?

Volume = 10 ÷ 1.225 = 8.16 m^3

PRESSURE

is the force per unit area.



$$\boxed{pressure = \frac{force}{area}}$$

$force = pressure \times area$

$area = \frac{force}{pressure}$

The main units of pressure are:

- Newtons per cm² (**N/cm²**)
- Newtons per m² (**N/m²**)

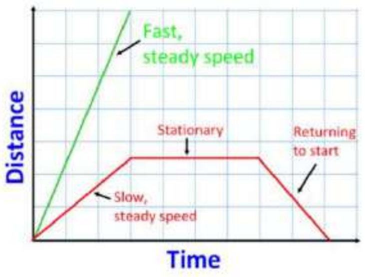
Example: A box creates a force of 100 Newtons on a table. Its pressure on the table is 15 N/m². Find the area of the base of the box.

Area = Force ÷ Pressure = 100 ÷ 15 = 6.67 m^2.

Example: A box is placed on the floor. The area of the box in contact with the floor is 2.4 m². The pressure exerted on the floor is 16 newtons/m². Work out the force exerted by the box on the floor.

Force = 16 × 2.4 = 38.4 N

DISTANCE - TIME GRAPH



- A **straight** diagonal line with positive gradient shows the object is moving at a **constant** speed.
- A **steeper** line shows the object is moving **faster.**
- A horizontal line shows that the object has **stopped moving.**
- Diagonal line going back towards the Time axis (negative gradient) shows the object is coming **closer to its starting position** (returning).
- **Gradient** of the line equals $\boxed{\textbf{speed} = \frac{distance}{time}}$.

Example: A train travels from Milton to Redville, stops for 30 minutes, then travels to Leek.

(a) How long did it take the train to travel from Milton to Redville? 2 hours

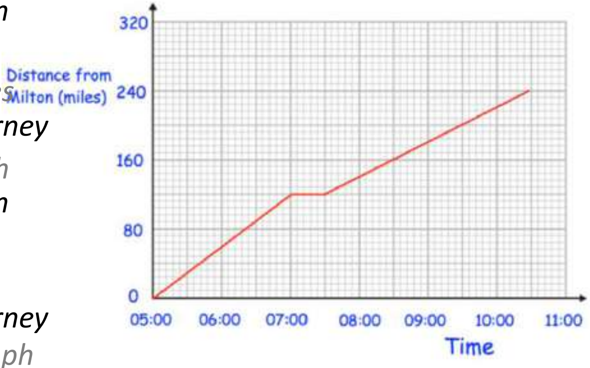
(b) How far is Redville from Milton? 120 miles

(c) Work out the speed of the train for the journey from Milton to Redville. 120 ÷ 2 = 60mph

(d) How long did it take the train to travel from Redville to Leek? 3 hours

(e) How far is Leek from Redville? 120 miles

(f) Work out the speed of the train for the journey from Redville to Leek. 120 ÷ 3 = 40mph



Year 9 Mathematics Knowledge Organiser – Unit 7: Accuracy and Measures

ERROR INTERVALS (BOUNDS)

All numbers rounded to the nearest whole number are half the whole number greater or smaller.

Example: What numbers can be rounded to 5?

all the numbers greater <u>or equal</u> 4.5: $4.5 \le x$ *all the numbers just below 5.5: $x < 5.5$*

All numbers rounded to the nearest tenth are half the tenth (*0.1÷2 = 0.05*) greater or smaller.

Example: What numbers can be rounded to 3.7?

all the numbers greater <u>or equal</u> 3.65: $3.65 \le x$ *all the numbers just below 3.75: $x < 3.75$*

The degree the number is rounded to (*tens, units, tenths*) is called the <u>degree of accuracy</u>.

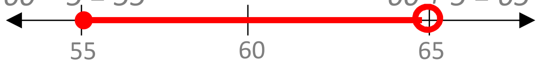
All numbers that can be rounded to a certain degree of accuracy are <u>up to half a degree of accuracy</u> (*half of ten, half of a unit or tenth*) greater or smaller.

Example: Number 60 is rounded to the nearest 10. Complete the error interval.

The degree of accuracy is 10.

10÷2= 5

60 – 5 = 55   *60 + 5 = 65*



$55 \le x < 65$

(all numbers greater or equal to 55 and less than 65)

Example: Number 1.4 is rounded to the nearest tenths. Complete the error interval.

