Sale
Grammar
School

Mathematics – Year 7

KNOWLEDGE ORGANISER

To support your revision for the End of Year Assessment

To effectively revise for a Maths assessment, you must finish, check, and correct many Maths questions.

- This document is to support your revision but remember the key with Maths revision is to **finish lots of questions**. Techniques like rewriting revision notes or copying from a revision guide, colour coding, and making posters can be enjoyable, but generally, they aren't the most effective use of revision time.
- Use your progress books and finish outstanding chapters or redo questions you struggled with.
- Www.corbettmaths.com has lots of helpful videos and worksheets you can use as well.
- Use notes, and work through examples and questions in your book.
- Don't use your calculator unless the question specifically asks for it, you need to practise non-calculator skills as well. But checking answers with a calculator is very useful.
- If your struggle with anything, come to the support session during Tuesday lunchtime, in M51 or ask your teacher.
- The full-colour version can be found on www.smlmaths.com.

Year 7 Mathematics Knowledge Organiser – Unit 1: Representing data

Keywords

MEAN	means the fair share (**total of values ÷ number of values**).
MEDIAN	is the **middle value** when the values are put in order.
MODE	is the **most common** value.
RANGE	is the **difference between the biggest and smallest** values.
Frequency	is the number of times an event happens.
Frequency table	is a table for a set of observations showing how frequently each event occurs.
Grouped data	is data grouped into non-overlapping classes or intervals.
Class	is an interval for grouping data.

MEAN, MEDIAN, MODE: **AVERAGES**

AVERAGES AND RANGE FROM LIST OF DATA

Example: Find all the averages and range from this list of data: 5, 6, 8, 9, 5, 8, 8, 7

Mean = total of values ÷ number of values

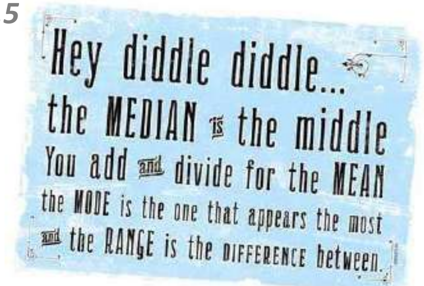
= (5 + 6 + 8 + 9 + 5 + 8 + 8 + 7) ÷ 8 = 56 ÷ 8 = ***7***

Mode (the most common value) = ***8***

Range = biggest – smallest value = *9 – 5 =* ***4***

Median:

1. arrange data from the smallest to the biggest:
   5, 5, 6, 7, 8, 8, 8, 9
2. find the middle value: *~~5~~, ~~5~~, ~~6~~, 7, 8, ~~8~~, ~~8~~, ~~9~~*
   - *the middle value is between 7 and 8*
   - *find mean of 7 and 8 = (7 + 8) ÷ 2 = 7.5*
   - *median =* ***7.5***



Hey diddle diddle...
the MEDIAN is the middle
You add and divide for the MEAN
the MODE is the one that appears the most
and the RANGE is the DIFFERENCE between.

Example: Choose 3 numbers with:

3 9 3 5 5 7 6 1

a) mean 6	*5,6,7*
b) mean 3 and range 4	*1,3,5*
c) mean 5 and median 3	*3,3,9*

AVERAGES FROM FREQUENCY TABLE

Example: A team plays 20 games, the coach records the number of goals they score in each game in a frequency table. Find averages and range.

Number of Goals	Frequency	Total number of goals
0	5	0 × 5 = 0
1	6	1 × 6 = 6
2	4	2 × 4 = 8
3	3	3 × 3 = 9
4	2	4 × 2 = 8
Total	20	31

Mode: the most common number *of goals is 1 (6 times in the table)*

Mode = 1

Range: *the highest value is 4 goals and the lowest 0 goals.*

Range = 4 – 0 = 4

Mean:

1. Create the third column and multiply (**value** × **frequency**) to find the total number of values (goals)
2. Find total of frequencies and total of the 3rd column.
3. Divide $\frac{\boldsymbol{total\ number\ of\ values\ (goals)}}{\boldsymbol{total\ frequencies}} = \frac{31}{20} = \mathbf{1.55}$

Median:

1. Work out the **position of the median =** $\frac{\boldsymbol{total\ frequency+1}}{\mathbf{2}} = \frac{20+}{2} =$ *10.5th position*
2. *There are 5 '0 goals' + 6 '1 goals', which makes 11 values. The median is 10.5th value,* ***median = 1****.*
   (imagine values in a list: 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2…)

10.5th value

AVERAGES FROM THE GROUPED DATA

Example: Find the estimate of mean, median and modal classes from the table below:

POCKET MONEY (£)	FREQUENCY (F)	MIDPOINT (X)	F × X = FX
0 < P ≤ 1	2	0.5	2 × 0.5 = 1
1 < P ≤ 2	5	1.5	5 × 1.5 = 7.5
2 < P ≤ 3	5	2.5	5 × 2.5 = 12.5
3 < P ≤ 4	9	3.5	9 × 3.5 = 31.5
4 < P ≤ 5	15	4.5	15 × 4.5 = 67.5
TOTAL	36	TOTAL	120

Modal class: ***4 < P ≤ 5*** *(the most common class, 15 times)*

Range: *£5 – £0 =* ***£5***

Mean:

1. Create 3rd column (**midpoint** of the classes)
2. Create 4th column (**midpoint** × **frequency**)
3. Find total of frequencies and total of the 4th column.
4. Divide $\frac{\boldsymbol{total\ number\ of\ values\ (£)}}{\boldsymbol{total\ frequencies}} = \frac{120}{36} =$ ***£3.33***

Median class:

1. Position of the median = $\frac{\boldsymbol{total\ frequency+1}}{\mathbf{2}} = \frac{36+1}{2} =$ *18.5th position*

POCKET MONEY (£)	FREQUENCY (F)	
0 < P ≤ 1	2	2
1 < P ≤ 2	5	2+5=7
2 < P ≤ 3	5	2+5+5=12
3 < P ≤ 4	9	2+5+5+9=21
4 < P ≤ 5	15	2+5+5+9+15=36

2. *Median is 18.5th value, that is in* ***median class 3 < P ≤ 4***

Year 7 Mathematics Knowledge Organiser – Unit 1: Representing data

TWO-WAY TABLES

Data collected in two-way tables is divided into more than one category.

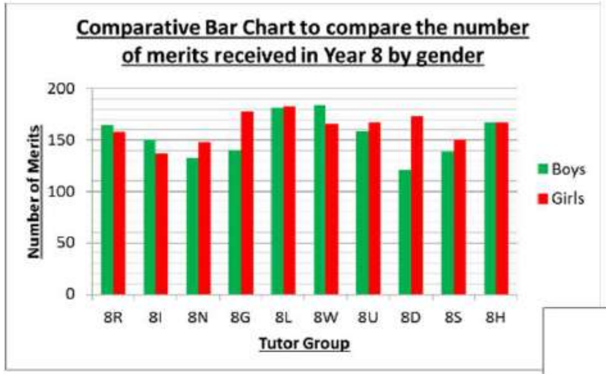
Example: Some college students were asked to choose which of the three subjects, English, Maths or Science they enjoyed the most. Complete the two-table below.

	E	M	S	Total
Girls	20	13	17	50
Boys	18	15	23	56
Total	38	28	40	106

- *the black numbers in the table are filled in from the text*
- *the red numbers are added by calculation*

BAR CHARTS

Bar charts represent statistical information. Bars, of **equal width**, represent frequencies by their height.

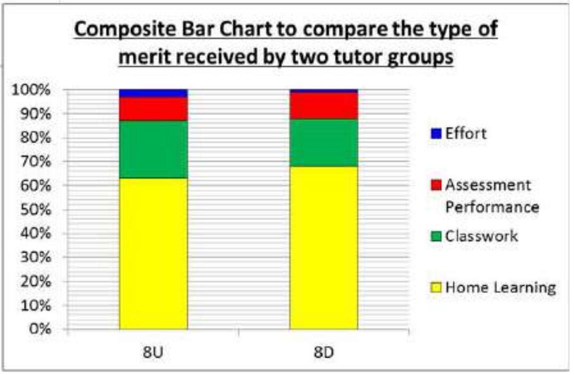


Comparative Bar Chart:

- Bars for each category side-by-side
- Gaps between each category

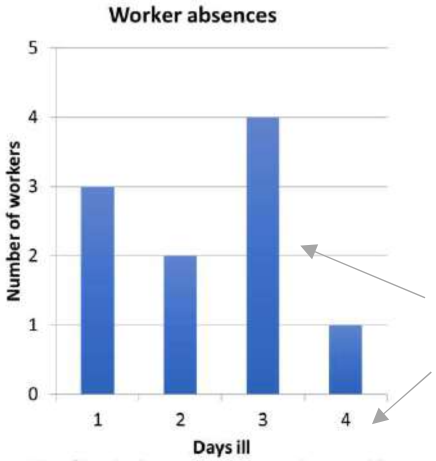
Composite Bar Chart:

Bars show the size of individual categories split into their separate parts.



