Mechanics A2 Maths 2021

(1.) $a = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $t = 0$ $u = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

If $t = 2$ $\quad v = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

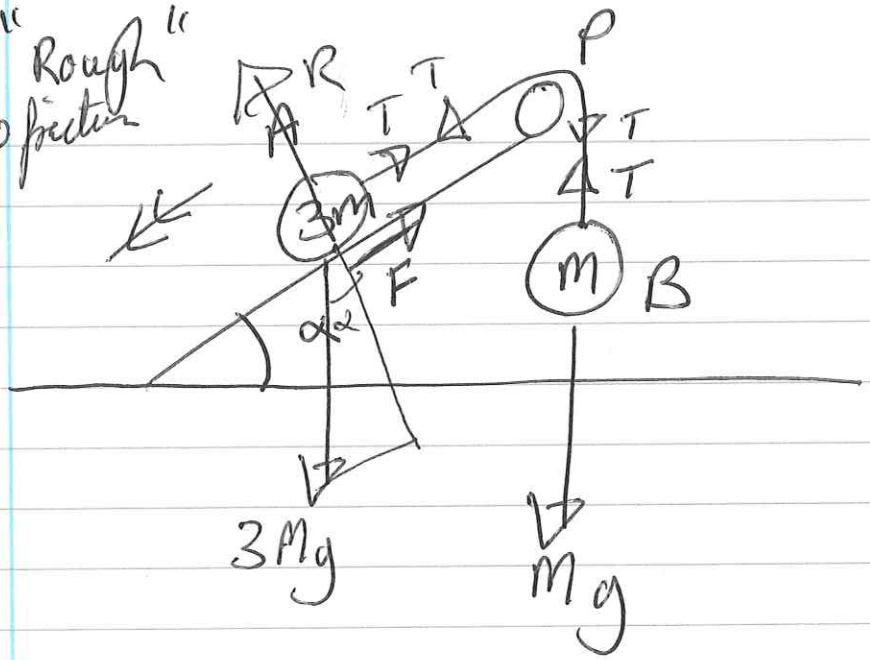
$s = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ at time zero

(b) When $t = 3$ $\quad s = ut + \frac{1}{2}at^2$

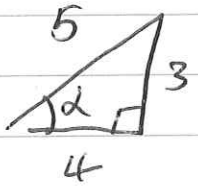
so $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ -3 \end{pmatrix} 3^2$

$= \begin{pmatrix} 22 \\ -25/2 \end{pmatrix}$ $\quad 22\underset{\sim}{i} - 12\frac{1}{2}\underset{\sim}{j}$