The degree of accuracy is 0.1.

0.1÷2= 0.05

1.4 – 0.05 = 1.35   *1.4 + 0.05 = 1.45*

$1.35 \le x < 1.45$

(all numbers greater or equal to 1.35 and less than 1.45)

Keywords

INTERVAL means all the numbers between two given numbers.

INEQUALITY is a comparison of two values, showing if one is less than, greater than, or simply not equal to another value.

INEQUALITY SYMBOLS

> greater than	*Example: $x > 10$, means all numbers greater **and excluding** 10*
≥ greater than or equal to	*Example: $x \ge 10$, means all numbers greater **and including** 10*
< less than	*Example: $x < 10$, means all numbers up to **and excluding** 10*
≤ less than or equal to	*Example: $x \le 10$, means all numbers up to **and including** 10*
≠ not equal	*Example: $3 \neq 5$*

OPERATIONS WITH BOUNDS

Example:

A = 34 cm to the nearest cm.

B = 11.2 cm to one decimal place.

C = 200 cm to one significant figure.

Calculate:

1. *the lower bound for **A + B***
2. *the upper bound for **C - B***
3. *the upper bound for **A × C***
4. *the lower bound for **C ÷ B***

UB (A) = 34.5 CM
LB (A) = 33.5 CM

UB (B) = 11.25 CM
LB (B) = 11.15 CM

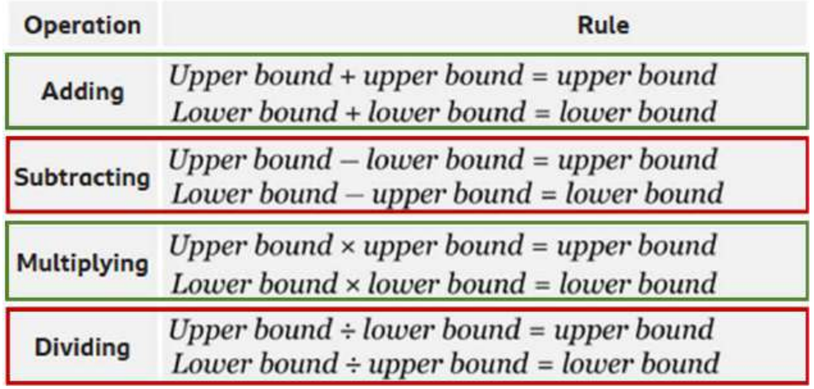
UB (C) = 250 CM
LC (C) = 150 CM

Lower bound for A + B =
LB(A) + LB(B) = 33.5 + 11.15 = 44.65 cm

Upper bound for C – B =
UB(C) - LB(B) = 250 - 11.15 = 238.85 cm

Upper bound for A × C =
UB(A) × UB(C) = 34.5 × 250 = 8625 cm^2

Lower bound for C ÷ B =
LB(C) ÷ UB(B) = 150 ÷ 11.25 = $13.\dot{3}$



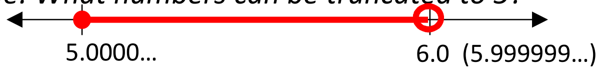
Operation	Rule
Adding	*Upper bound + upper bound = upper bound* *Lower bound + lower bound = lower bound*
Subtracting	*Upper bound – lower bound = upper bound* *Lower bound – upper bound = lower bound*
Multiplying	*Upper bound × upper bound = upper bound* *Lower bound × lower bound = lower bound*
Dividing	*Upper bound ÷ lower bound = upper bound* *Lower bound ÷ upper bound = lower bound*

ERROR INTERVALS (TRUNCATION)

Truncating means shortening a number at a particular place value and filling in any zeros to keep it the same size.
(It can be thought of as rounding down to a degree of accuracy.)

All numbers truncated to certain place values (*ones*, *10s*, *tenths..*) can be <u>up to the whole place value</u> greater.

Example: What numbers can be truncated to 5?

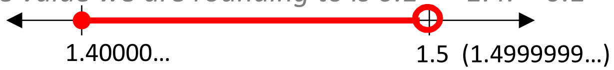


$5 \le x < 6$

(all numbers greater or equal to 5 and less than 6)

Example: Number 1.4 is truncated to the nearest tenths. Complete the error interval.