AVERAGES FROM BAR CHART

Example: Find mean, median, mode and range from this bar chart.



1. Mode is represented by the highest bar
   *Mode: = **3 days***
2. Range *= 4 - 1 = **3 days***
3. To find the mean and median use the table:

Days ill	Workers	Total days
1	3	1 × 3 = 3
2	2	2 × 2 = 4
3	4	3 × 4 = 12
4	1	4 × 1 = 4
Totals	10	23

$$\text{Mean} = \frac{\textbf{total number of values (day)}}{\textbf{total frequencies (workers)}} = \frac{23}{10} = \mathbf{2.3}$$

Median:

- the position of the median $= \frac{total\ frequency + 1}{2} = \frac{10+1}{2} = 5.5^{th}$ *position*
- *imagine values in a list: 1, 1, 1, 2, 2, 3, 3, 3, 3, 4*
  5.5^{th} value lies between 2 and 3

Days ill	Workers	
1	3	**3**
2	2	3+2=**5**
3	4	3+2+4=**9**
4	1	3+2+4+1=**10**

- *or, 5.5^{th} position is located between 2^{nd} and 3^{rd} row of the table*
- *median = (2 + 3)÷2 = **2.5***

Year 7 Mathematics Knowledge Organiser – Unit 1: Representing data

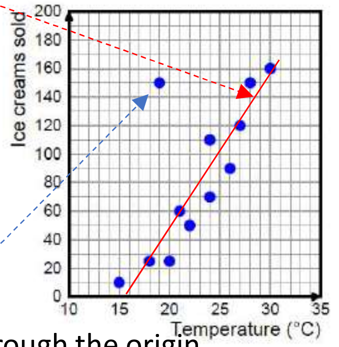
SCATTER GRAPHS

Scatter graph is a graph in which the values of two variables are plotted along two axes.

LINE OF THE BEST FIT is a line drawn on a scatter graph to represent the best estimate of an linear relationship between the variables.

- It should not go too much further beyond the outer points.
- It does not have to run through the origin.
- Approximately the same amount of points should be above and below the line.

Temperature vs Ice creams sold

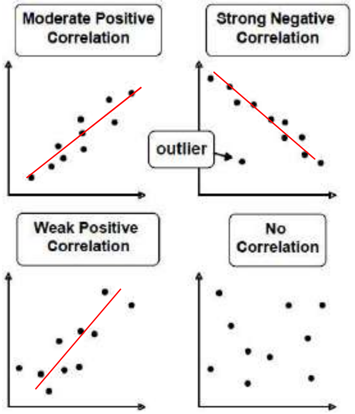


OUTLIER is an extreme value which is much higher or much lower than the rest of the values.

CORRELATION is a measure of the strength of the relationship between two variables.

Strong correlation implies close relationship (the points make a relatively straight line).

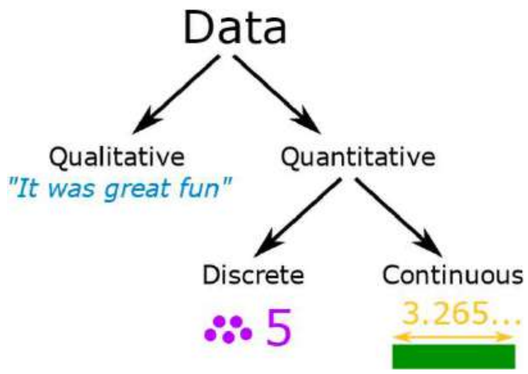
Weak correlation: the points are further apart from each other.



Positive correlation: as one variable increases, so does the other. The line of the best fit has positive gradient (going uphill).

Negative correlation: as one quantity increases the other decreases. The line of the best fit has negative gradient (going downhill).

Keywords



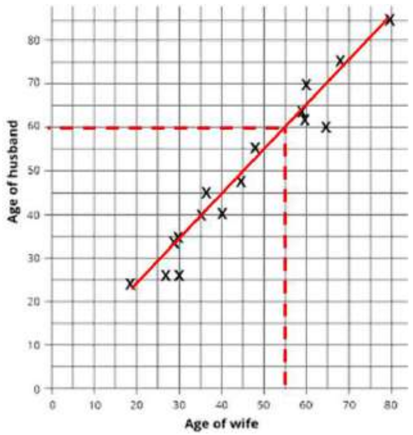
Discrete data is counted, it can only take certain values.
Example: the number of students in a class

Continuous data is measured, it can take any value (within a range).
Example: a person's height.

Example: Based on the scatter graph below, predict the age of the husband of a 55 year old woman.

1. *Draw a line of the best fit.*
2. *Draw a line from number 55 on the x-axis (age of wife axis) towards the line of the best fit.*
3. *Read the corresponding age of the husband from y-axis.*

Answer: 60



PIE CHARTS

A type of graph in which a circle is divided into sectors that each represent a proportion of the whole.

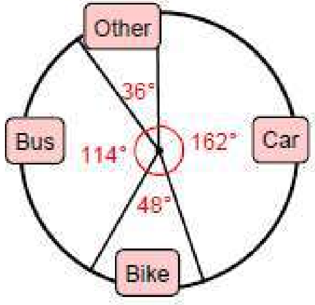
The sum of the angles in pie chart is **360** degrees.

Transport	Frequency	Angle
Car	27	162°
Bike	8	48°
Bus	19	114°
Other	6	36°
TOTAL	60	360°

×60

Example: Create a pie chart from the table above.

- You will always need to calculate the total for the frequency: *27 + 8 + 19 + 6 = 60*
- Calculate the angle that represents one item in the list by sharing 360 into total of the frequency: *360 ÷ 60 = 6 (means one car or bike or bus or other is represented by 6˚)*
- To calculate the angle of each category, multiply the frequency of the category by angle that represents one item only: *for example angle for all cars = 27 cars × 6˚ = 162˚*
- Always check if your angles are correct by seeing if they add up to 360: *162 + 48 + 114 + 36 = 360˚*
- Draw a pie chart using a protractor
- Label the pie chart



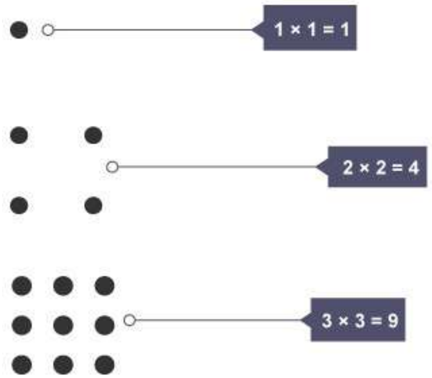
Year 7 Mathematics Knowledge Organiser – Unit 2: Number skills

Keywords

Integer	is any of the positive or negative whole numbers and zero.	*Example: ...-2, -1, 0, +1, +2 ...*
Even number	is an integer that is divisible by 2.	*Example: 2, 4, 6, 8, 10, 12, 14, …*
Odd number	is an integer that has a remainder of 1 when divided by 2.	*Example: 1, 3, 5, 7, 9, 11, 13, 15, …*
Product	is the result of multiplying one number by another.	
Index notation	is used as a shortcut for multiplication when a number is being multiplied by itself. *Example:* $5^4 = 5 \times 5 \times 5 \times 5$	

SQUARE NUMBERS

A **square number** is the result of multiplying integer by itself.
It is called a square number because it gives the area of a square whose side length is an integer.

