Jan 08 C3

1) $f(x) = x^3 + 4 \qquad g(x) = 2x - 5$

i) $fg(x) = (2x-5)^3 + 4$

$fg(1) = (2-5)^3 + 4 = -27 + 4$

$= -23$

ii) $f^{-1}(x) = \sqrt[3]{x-4}$

$f^{-1}(12) = \sqrt[3]{8} = 2$

2i) $x_1 = 3$

$x_2 = 2.864$

$x_3 = 2.878$

$x_4 = 2.877$

$x_5 = 2.87680487$

$x_6 = 2.87678649$

$x_7 = 2.87678779$

$x = 2.877$ (3dp)

( ) $x = \sqrt[3]{31 - \frac{5}{2}x}$

$x^3 = 31 - \frac{5}{2}x$

$2x^3 = 62 - 5x$

$2x^3 + 5x - 62 = 0$

3a) $\sec\left(\frac{1}{2}\alpha\right) = 4$

$\cos\left(\frac{1}{2}\alpha\right) = \frac{1}{4}$

$\frac{1}{2}\alpha = 75.5, \quad 284.5$

$\underline{\alpha = 151.0}$

b) $\tan\beta = 7\cot\beta$

$\tan\beta = \frac{7}{\tan\beta}$

$\tan^2\beta = 7$

$\tan\beta = \pm\sqrt{7}$

$\beta = \underline{69.3^\circ}, \cancel{-69.3} \ \underline{110.7^\circ}$

4) $V = (h^6 + 16)^{1/2} - 4$

i) $\frac{dV}{dh} = 3h^5(h^6 + 16)^{-1/2}$

At $h = 2$

$\frac{dV}{dh} = 96(64+16)^{-1/2} = \frac{96}{\sqrt{80}} = 10.7$

ii) $\frac{dV}{dt} = 8$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

at $h = 2$ $\frac{dh}{dV} = \frac{\sqrt{80}}{96}$

$\frac{dh}{dt} = 8 \times \frac{\sqrt{80}}{96} = 0.745$

$= 0.75 \ (2sf)$

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5a) $\int (3x+7)^9 \, dx$

$= \frac{(3x+7)^{10}}{30} + k$

$\frac{1}{4x}$

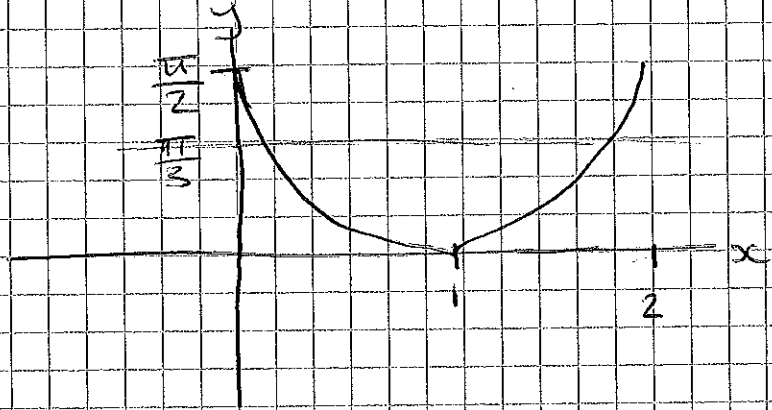
b) $\pi \int_3^6 \left(\frac{1}{2\sqrt{x}}\right)^2 dx = \pi \int_3^6 \frac{x^{-1}}{4} \, dx$

$= \frac{\pi}{4}\left[\ln x\right]_3^6 = \frac{\pi}{4}(\ln 6 - \ln 3)$

$= \frac{\pi \ln 2}{4}$

i) translation 1 unit in negative x direction and reflection in x axis.

ii)



iii) $-\sin^{-1}(x-1) = \frac{1}{3}\pi$

$\sin^{-1}(x-1) = -\frac{1}{3}\pi$

$(x-1) = \sin\left(-\frac{1}{3}\pi\right)$

$x = -\frac{\sqrt{3}}{2} + 1$

or $x = 2 - \left(-\frac{\sqrt{3}}{2} + 1\right)$

$= \cancel{\frac{3-\sqrt{3}}{2}} \quad 1 \pm \frac{\sqrt{3}}{2}$

7) $y = \dfrac{xe^{2x}}{x+k}$

i) $\dfrac{d(xe^{2x})}{dx} = e^{2x} + 2xe^{2x} = e^{2x}(1+2x)$

$$\frac{dy}{dx} = \frac{e^{2x}(1+2x)(x+k) - xe^{2x}}{(x+k)^2}$$

$$= \frac{e^{2x}\left((1+2x)(x+k) - x\right)}{(x+k)^2}$$

$$= \frac{e^{2x}\left(\cancel{x} + 2x^2 + 2xk + k - \cancel{x}\right)}{(x+k)^2}$$

$$= \frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$$

ii) St pt at $\dfrac{dy}{dx} = 0$

$$\frac{e^{2x}(2x^2 + 2kx + k)}{\cancel{(x+k)^2}} = 0$$

$\cancel{e^{2x} = 0}$ or $2x^2 + 2kx + k = 0$

when $b^2 - 4ac = 0$

$4k^2 - 8k = 0$

$4k(k-2) = 0$

$\cancel{k = 0}$ $\quad$ $k = 2$.