The place value we are rounding to is 0.1   *1.4. + 0.1 = 1.5*



$1.4 \le x < 1.5$

(all numbers greater or equal to 1.4 and less than 1.5)

Year 9 Mathematics Knowledge Organiser – Unit 8: Graphical solutions

Keywords

SIMULTANEOUS EQUATIONS Simultaneous equations are two or more equations with two or more unknown variables.
Simultaneous equations can be solved if the number of unknown variables is equal to or less than the number of equations.

SIMULTANEOUS EQUATIONS

Solution by elimination:

Example 1: Solve $6x + y = 15$ ①
$4x + y = 11$ ②

Equations are labelled to make the steps clearer.

Step 1: Eliminate the variable with the same co-efficient (SUBTRACT equations)

①-②
$$\begin{aligned}6x + y &= 15\\ \underline{4x + y} &= \underline{11}\\ 2x &= 4\\ \mathbf{x} &= \mathbf{2}\end{aligned}$$

Variable y has the same coefficient = 1, it can be eliminated by subtracting equation ② from equation ①
.

Step 2: To find y, substitute x = 2 into one of the original equations

①
$$\begin{aligned}(6\times2) + y &= 15\\ 12 + y &= 15 \quad /(-20)\\ \mathbf{y} &= \mathbf{3}\end{aligned}$$

Step 3: Check your answers by substitution

Example 2: Solve $4x + 3y = 32$ ①
$5x – 3y = 13$ ②

Step 1: Eliminate the variable with the same co-efficient (ADDING equations)

①+②
$$\begin{aligned}4x + 3y &= 32\\ \underline{5x - 3y} &= \underline{13}\\ 9x &= 45 \quad /(\div 9)\\ \mathbf{x} &= \mathbf{5}\end{aligned}$$

Variable y has the same coefficient = 3, it can be eliminated by adding equations.

Step 2: To find y, substitute x = 5 into one of the original equations

①
$$\begin{aligned}(4\times5) + 3y &= 32\\ 20 + 3y &= 32 \quad /(-20)\\ 3y &= 12 \quad /(\div 3)\\ \mathbf{y} &= \mathbf{4}\end{aligned}$$

Step 3: Check your answers by substitution

Example 3: Solve $3x + 4y = 7$ ①
$5x - 2y = 16$ ②

Step 1: When neither of the coefficients are the same we multiply one or both equations to make them the same…

② $\times 2$ $10x - 4y = 32$

If the second equation will be multiplied by 2, the coefficient of y will be the same and the y variable can be eliminated.

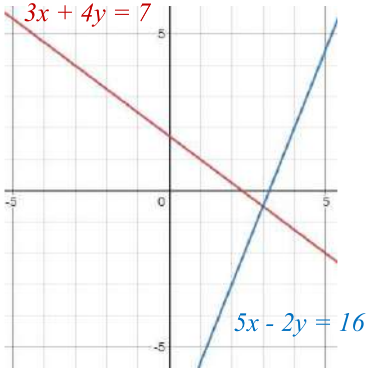
Step 2: Elimination

①+②
$$\begin{aligned}3x + 4y &= 7 \quad ①\\ \underline{10x - 4y} &= \underline{32} \quad ②\\ 13x &= 39 \quad /(\div 13)\\ \mathbf{x} &= \mathbf{3}\end{aligned}$$

Step 3: Find y

①
$$\begin{aligned}3\times3 + 4y &= 7\\ 9 + 4y &= 7 \quad /(-9)\\ 4y &= -2 \quad /(\div 4)\\ \mathbf{y} &= \mathbf{-1/2}\end{aligned}$$

Step 4: Check your answers by substitution



Solution by substitution

Example: Solve $2x + y = 3$ ①
$5x + 2y = 8$ ②

Step 1: Isolate one variable from one equation

① $y = 3 – 2x$

Step 2: Substitute into other equation

②
$$\begin{aligned}5x + 2(3 - 2x) &= 8\\ 5x + 6 – 4x &= 8\\ x + 6 &= 8 \quad /(-6)\\ \mathbf{x} &= \mathbf{2}\end{aligned}$$