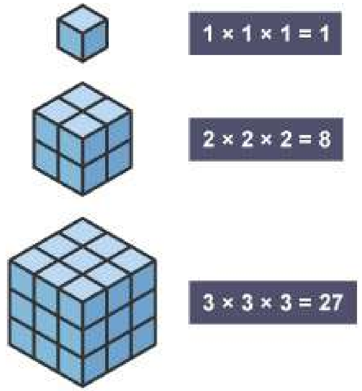


Notation of the square number: x^2

Example: $3^2 = 3 \times 3 = 9$

CUBE NUMBERS

A **cube number** is the result of multiplying integer by itself, and by itself again.
It is called a cube number because it gives the volume of a cube whose side length is an integer.



Notation of the cube number: x^3

Example: $5^3 = 5 \times 5 \times 5 = 125$

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2	**1**	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**	**121**	**144**
x^3	**1**	**8**	**27**	**64**	**125**	216	343	512	729	1000	1331	1728

* You should remember the first twelve square numbers and the first five cube numbers.

Examples:

$4^2 =$ $4 \times 4 =$ 16

$(-4)^2 =$ $(-4) \times (-4) =$ 16

$-4^2 =$ $-(4 \times 4) =$ -16

Examples:

$4^3 =$ $4 \times 4 \times 4 =$ 64

$(-4)^3 =$ $(-4) \times (-4) \times (-4) =$ -64

$-4^3 =$ $-(4 \times 4 \times 4) =$ -64

ROOTS

A **square root** of a number is a value that, when multiplied by itself, gives the number.
Example: $4 \times 4 = 16$, *so a* ***square root*** *of 16 is 4.*
Notation for the square root is $\sqrt{}$

The **cube root** of a number is a special value that, when used in a multiplication three times, gives that number.
Example: $3 \times 3 \times 3 = 27$, *so the* ***cube root*** *of 27 is 3.*
Notation for the square root is $\sqrt[3]{}$

Example:

$\sqrt{81} = 9$

$\sqrt[3]{27} = 3$

Extension:

$-\sqrt{81} = -9$

$\sqrt[3]{-27} = -3$

Note: It is impossible to find square root of a negative number, for example $\sqrt{-81}$.

More examples:
Evaluate

$7^2 = 49$

$5^3 = 125$

$\sqrt{36} = 6$

$\sqrt[3]{8} = 2$

More extensions:
Evaluate

$(-7)^2 = 49$

$(-5)^3 = -125$

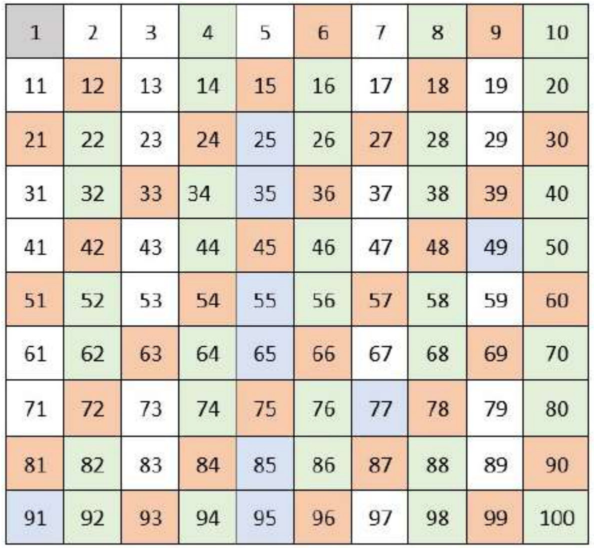
$-\sqrt{36} = -6$

$\sqrt[3]{-8} = -2$

Year 7 Mathematics Knowledge Organiser – Unit 2: Number skills

PRIME NUMBER

Prime numbers (*not shaded numbers on the hundred square below*) are whole numbers greater than 1 that have **exactly** two factors, themselves and 1.
Composite numbers (*shaded numbers on the 100 square below*) are integers that are divisible without remainder by at least one positive integer other than themselves and one.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Example:
***15** is divisible by 1, 15, 3, 5, therefore it is **not** a prime number.*
***2** is divisible by 1 and 2 only, therefore it **is** a prime number (the only even prime number)*

Important note: Number **1** is not prime nor composite number.
Prime numbers that you should remember are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

FACTORS

Factors are numbers that divide another number without leaving a remainder.

Examples:
Find factors of number 32
1 × 32
2 × 16
4 × 8
Factors of number 32 are: 1, 2, 4, 8, 16 and 32

Find factors of number 12

Answer: 1, 2, 3, 4, 6, 12

USING CALCULATOR

To work out 34^2:
- Enter ***34***
- Press the x^2 button
- Press =

You should get the answer 1156

To work out 26^3:
- Enter ***26***
- Press the x^3 button
- Press =

You should get the answer 17576

To work out $\sqrt[3]{2197}$:
- Press SHIFT
- Press the $\sqrt{\square}$ button
- Enter ***2197***
- Press =

You should get the answer 13

To work out $\sqrt{225}$:
- Press the $\sqrt{\square}$ button
- Enter ***225***
- Press =

You should get the answer 15

LCM or The Lowest / Least Common Multiple

LCM is smallest positive number that is a multiple of two or more numbers.

Example:
Find the lowest common multiple of 6 and 9.
Multiples of 6 are: 6, 12, 18, 24, 30, 36…
Multiples of 9 are: 9, 18, 27, 36, 45, 54, …
Common multiples of 6 and 9 are: 18, 36 …
The LCM is 18

HCF or the Highest Common Factor

HCF is the greatest number that is a factor of two (or more) other numbers.

Examples:
Find the highest common factor of 18 and 24.
Factors of 18 are: 1, 2, 3, 6, 9, 18
Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24
Common factors of 18 and 24 are: 1, 2, 3 and 6
The HCF is 6

Find the HCF and LCM of 12 and 15

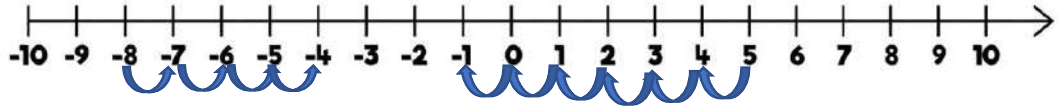
Multiples of 12 are: 12, 24, 36, 48, 60, 72…
Multiples of 15 are: 15, 30, 45, 60, 75…
The LCM is 60

Factors of 12 are: 1, 2, 3, 4, 6, 12
Factors of 15 are: 1, 3, 5, 15
The HCF is 3

Year 7 Mathematics Knowledge Organiser – Unit 2: Number skills

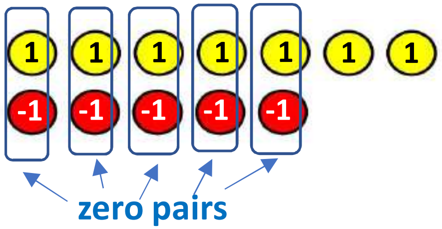
DIRECTED NUMBER

One of the methods which can help you to solve calculation with negative numbers, is a **number line**



$(-8) + 4 = -4$ $5 - 6 = -1$

Second method is using **counters**.



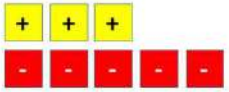
This represents calculation $7 + (-5)$

There are five **zero pairs**.
Removing the zero pairs leaves us with **2**.
Therefore: $7 + (-5) = 2$

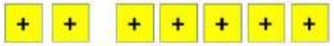
This leads us to realise that $\mathbf{7 + (-5)}$ gives the same answer as $\mathbf{7 - 5}$.
So **7 – 5 = 7 + (-5)**
Consequently also **7 – (-5) = 7 + (+5) = 7 + 5**

The subtraction symbol '–' means addition of the 'additive inverse'

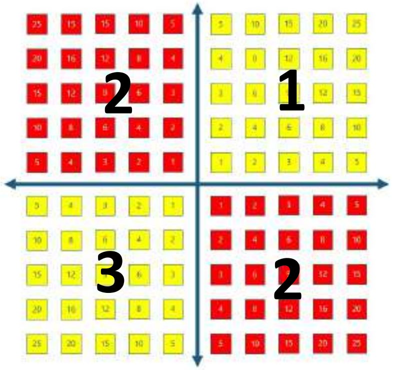
3 – 5= 3 + (⁻5) = ⁻2



2 – (⁻5) = 2 + 5 = 7



MULTIPLICATION AND DIVISION OF DIRECTED NUMBERS

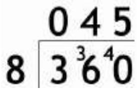


1. × or ÷ of positive numbers gives POSITIVE answer
   Example: 2 × 3 = 6
2. × or ÷ of positive AND negative numbers gives NEGATIVE answer
   Example: $3 \times (-2) = -6$ *or* $(-6) \div 3 = -2$
3. × or ÷ of negative numbers gives POSITIVE answer
   Example: $(-6) \div (-2) = 3$

DIVISION

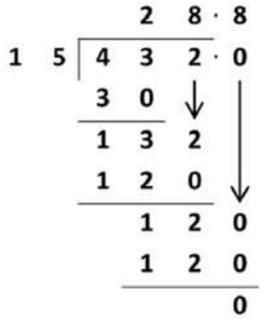
To divide two numbers, we use the bus stop method.