(2) "Rough" so friction



$\tan\alpha = 3/4$



$\mu = 1/6$

$\sin\alpha = 3/5$
$\cos\alpha = 4/5$

$\underline{F = ma}$

(a) Consider A ⇄ to plane

$3ma = 3mg\sin\alpha - F - T$

(b) Consider B

$ma = T - mg$ so $T = ma + mg$

A ⊥ to plane

$3mg\cos\alpha = R$

$F = \mu R = 1/6\ 3mg\cos\alpha$
$= 1/2\ mg\cos\alpha$

so $3ma = 3mg \times 3/5 - 1/2 \times 4/5 \times mg - T$

$3ma = \frac{9mg}{5} - \frac{2}{5}mg - ma - mg$

$3a = \frac{9}{5}g - \frac{2}{5}g - a - g$

$4a = \frac{2}{5}g \qquad a = \frac{g}{10}$

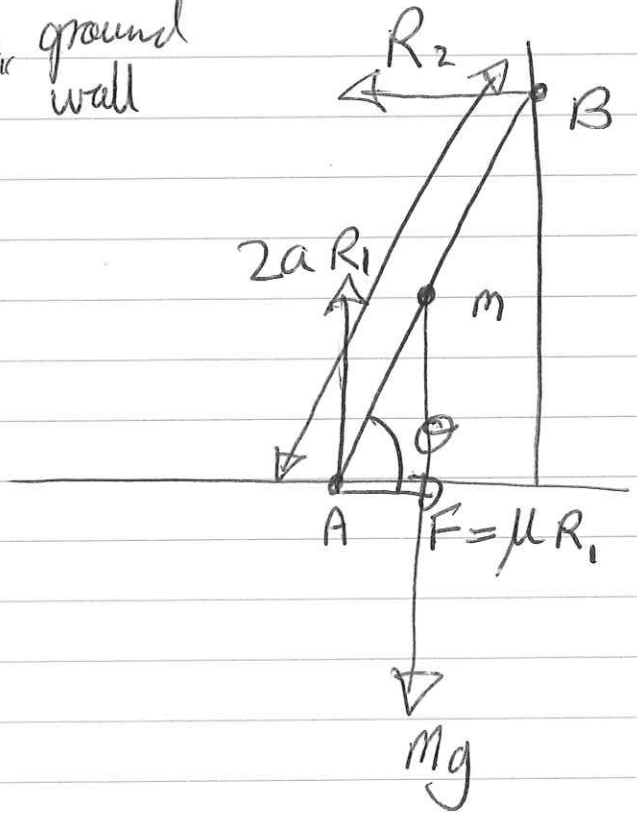
(c)

Constant acceleration so constant gradient of $\frac{g}{10}$.

Initially at rest, so starts at the origin

(d) As there are different amounts of string either side of the pulley, there will similarly be different masses either side of the pulley.

(3) "Rough" ground
"Smooth" wall



$R\uparrow \qquad R_1 = Mg$

$B\rightarrow \quad aMg\cos\theta + \mu R\, 2a\sin\theta \geq Mg\, 2a\cos\theta$

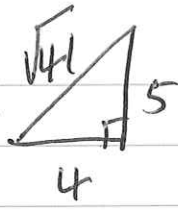
$\mu R\, 2a\sin\theta \geq Mg\, a\cos\theta$

$\mu Mg\, 2a\sin\theta \geq Mg\, a\cos\theta$

$\mu \geq \frac{1}{2}\cot\theta$

(3)

$\tan\theta = 5/4$



$\sin\theta = \frac{5}{\sqrt{41}}$

$\cos\theta = \frac{4}{\sqrt{41}}$

Now $\xrightarrow{\quad}$ KMy so friction working the other way $\xleftarrow{\quad}$ $\frac{1}{2}$My

(A) $a\cos\theta\, My = 2a\sin\theta\, R_2$

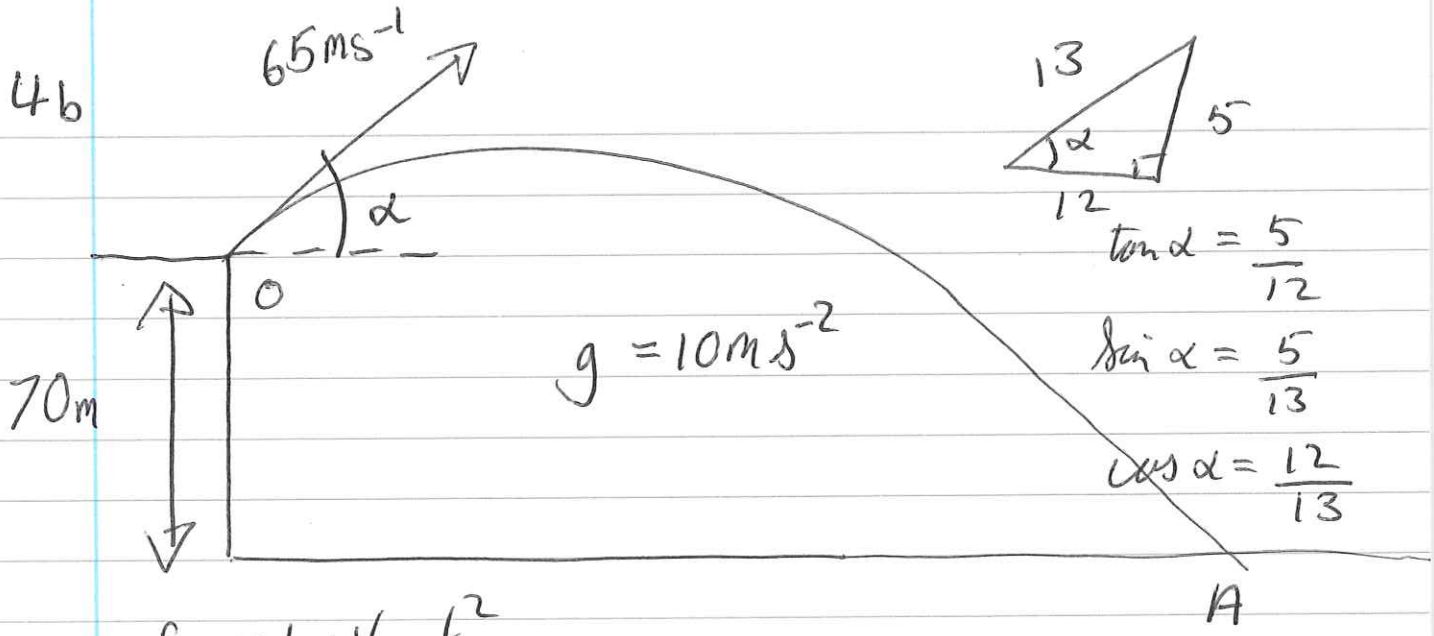
$$R_2 = \frac{a\cos\theta My}{2a\sin\theta} = \frac{1}{2} \times \frac{4}{\sqrt{41}} \times \frac{\sqrt{41}}{5} My$$