Step 3: Substitute x = 2 into one of the original equations

①
$$\begin{aligned}2\times2 + y &= 3\\ 4 + y &= 3 \quad /(-4)\\ \mathbf{y} &= \mathbf{-1}\end{aligned}$$

Step 3: Check your answers by substitution

Solve $x^2 + y^2 = 29$ ①
$y – x = 3$ ②

Step 1: Eliminate one variable:

② $y = 3 + x$

Step 2: Substitute and rearrange

①
$$\begin{aligned}x^2 + (3+x)^2 &= 29\\ x^2 + 9 + 6x + x^2 &= 29\\ 2x^2 + 6x – 20 &= 0 \quad /(\div 2)\\ x^2 + 3x – 10 &= 0\end{aligned}$$

Step 3: Solve quadratic

① $(x + 5)(x - 2) = 0$

$\mathbf{x = -5 \text{ or } x = 2}$

Step 4: Substitute

② $x = -5 \Rightarrow \quad y = 3 - 5 = -2$
$x = 2 \Rightarrow \quad y = 3+2= 5$

SOLUTION ***(-5, -2) and (2, 5)***

Year 9 Mathematics Knowledge Organiser – Unit 8: Graphical solutions

Keywords

EQUATION OF THE STRAIGHT LINE → $y = mx + c$ → $ax + by = c$

$y = mx + c$ — m is the gradient of a line, that is the steepness of the line, c is the y-axis intercept, that is the value of y when x = 0.

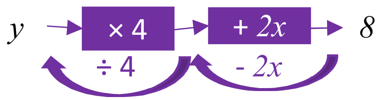
$ax + by = c$

REARRANGING $ax + by = c$ EQUATION INTO $y = mx + c$

Rearrange the equation to make y the subject.

1. Find what operations are performed on y.
2. Use inverse operations (you can always imagine an equation as a function machine to help you understand what is happening with a variable and how to undo it).

Example: Rearrange $2x + 4y = 8$



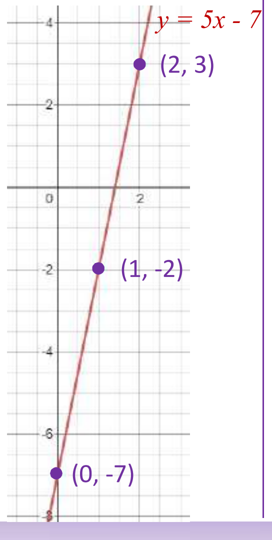
$$2x + 4y = 8$$
$$-2x \quad -2x$$
$$4y = 8 - 2x$$
$$\div 4 \quad \div 4$$
$$y = 2 - \frac{1}{2}x$$

PLOTTING GRAPH OF THE STRAIGHT LINE

1) Choose three values for x, for example $x=0$, $x=1$, $x=2$.
2) Find y values substituting x values into the equation.
3) Write down coordinates (x,y).
4) Plot the (x,y) points.
5) Draw and label the line.

Example: Plot $y = 5x - 7$

x	y	(x,y)
0	$y = 5\times0 - 7 = -7$	(0, -7)
1	$y = 5\times1 - 7 = -2$	(1, -2)
2	$y = 5\times2 - 7 = 3$	(2, 3)



FINDING THE EQUATION OF THE LINE BETWEEN TWO POINTS

Gradient of the line $\boxed{m = \frac{\text{change in } y}{\text{change in } x}}$

Example: What is an equation for the line that passes through the points (1,3) and (3, 7)?

1. Find the gradient of the equation.

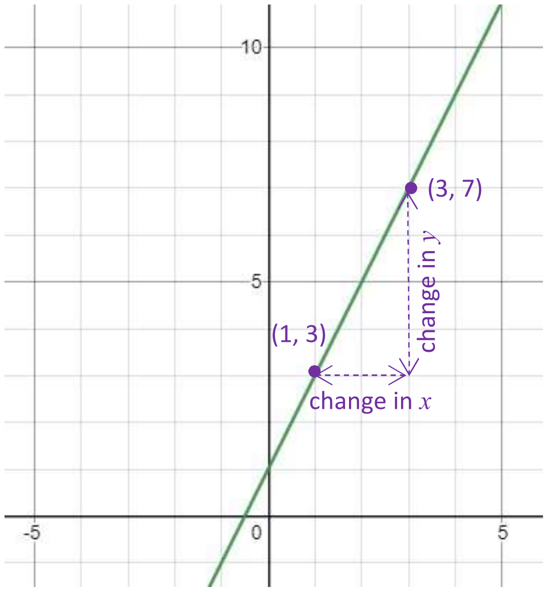
   $m = \frac{\text{change in } y}{\text{change in } x} = \frac{7-3}{3-1} = \frac{4}{2} = 2$
2. Substitute gradient in $y = mx + c$.