Short division



Long division

432 ÷ 15 becomes = 28.8

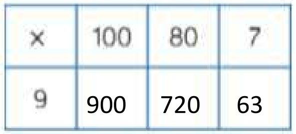


Keywords

Quotient: the result of a division
Dividend: the number being divided
Divisor: the number we divide by

MULTIPLICATION

To multiply two numbers together, the grid method is useful to ensure that the calculation is completed correctly.
The second method to use is column method.



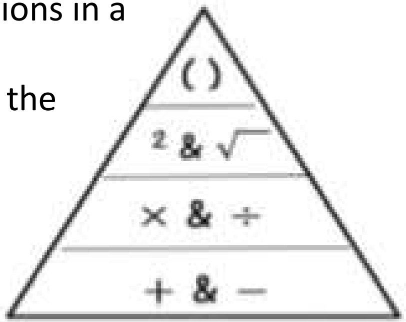
×	100	80	7
9	900	720	63

900 + 720 + 63 = 1683

BIDMAS

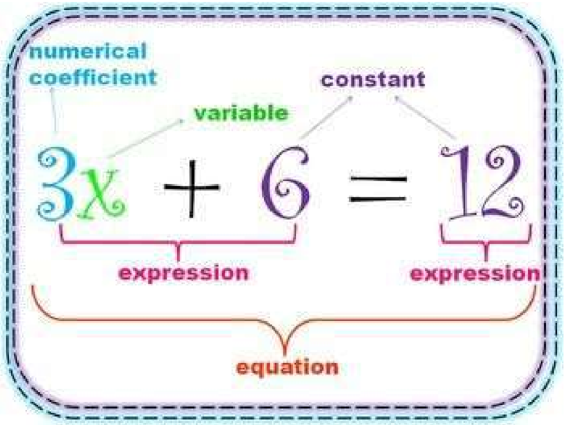
The **order** in which we complete operations in a sum is important.
If operations of the same priority are in the the same sum, we work from left to right

Example: *10 – 3 + 5 = 12,*
first 10 – 3 = 7,
then 7 + 5 = 12.



Year 7 Mathematics Knowledge Organiser – Unit 3: Basic Algebra

Keywords



A **variable** is a letter or symbol that represents an unknown value.

When variables are used with other numbers, parentheses, or operations, they create an **algebraic expression**.

Equation is algebraic expression with equal sign, which can be solved (value of variable is found).

A **coefficient** is the number multiplied by the variable in an algebraic expression.

A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division, including + or – symbol in front of it.

A **constant** is a number that cannot change its value.

Identity is an equation that is true no matter what values are chosen. (symbol ≡)

A **formula** is an equation linking sets of physical variables.

SIMPLIFYING EXPRESSIONS

Multiplication of a number and variable is written without multiplication symbol, numbers first, letters in alphabetical order:

Example: $3 \times x = 3x$

$y \times 6 \times x = 6xy$

The division is written as a fraction:

Example: $6 \div x = \frac{6}{x}$

Multiplying and dividing variables

Examples: $x \times x \times x = x^3$

$2 \times x \times y \times 3 =$

$2 \times 3 \times x \times y = 6xy$

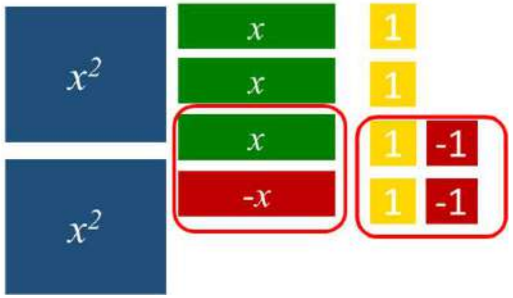
$6x \div 2 = 3x$

COLLECTING LIKE TERMS

'Like terms' are terms whose **variables** (and their powers) are the same, the **coefficients** can be different.

$3x + 2x^2 - x + 4 - 2 =$

$2x^2 + 2x + 2$



Examples:

$x + x + 2x = 4x$

$x + 4 + 3x - 5 = 4x - 1$

$x + 3y + 2x - 2y + 3 = 3x + y + 3$

$9x^2 - 2x - 5x^2 - 5x = 4x^2 - 7x$

$2x^2 + xy + 3x^2 + xy = 5x^2 + 2xy$

WRITING EXPRESSIONS AND EQUATIONS

The word phrases can be translated into algebraic expressions or equations with variables.

Examples:

- *Eight less than the **quotient** of a number and two*
  $\frac{x}{2} - 8$
- *Nine times the **sum** of a number and fifteen*
  $9(n + 15)$
- *The sum of **twice** a number and seven*
  $2x + 7$
- *One plus the **product** of a number and five*
  $1 + 5x$
- *A number **less than** twenty-five*
  $25 - x$
- *I think of a number (x). When I multiply the number by two ($2x$) and add 3 ($2x + 3$), the answer is 11* $2x + 3 = 11$
- *I think of a number (x). When I add 3 ($x+3$) and multiply the result by 2 ($2(x + 3)$), the answer is 11*
  $2(x+3) = 11$

Year 7 Mathematics Knowledge Organiser – Unit 3: Basic algebra

Keywords

Expanding brackets means to remove the brackets.
Factorising means putting brackets back into expressions.
Factors of a number are the numbers that divide the original number without remainder.
Writing a number as a product of factors is called a **factorisation** of the number.
The **Highest Common Factor (HCF)** is the largest common factor (the factor that two or more numbers have in common).

WRITING FORMULAE

Formula must have it subject of the formula

Example:

- *Guy, Eric and Luke go Christmas shopping. Write a formula calculating how much money **T** each man has left after shopping.*
  (a) Guy had £20 and spent £y on presents.
  $T = 20 - y$
  (b) Eric had £m and spent £12 on presents.
  $T = m - 12$
  (c) Luke had £a and spent half £b on presents
  $T = a - \frac{1}{2}b$
- *Adult tickets to the cinema cost £7. Child tickets cost £4. Write a formula for the total cost C of taking 'a' adults and 'c' children to the cinema.* $C = 7a + 4c$
- *A phone company charges a monthly fee of £10.25 and £0.12 per minute. Write a formula for the monthly bill, **b** for **m** minutes.*
  $b = 10.25 + 0.12m$
- *A delivery company charges according to the weight of each parcel. They charge £2 per kilogram, plus a 50p handling fee. Write a formula for the cost C of sending a parcel that weighs k kilograms.* $C = 2k + 0.5$

EXPANDING SINGLE BRACKETS

Multiply everything in the brackets by a number or variable in front of the bracket

Examples: Expand

$4(a + 6) = 4a + 24$

$-2(b - 4) = -2b + 8$

$c(2c - 5) = 2c^2 - 5c$

$2d(3d - e) = 6d^2 - 2de$

Grid method

	a	+ 6
4	4a	+24

	b	-4
-2	-2b	+8

FACTORISING

- Find the HCF of the terms in the brackets (the highest numerical factor and the highest power of the variable).
- Put the HCF in front of the brackets. Terms divided by HCF stay in the brackets.
- Check your answers by expanding brackets.

Examples: Factorise

$4x + 12 = 4(x + 3)$

$7x^2 + 3x = x(7x + 3)$

$8x^2 + 16x = 8x(x + 2)$

SUBSTITUTION

If we are told what number a variable represents, we can **substitute** this into expressions to find their value.

Examples:
Find the value of expressions when $x = 5, y = 4$

$7x = 7 \times 5 = 35$

$3(x + 1) = 3 \times (5 + 1) = 3 \times 6 = 18$

$\frac{2(x-1)}{4} = \frac{2(5-1)}{4} = \frac{2\times4}{4} = \frac{8}{4} = 2$

$x^2 = 5^2 = 25$

$2x^2 = 2 \times 5^2 = 2 \times 25 = 50$

$(2x)^2 = (2 \times 5)^2 = 10^2 = 100$

$xy = 5 \times 4 = 20$

Year 7 Mathematics Knowledge Organiser – Unit 4: Fractions

Keywords

Fractions represent equal parts of a whole. *Example:* $\frac{3}{4}$

3 ← the **numerator** says how many parts we have.