$2x^2 + 4x + 2 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$ $\quad$ $x = -1$ $\quad$ $y = -e^{-2}$

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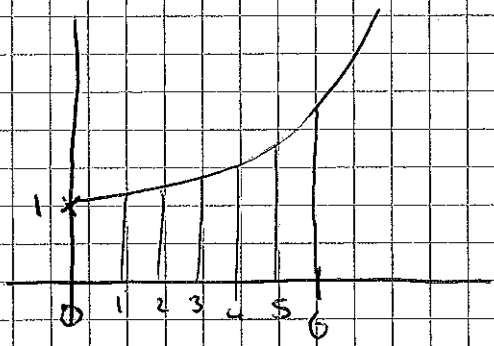
8) $\int_0^6 2^{x} \, dx$



6 strips

$y_0 = \cancel{2}\ 1$

$y_1 = 2$

$y_2 = 4$

$y_3 = 8$

$y_4 = 16$

$y_5 = 32$

$y_6 = 64$

$$\int = \frac{1}{3} \times 1 \left(1 + 64 + 4(2 + 8 + 32) + 2(4 + 16) \right)$$

$$= \cancel{87.6} \quad \underline{\underline{91}}$$

ii) $\cancel{e}\ 2^{x} = e^{kx}$

$e^{k} = 2$

$\ln e^{k} = \ln 2$

$k = \ln 2$

$2^{x} = e^{\ln 2x} = e^{x \ln 2} = \cancel{e^{\ln 2^{x}}}$

$$\int_0^6 e^{x \ln 2} \, dx$$

$$= \left[\cancel{\ln 2}\ \frac{e^{x \times \ln 2}}{\ln 2} \right]_0^6 = \cancel{\ln 2} \left(\frac{e^{\ln 2 \times 6} - e^{0}}{\ln 2} \right)$$

$$= \cancel{\ln 2}\ \frac{(63)}{\ln 2}$$

$\cancel{= 90.87}$

$$\int_0^6 2^x = \frac{63}{\ln 2}$$

~~90~~

$$91 \approxeq \frac{63}{\ln 2}$$

$$\ln 2 \approxeq \frac{63}{91}$$

$$\ln 2 \approx \frac{9}{13}$$ as required

9i) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$\cos(\theta+60) = \cos\theta\cos 60 - \sin\theta\sin 60$$

$$= \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$

~~4cos~~

$$\cos(\theta+30) = \cos\theta\cos 30 - \sin\theta\sin 30$$

$$= \frac{\sqrt{3}}{2}\cos\theta - \cancel{\theta}\frac{1}{2}\sin\theta$$

$$4\cos(\theta+60)\cos(\theta+30)$$

$$= 4\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$$

$$= 4\left(\frac{\sqrt{3}}{4}\cos^2\theta - \frac{1}{4}\sin\theta\cos\theta - \frac{3}{4}\sin\theta\cos\theta + \frac{\sqrt{3}}{4}\sin^2\theta\right)$$

$$= \sqrt{3}\,(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) \boxed{-4\sin\theta\cos\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\boxed{2\sin 2\theta} = 4\sin\theta\cos\theta$$

$$= \sqrt{3} - 2\sin 2\theta$$

~~0.39837~~ Jan 08.

when $\theta = 22.5$

ii) $4\cos 82.5 \cos 52.5 = \sqrt{3} - 2\sin 45$

$= \sqrt{3} - \dfrac{2}{\sqrt{2}}$

iii) $\sqrt{3} - 2\sin 2\theta = 1$

$-2\sin 2\theta = 1 - \sqrt{3}$

$\sin 2\theta = \dfrac{-1+\sqrt{3}}{2}$

$2\theta = 21.5^\circ, 158.5$

$\theta = \underline{10.7^\circ}, \underline{79.3^\circ}$

iv) $\sqrt{3} - 2\sin 2\theta = k$

$-2\sin 2\theta = k - \sqrt{3}$

$\sin 2\theta = \dfrac{-k+\sqrt{3}}{2}$

no solns if a) $\dfrac{-k+\sqrt{3}}{2} > 1$

$-k > 2 - \sqrt{3}$

$k < \underline{\sqrt{3} - 2}$

or b) $\dfrac{-k+\sqrt{3}}{2} < -1$

$-k + \sqrt{3} < -2$

$-k < -2 - \sqrt{3}$

$k > \underline{2 + \sqrt{3}}$