   $y = 2x + c$
3. Substitute coordinates of one point into the equation and find y intercept.

   $(3,7)$ $\quad 7 = 2\times3 + c$

   $7 = 6 + c$

   $c = 1$
4. Write the equation. $\quad y = 2x + 1$



PARALLEL AND PERPENDICULAR LINES

Two lines are parallel if their gradients are **equal**.
Two lines are perpendicular if the gradient of one is the **negative reciprocal** of the other.

Example: Find the equation of a line parallel to $y = 2x + 3$, running through points (4, 3).

1. Gradient of the parallel line is the same, but the y-intercept is unknown.

   $y = 2x + c$
2. Substitute the coordinates of the point into the equation and find the y-*intercept.*

   $(4,3)$ $\quad 3 = 2\times4 + c$

   $3 = 8 + c$

   $c = -5$
3. Write the equation. $\quad y = 2x -5$

Example: Find the equation of a line perpendicular to $y = 3x + 2$, running through point (9, 10).

1. Gradient of the perpendicular line is the negative reciprocal of the gradient of the original line, the y-intercept is unknown.

   the negative reciprocal of 3 is $-\frac{1}{3}$

   $y = -\frac{1}{3}x + c$
2. Substitute the coordinates of the point into the equation and find the y-*intercept.*

   $(9,10)$ $\quad 10 = -\frac{1}{3}\times9 + c$

   $10 = -3 + c$

   $c = 13$
3. Write the equation. $\quad y =-\frac{1}{3}x + 13$

Year 9 Mathematics Knowledge Organiser – Unit 8: Graphical solutions

SOLVING SIMULTANEOUS EQUATIONS GRAPHICALLY

Plot the graphs of the equations.
Identify the crossing point(s).

Example: Solve simultaneous equations graphically

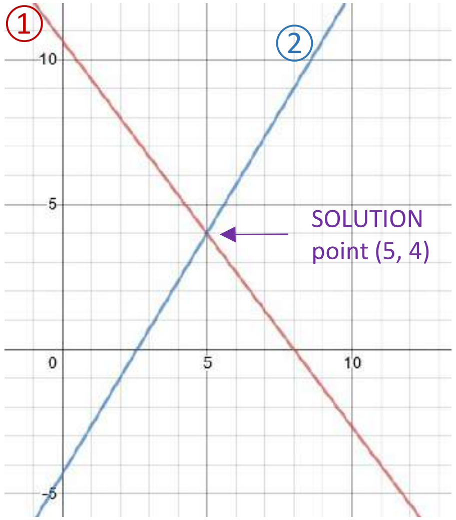
$4x + 3y = 32$ ①
$5x - 3y = 13$ ②

① $y = \frac{32-}{3}$

x	0	2	5
y	10.7	8	4

② $y = \frac{13-}{-3}$

x	0	2	5
y	-4.3	-1	4

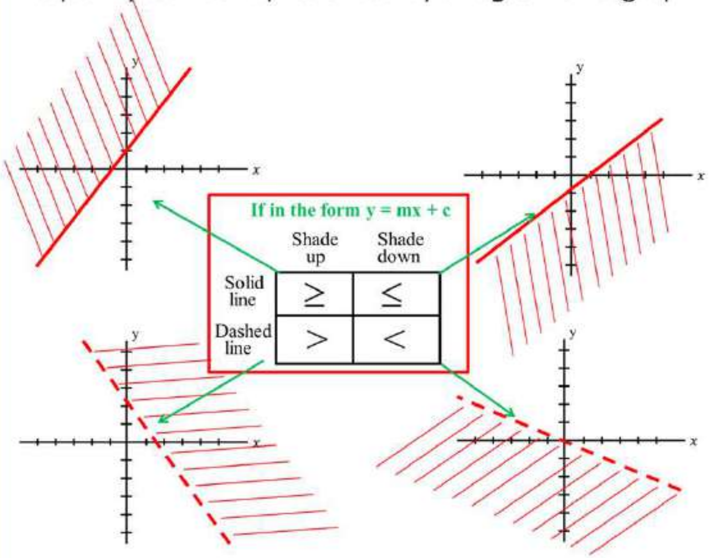


SOLUTION point (5, 4)

Solution is point (5, 4)
$x = 5$ $y = 4$

SOLVING LINEAR INEQUALITIES IN TWO VARIABLES

When an inequality involves two variables, the inequality can be represented by a ***region*** on a graph.