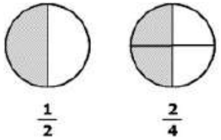
4 ← the **denominator** says how many equal parts the whole is divided into

Mixed number is a number consisting of an integer and a proper fraction. *Example:* $1\frac{1}{4}$

Improper fraction is a fraction in which the numerator is greater than the denominator. *Example:* $\frac{5}{4}$

Common denominator is a common multiple of the denominators of several fractions.

Equivalent fractions are fractions which have the same value. *Example:* $\frac{1}{2}$ *and* $\frac{2}{4}$ *are equivalent fractions.*

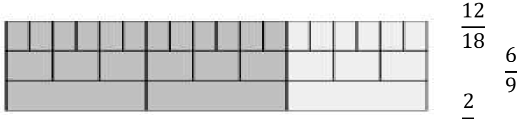


SIMPLIFYING FRACTIONS

Divide numerator and denominator by the same factor:

Example: Simplify/cancel down $\frac{12}{30}$ *fully*

$$\frac{12}{18} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{9} \overset{\div 3}{\underset{\div 3}{=}} \frac{2}{3}$$



All of these fractions are **equivalent**.

ORDERING FRACTIONS

Convert fractions into equivalent fractions with the lowest common denominator and compare.

Example: Put these fractions in ascending order

$\frac{2}{3}$ $\frac{3}{5}$ $\frac{1}{2}$

The lowest common multiple of 3, 5, and 2 is 30

$\frac{2}{3}=\frac{20}{30}$ $\frac{3}{5}=\frac{18}{30}$ $\frac{1}{2}=\frac{15}{30}$

The correct order is: $\frac{1}{2}$ $\frac{3}{5}$ $\frac{2}{3}$

CONVERTING MIXED NUMBERS TO IMPROPER FRACTIONS

Example: Convert $2\frac{3}{4}$ *into improper fraction*

$2\frac{3}{4}$

$=\frac{4}{4}+\frac{4}{4}+\frac{3}{4}=\frac{11}{4}$

- Imagine mixed number as an addition

$2\frac{3}{4}=2+\frac{3}{4}$

- Change integer part into fraction with the same denominator

$2+\frac{3}{4}=\frac{8}{4}+\frac{3}{4}$

- Add the fractions

$2\frac{3}{4}=\frac{8}{4}+\frac{3}{4}=\frac{8+3}{4}=\frac{11}{4}$

Different method:

$$\overset{+}{\underset{\times}{}}\, 2\frac{3}{4}=\frac{(4\times 2)+3}{4}=\frac{8+3}{4}=\frac{11}{4}$$

CONVERTING IMPROPER FRACTIONS TO MIXED NUMBERS

Example: Convert $\frac{11}{4}$ *into improper fraction*

- Divide numerator by denominator
- The answer is the integer part and the remainder is the numerator of fractional part
- Denominator stays the same

$11 \div 4=2\ r3$

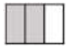
$2\frac{3}{4}$

ADDING AND SUBTRACTING FRACTIONS

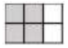
Example: Add $\frac{2}{3}$ *and* $\frac{1}{2}$

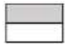
1. Convert fractions into fractions with common denominator.

   The lowest common denominator is 6

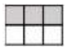


$\frac{2}{3}=\frac{4}{6}$



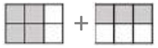


$\frac{1}{2}=\frac{3}{6}$



2. Add or subtract numerators and simplify, if needed. Convert improper fractions into mixed numbers.

$$\frac{2}{3}+\frac{1}{2}=\frac{4}{6}+\frac{3}{6}=\frac{7}{6}=1\frac{1}{6}$$



<u>To add or subtract mixed numbers</u>, convert mixed numbers to improper fractions first, than add or subtract.

Example: Subtract $1\frac{3}{4}$ *and* $1\frac{1}{2}$

$$1\frac{3}{4}-1\frac{1}{2}=\frac{7}{4}-\frac{3}{2}=\frac{7}{4}-\frac{6}{4}=\frac{1}{4}$$

converting into improper fractions

common denominator

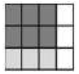
Year 7 Mathematics Knowledge Organiser – Unit 4: Fractions

MULTIPLYING FRACTIONS

Multiply numerators and denominators separately and simplify the answer if possible.

Example: $\frac{3}{4} \times \frac{2}{3}$

$\frac{3}{4}$ 

$\frac{2}{3}$ *of* $\frac{3}{4} = \frac{6}{12}$ 

$$\frac{3}{4} \times \frac{2}{3} = \frac{3\times2}{4\times3} = \frac{6}{12} = \frac{1}{2}$$

×

×

<u>To multiply fraction by the whole number</u>, convert whole number into the fraction and multiply.

Example:

$$\frac{2}{3} \times 4 = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

$$\frac{2}{3} \times \frac{4}{1} = \frac{2\times4}{3\times1} = \frac{8}{3} = 2\frac{2}{3}$$

<u>To multiply mixed numbers</u>, convert mixed numbers to improper fractions and multiply. Convert the answer back to mixed number.

Example:

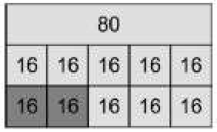
$$2\frac{1}{3} \times 3\frac{1}{2} = \frac{7}{3} \times \frac{7}{2} = \frac{7\times7}{3\times2} = \frac{49}{6} = 8\frac{1}{6}$$

FRACTION OF A NUMBER

Divide a number by denominator and multiply by numerator.

Example: Find $\frac{2}{5}$ *of £80*

(£80 ÷ 5) × 2= £16 × 2 = £32



ONE NUMBER AS A FRACTION OF ANOTHER

To write one number (x) as a fraction of another number (y), write number x as a numerator and number y as denominator of the fraction and simplify.

Example: Express 35 as fraction of 80 $\frac{35}{80} = \frac{7}{16}$

RECIPROCAL OR MULTIPLICATIVE INVERSE

The reciprocal of a **number n** is 1÷ n = $\frac{1}{n}$

Example: the reciprocal of 5 is $\frac{1}{5}$

To find the reciprocal of **a fraction**, flip the fraction.

Example: the reciprocal of $\frac{3}{4}$ *is* $\frac{4}{3}$

Any number multiplied by its reciprocal is always equal to 1.

Example: $5 \times \frac{1}{5} = \frac{5}{5} = 1$ $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

DIVIDING FRACTIONS

To divide two fractions, multiply the first fraction by the **reciprocal** of the second one.

Example: $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = \frac{2}{1} = 2$

Different method: $\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \div \frac{2}{8} = \frac{5\div2}{8\div8} = \frac{5\div2}{1} = \frac{5}{2}$

<u>Dividing fractions and whole numbers</u>. Convert the whole number into a fraction and divide.

Example:

$$3 \div \frac{1}{4} = \frac{3}{1} \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$$

$$\frac{2}{5} \div 2 = \frac{2}{5} \div \frac{2}{1} = \frac{2}{5} \times \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$$

<u>Dividing mixed numbers</u>: change the mixed numbers to an improper fractions and divide.

Example: $2\frac{2}{5} \div \frac{1}{2} = \frac{12}{5} \div \frac{1}{2} = \frac{12}{5} \times \frac{2}{1} = \frac{24}{5} = 4\frac{4}{5}$

CONVERTING BETWEEN FRACTIONS, TERMINATING DECIMALS AND PERCENTAGES

D ⟶ F

1. Write the decimal as a fraction 'over one'.
2. Convert the fraction into a fraction with whole number in the numerator by multiplying both numerator and denominator by the multiple of 10.
3. Simplify.

Example: Convert 0.84 into a fraction.

$0.84 = \frac{0.84}{1} = \frac{84}{100} = \frac{21}{25}$

F ⟶ D

1. Divide the numerator by denominator.
2. Sometimes, you can help yourself by converting the fraction into the fraction with multiple of 10 in the denominator.

Example1: Convert $\frac{2}{9}$ *to decimal*

2 ÷ 9 = 0.22222…..