$$= \frac{2}{5} My$$

$$KMy = \frac{2}{5} My + \frac{1}{2} My$$

$$K = 9/10$$

4b



$S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$v = u + at$

Consider vertical motion

$S = -70 \quad g = -10 \quad u = 65\sin\alpha$

$S = ut + \frac{1}{2}at^2$

$-70 = (65\sin\alpha)t + \frac{1}{2} \times -10 \times t^2$

$5t^2 - 65 \times \frac{5}{13} t - 70 = 0$

$5t^2 - 25t - 70 = 0$

$t^2 - 5t - 14 = 0$

$(t-7)(t+2) = 0$

$t = 7$ or ~~$t = -2$~~

$t = 7$

4c The shape of the object has been ignored.

(5a) $\underset{\sim}{v} = 3t^{1/2}\,\underset{\sim}{i} - 2t\,\underset{\sim}{j}$

$\underset{\sim}{a} = \frac{3}{2}t^{-1/2}\,\underset{\sim}{i} - 2\,\underset{\sim}{j}$

(b) $\begin{pmatrix} k \\ -k \end{pmatrix}$ is same direction as $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

so $\begin{pmatrix} k \\ -k \end{pmatrix} = \begin{pmatrix} 3t^{1/2} \\ -2t \end{pmatrix} \Rightarrow \begin{matrix} k = 3t^{1/2} \\ k = 2t \end{matrix}$

so $3t^{1/2} = 2t \qquad 0 = 2t - 3t^{1/2}$

$0 = t^{1/2}(2t^{1/2} - 3)$

$t^{1/2} = 0$ not possible

$2t^{1/2} - 3 = 0 \Rightarrow t^{1/2} = \frac{3}{2} \quad t = \frac{9}{4}$

(c) $\underset{\sim}{r}$ is position vector $t = 1$ $\underset{\sim}{r} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\underset{\sim}{r} = (2t^{3/2} + c)\,\underset{\sim}{i} - (t^2 + k)\,\underset{\sim}{j}$

$t = 1 \quad (2 + c)\,\underset{\sim}{i} - (1 + k)\,\underset{\sim}{j}$

so $2 + c = 0 \Rightarrow c = -2$

$-(1 + k) = -1 \Rightarrow k = 0$

so $\underset{\sim}{r} = (2t^{3/2} - 2)\,\underset{\sim}{i} - t^2\,\underset{\sim}{j}$

5d Speed $= 10$

$\underset{\sim}{v} = 3t^{1/2}\underset{\sim}{i} - 2t\underset{\sim}{j}$

Speed $= |v|$

$$\sqrt{(3t^{1/2})^2 + (2t)^2}$$

$$\sqrt{9t + 4t^2} = 10$$

$$100 = 9t + 4t^2$$

$$4t^2 + 9t - 100 = 0$$

$t = 4$ or ~~$-25/4$~~

$$\underset{\sim}{r} = \begin{pmatrix} 2 \times 4^{3/2} \\ -4^2 \end{pmatrix} - 2 = \begin{pmatrix} 14 \\ -4^2 \end{pmatrix}$$

$$\sqrt{14^2 + 16^2} = 2\sqrt{113}$$