If in the form y = mx + c

	Shade up	Shade down
Solid line	≥	≤
Dashed line	>	<

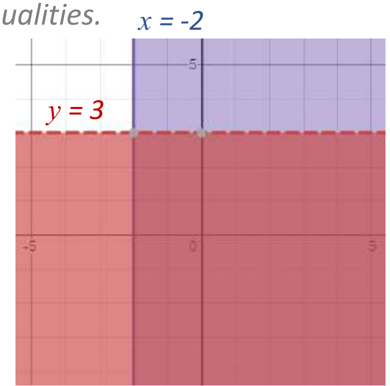
Example: Show the region that satisfies both inequalities $x \geq -2$ and $y < 3$.

1. *Plot graphs of $x = 2$ and $y = 3$, making sure you are using dashed or solid lines correctly.*
2. *Shade the regions that satisfy inequalities.*

   *The region to the right from the **solid** line $x = -2$ satisfies inequality $x \geq -2$.*

   *Region bellow **dashed** line $y = 3$ satisfies inequality $y < 3$.*

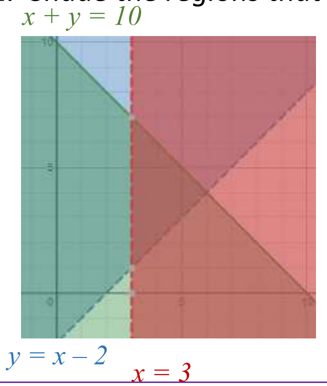
   Cross section of these two regions satisfies both inequalities.



Example: Find the region that satisfies the inequalities

$y > x - 2$ $x + y \leq 10$ $x > 3$

1. *Plot graphs of $y = x - 2$, $x + y = 10$ and $x = 3$, making sure you are using dashed or solid lines correctly.*
2. *Shade the regions that satisfy inequalities.*



*The region to the **right** from the dashed line $x = 3$ satisfies inequality $x > 3$.*

*Region **bellow** solid line $x + y = 10$ satisfies inequality $x + y \leq 10$.*

*The region **above** the dashed line $y = x - 2$ satisfies inequality $y > x - 2$.*

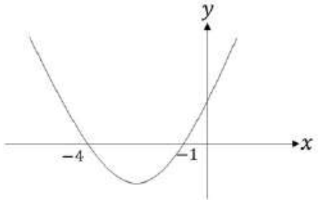
Cross section of these regions satisfies all three inequalities.

SOLVING SIMULTANEOUS INEQUALITIES WITH QUADRATIC GRAPHS

Example: Shade region that satisfies the inequality $y \geq x^2 + 5x + 4$

1. Find roots by factorising quadratic.

$x^2 + 5x + 4 = 0$
$(x + 4)(x + 1) = 0$
$x = -4$ and $x = -1$



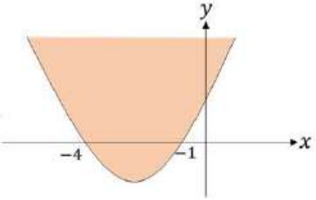
2. Sketch the graph of the quadratic using the full line.

Parabola crosses

- *The x-axis at $x = -4$ and $x = -1$*
- *The y axis at $y = 4$.*

3. Find which region satisfies the given inequality $y \geq x^2 + 5x + 4$, and shade it.

Shade the region above the graph.



Example: Shade region that satisfies both inequalities $y \geq x^2 + 5x + 4$ and $y < x + 4$.

1. *The region for $y \geq x^2 + 5x + 4$ is shaded in example above.*
2. *Sketch graph of $y = x + 4$ using dashed line.*
3. *Shade region below $y = x + 4$*
4. *Cross section of these regions satisfies both inequalities*