Example2: Convert $\frac{2}{25}$ *to decimal*

$\frac{2}{25} = \frac{8}{100}$ *8 ÷ 100 = 0.08*

Multiply by 100
D ⟶ P
D ⟵ P
Divide by 100

Examples:

0.04 = (0.04 ×100) % = 4%

1.2 = (1.2 × 100)% = 120%

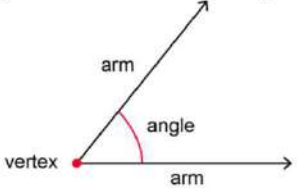
23% = $\frac{23}{100}$ *= 0.23*

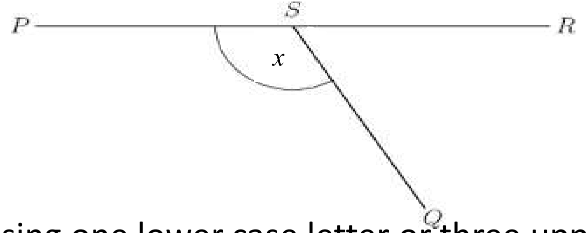
5% = $\frac{5}{100}$ *= 0.05*

Year 7 Mathematics Knowledge Organiser – Unit 5: Angles

Keywords

An **ANGLE** is the **amount of turn** between two **straight lines** joined (or **intersected**) at a point called a **VERTEX**. The two lines are called **arms.** Angles are commonly marked by an **arc** (part of a circle) between arms.

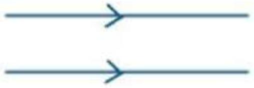




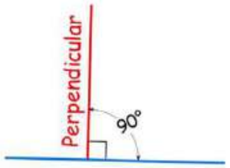
Angles are labelled using one lower case letter or three upper case letters, in which case the letter in the middle always represents the vertex.

Example: Angle x is labelled as $\angle PSQ$ or $\angle QSP$

PARALLEL LINES are lines that **never cross** each other - they keep the same distance apart from each other. They are marked with arrows.

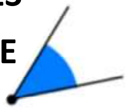


Lines that cross (intersect) each other at **right angles** (90°) are called **PERPENDICULAR LINES**.



TYPES OF ANGLES

An **ACUTE ANGLE** is less than 90°.



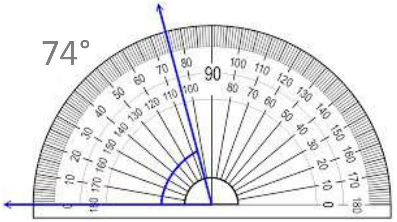
A **RIGHT ANGLE** is exactly 90°.

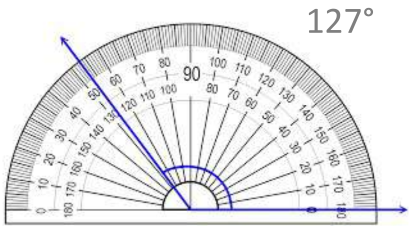
An **OBTUSE ANGLE** is greater than 90° but less than 180°.

A **REFLEX ANGLE** is greater than 180° but less than 360°.

MEASURING ANGLES

1. Place the midpoint of the protractor on the VERTEX of the angle.
2. Line up one arm of the angle with the zero line of the protractor (where you see the number 0).
3. Read the degrees where the other arm crosses the number scale.





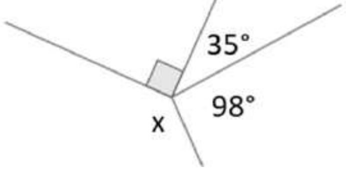
ANGLES AROUND THE POINT

Angles at a point add up to 360°.

Example: Find the missing angle x.

x = 360 – (98 + 35 + 90) = 360 - 223

x = 137°



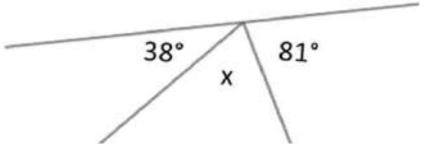
ANGLES ON THE STRAIGHT LINE

Angles on a straight line add up to 180°.

Example: Find the missing angle x.

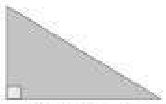
x = 180 – (38 + 81) = 180 - 119

x = 61°

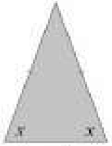


TRIANGLES

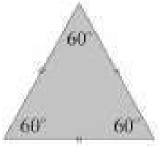
Right Angle Triangles have a 90° angle in.
Isosceles Triangles have 2 equal sides and 2 equal base angles.
Equilateral Triangles have 3 equal sides and 3 equal angles (60°).
Scalene Triangles have different sides and different angles.



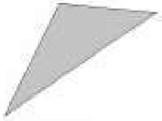
Right Angled



Isosceles



Equilateral



Scalene

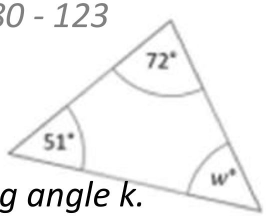
ANGLES IN THE TRIANGLE

Angles in the triangle add up to 180°.

Example: Find the missing angle w.

w = 180 – (72 + 51) = 180 - 123

w = 57°



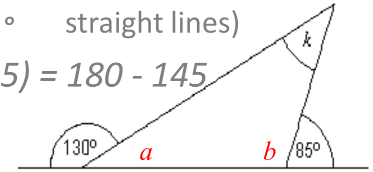
Example: Find the missing angle k.

a = 180 – 130 = 50° (angles on the straight lines)

b = 180 – 85 = 95°

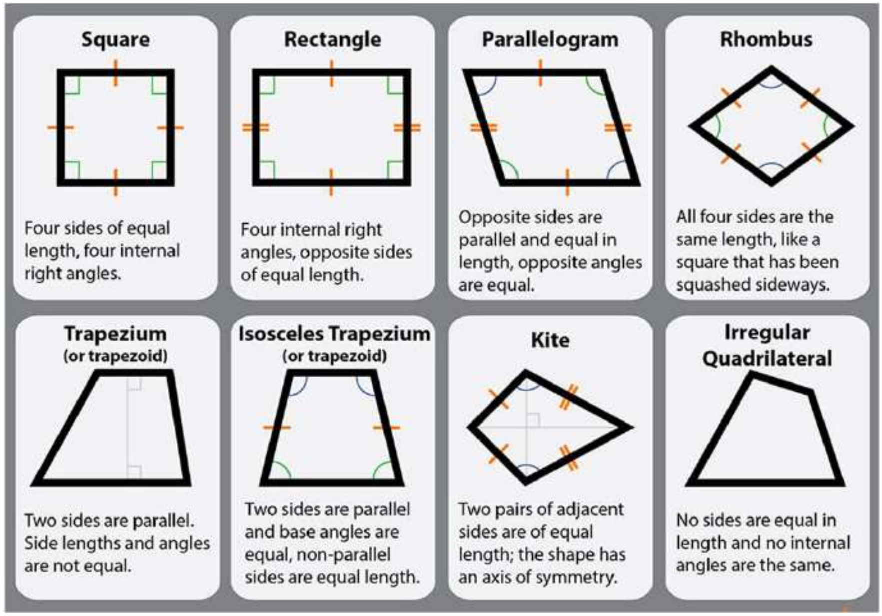
w = 180 – (50 + 95) = 180 - 145

w = 35°



Year 7 Mathematics Knowledge Organiser – Unit 5: Angles

QUADRILATERALS



Square
Four sides of equal length, four internal right angles.

Rectangle
Four internal right angles, opposite sides of equal length.

Parallelogram
Opposite sides are parallel and equal in length, opposite angles are equal.

Rhombus
All four sides are the same length, like a square that has been squashed sideways.

Trapezium (or trapezoid)
Two sides are parallel. Side lengths and angles are not equal.

Isosceles Trapezium (or trapezoid)
Two sides are parallel and base angles are equal, non-parallel sides are equal length.

Kite
Two pairs of adjacent sides are of equal length; the shape has an axis of symmetry.

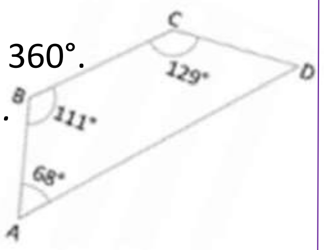
Irregular Quadrilateral
No sides are equal in length and no internal angles are the same.

ANGLES IN THE QUADRILATERAL

Angles in the quadrilateral add up to 360°.

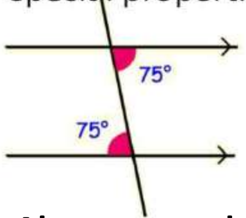
Example: Find the missing angle CDA.

∠CDA = 360 – (111 + 129 + 68)
= 360 – 308 = 52°

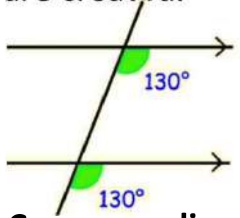


ANGLES IN PARALLEL LINES

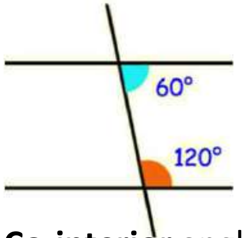
When a transversal (*a line that crosses at least two other lines*) intersects **parallel lines**, angles with special properties are created.



Alternate angles are equal



Corresponding angles are equal



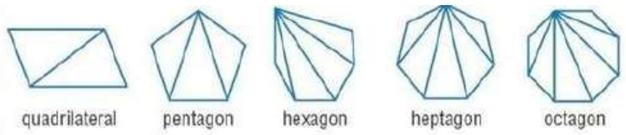
Co-interior angles sum to 180°

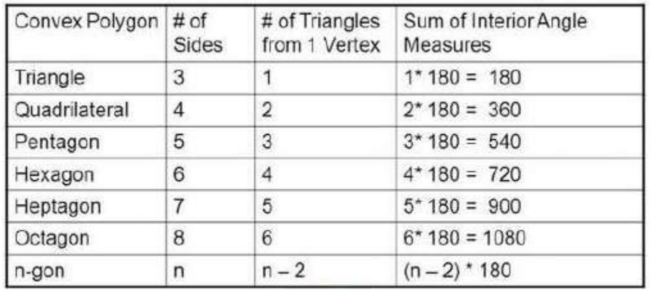
ANGLES IN THE POLYGONS (*a polygon is a 2-D shape made of straight lines*)

The sum of the <u>interior angles</u> of a polygon of n sides is found by the formula

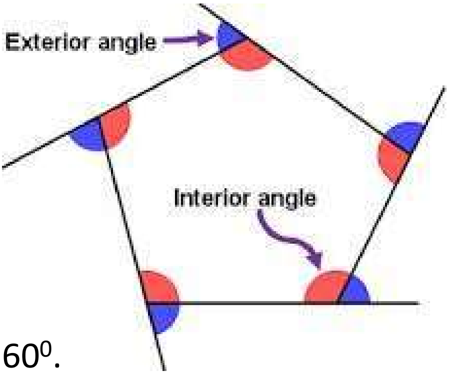
$$(n-2)\times 180$$

(n-2) represents min. number of triangles, each n-sided polygon can be divided into





Convex Polygon	# of Sides	# of Triangles from 1 Vertex	Sum of Interior Angle Measures
Triangle	3	1	1* 180 = 180
Quadrilateral	4	2	2* 180 = 360
Pentagon	5	3	3* 180 = 540
Hexagon	6	4	4* 180 = 720
Heptagon	7	5	5* 180 = 900
Octagon	8	6	6* 180 = 1080
n-gon	n	n – 2	(n – 2) * 180



The sum of the <u>exterior angles</u> of a polygon is always 360^0.
Adjacent interior and exterior angles in polygons add up to 180°.

<u>Regular polygon with n sides</u>

In a regular polygon all sides and angles are equal.

In a regular polygon, the size of one interior angle is equal to $\frac{sum}{n} = \frac{(n-2)\times 180}{n}$

In a regular polygon, the size of one exterior angle is equal to $\frac{360}{n}$

Example: Find interior and exterior angles in a <u>regular</u> pentagon.

Pentagon can be divided into 3 triangles, so sum of interior angles = (5 – 2) × 180 = 3 × 180 = 540°

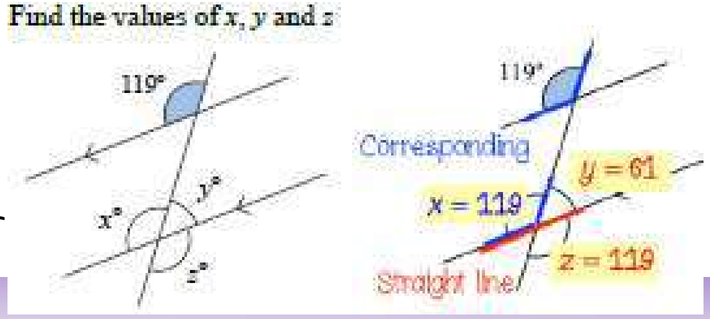
In regular pentagon, each interior angle = 540 ÷ 5 = 108°

Sum of exterior angles = 360°, so each ext. angle in regular pentagon = 360 ÷ 5 = 72°

VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are equal.

Example:



Year 7 Mathematics Knowledge Organiser – Unit 6: Decimals and Percentages

Keywords

Place Value is the value of each digit in the number based on its position.

Estimate means to find a value that is close enough to the right answer, usually with some thought or calculation involved. We use symbol ≈ to mark estimation. *Example:* $x \approx 10$ *means* x *is approximately equal to 10.*

Ascending means increasing in value, ordered from the smallest to the largest value.

Descending means decreasing in value, ordered from the largest to the smallest value.

Place Value Table

1000 thousands	100 hundreds	10 tens	1 units	.	0.1 tenths	0.01 hundredths	0.001 thousandths

ROUNDING TO DECIMAL PLACES

1. Find the digit with the value that you are rounding to.
2. If the decision digit (the next digit to the right) is having value
   - 5 or more – round up,
   - 4 or less – round down.
3. Do not write any more digits after the rounded number, not even 0s.

Example: Round 15.43 to the nearest tenths, or to 1 decimal place

$15.43 \approx 15.4$

the digit with the value that you are rounding to

the decision digit is having value 4 or less, that means rounding down

Note: Answer 15.43 rounded to 1dp is 15.40 is incorrect.

ADDING AND SUBTRACTING DECIMALS

1. Write one decimal beneath the other making sure that the digits with the same value are underneath each other.
2. Add 0s after the decimal point (place value holders) so that all the numbers have the same number of digits after the decimal point.
3. Add/subtract as you would with the whole numbers.
4. Copy the position of the decimal point into your answer.
5. Check your answer using estimation.

Example: Evaluate 65.3 – 42.45

$$\begin{array}{rr} & 65.30 \\ - & 42.45 \\ \hline & 22.85 \end{array}$$

Check by estimation: $65.3 \approx 65$

$42.45 \approx 43$

65 – 43 = 22, which is very close to 22.85

ORDERING DECIMALS

1. Align numbers using the place value table or by writing one decimal beneath the other making sure that the digits with the same value are underneath each other.
2. Compare the digits starting with the digits that have the biggest value (from the left).

Example: Order numbers 0.21, 0.201 and 2.1 in descending order.

0 . 2 1

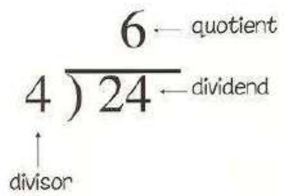
0 . 2 0 1 *(the smallest number)*

2 . 1 *(the highest number - the biggest units digit)*

Answer: 2.1, 0.21, 0.201

Year 7 Mathematics Knowledge Organiser – Unit 6: Decimals and Percentages

Factors	are numbers which multiplied together get another number.	*Example: 2 and 3 are factors of 6, because 2 × 3 = 6*
Product	is the answer when two or more values are multiplied together.	*Example: 6 is a product of 2 and 3, because 2 × 3 = 6*
Dividend	a number to be divided.	
Divisor	a number by which another number is to be divided.	
Quotient	the answer after one number is divided by another.	



MULTIPLYING DECIMALS

1. Multiply decimals by powers of 10 (*10, 100, 1000…*) to change them into whole numbers.
2. Multiply the whole numbers.
3. 'Undo' the first step by using inverse operation - dividing the answer by the same powers of 10.

Example: Evaluate 12.3 × 6.11

12.3 × 10 = 123

6.11 × 100 = 611

$$\begin{array}{rr} & 1\,2\,3 \\ \times & 6\,1\,1 \\ \hline & 1\,2\,3 \\ & 1\,2\,3\,0 \\ + & 7\,3\,8\,0\,0 \\ \hline & 7\,5\,1\,5\,3 \end{array}$$

75153 ÷ 10 ÷ 100 = 75.153

Check by estimation: *12.3 ≈ 12*

6.11 ≈ 6

12 × 6 = 72, which is very close to 75.153

Using multiplication of fractions to multiply decimals

1. Convert decimals into fractions.
2. Multiply fractions.
3. Convert the answer (fraction) back to decimal.

Example: Evaluate 12.3 × 6.11

$$\frac{123}{10} \times \frac{611}{100} = \frac{75153}{1000} = 75.153$$

Check your answer using estimation.

DIVIDING DECIMALS

1. Rewrite division as a fraction.
2. Find equivalent fraction with the whole number in denominator.
3. Divide numerator by denominator using bus method.

Example: Evaluate 0.12 ÷ 0.3

$$0.12 \div 0.3 = \frac{0.12}{0.3} = \frac{1.2}{3} = 1.2 \div 3 \ldots\ldots \quad \begin{array}{r|l} & 0\ .\ 4 \\ \hline 3 & 1\ .\ 2 \end{array}$$

$$0.12 \div 0.3 = 1.2 \div 3 = 0.4$$

Using division of fractions to divide decimals

1. Convert decimals to fractions.
2. Divide fractions.
3. Convert the answer back to decimal.

Example: Evaluate 0.8 ÷ 0.01

$$0.8 \div 0.01 = \frac{8}{10} \div \frac{1}{100} = \frac{8}{10} \times \frac{100}{1} = \frac{800}{10} = 80$$

RECURRING DECIMALS

Recurring decimal is a decimal number that has digits that repeat forever. The part that repeats is shown by placing a dot above the digit that repeats, or dots over the first and last digit of the repeating pattern.

Examples:

$$\frac{1}{3} = 0.333\ldots = 0.\dot{3}$$

$$\frac{1}{7} = 0.\underbrace{142857}\underbrace{142857}\ldots = 0.\dot{1}4285\dot{7}$$

$0.\dot{2}\dot{3} = 0.232323....$

$0.5\dot{3} = 0.533333.....$

$0.\dot{1}2\dot{3} = 0.123123123...$

Year 7 Mathematics Knowledge Organiser – Unit 6: Decimals and Percentages

'Per cent'	means out of 100. *Example:* 3% means 3 out of 100, which can be written in a form of a fraction $\frac{3}{100}$ or as a decimal 0.03
Multiplier	is a decimal that represents the percentage change.
VAT	stands for Value Added Tax. This is 20% tax added on to the price of most of the things that you can buy.

Increase/decrease or reduce means to make something bigger / smaller (in size or quantity).

% OF AN AMOUNT

Finding 'easy' %s (without calculator)

50% by halving an amount

25% by dividing an amount by 4

10% by dividing an amount by 10

5% by halving 10%

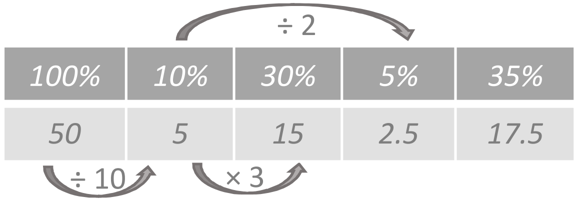
1% by dividing an amount by 100

…and adding them together

Example: 35% of 50

10% of 50 = 5

5% of 50 = 2.5



100%	*10%*	*30%*	*5%*	*35%*
50	*5*	*15*	*2.5*	*17.5*

Using multiplier (with calculator)

1. Change % into decimal (multiplier).
2. Multiply.

Example: 35% of 50 = 0.35 × 50 = 17.5

'of' means '×'

% INCREASE AND DECREASE

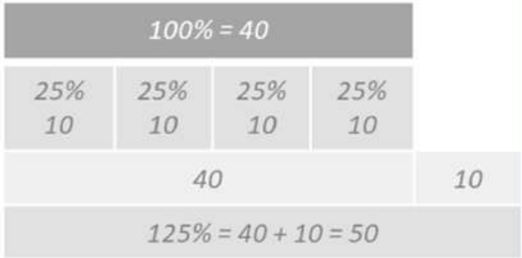
Finding % of an amount and adding (increase) or subtracting (decrease)

Example1:

Increase 40 by 25%.

25% of 40 = 10

40 + 10 = 50

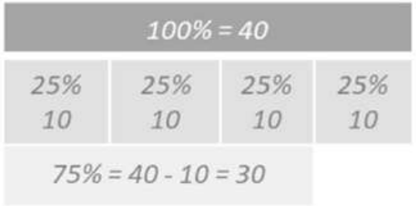


Example2:

Decrease 40 by 25%.

25% of 40 = 10

40 - 10 = 30



Using multiplier

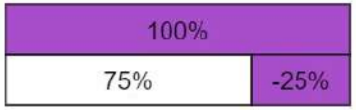
Find the multiplier and **multiply**.

Example1: Increase 40 by 25%.

Multiplier = (100 + 25)% = 125% = 1.25

40 × 1.25 = 50

Example2: Decrease 40 by 25%.



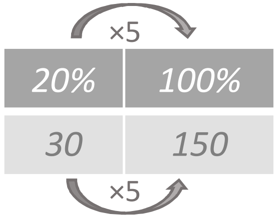
Multiplier = (100 - 25)% = 75% = 0.75

40 × 0.75 = 30

FINDING AN ORIGINAL AMOUNT

Using multiplication table

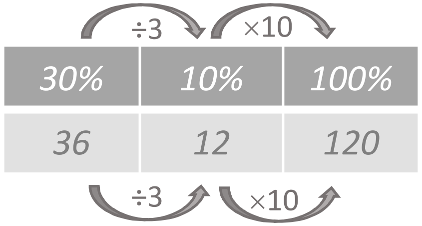
Example1: 20% of an amount is £30, What is the total amount?



20%	*100%*
30	*150*

The answer: 150

Example2: 30% of the members of the tennis club are pensioners. There are 36 pensioners. How many members are there in total?



30%	*10%*	*100%*
36	*12*	*120*

The answer: 120

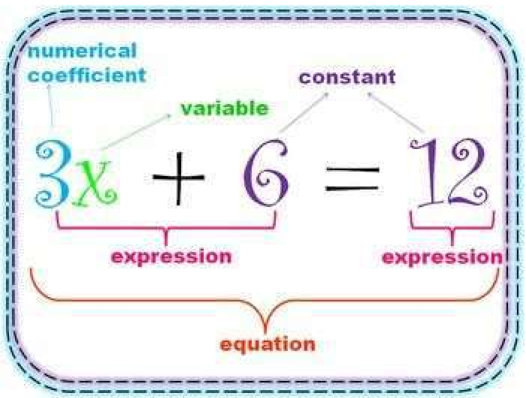
EXPRESSING ONE NUMBER AS % OF ANOTHER

Example: What is 17 as a percentage of 25?

$\frac{17}{25} \times 100 = 68$

17 is 68% of 25.

Year 7 Mathematics Knowledge Organiser – Unit 7: Solving equations

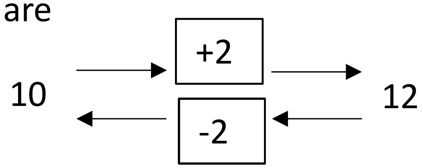


EQUATION is algebraic expression with equal sign, which can be solved (value of variable is found).

INVERSE OPERATIONS are opposite operations – one reverses the effect of the other.

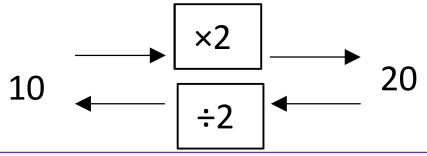
Two basic pairs of inverse operations are

ADDITION and **SUBTRACTION**



Example: $10 + 2 - 2 = 10$

MULTIPLICATION and **DIVISION**



Example: $10 \times 2 \div 2 = 10$

SOLVING ONE STEP EQUATIONS

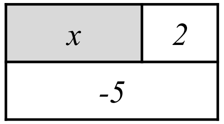
An equation is like a balance scale because it shows that two quantities are equal.

What you do to one side of the equation must also be done to the other side to keep it balanced.

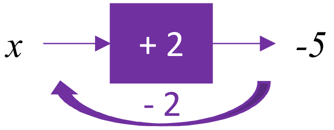
To solve one step equations, you need to ask three questions about the equation:

- What is the variable?
- What operation is performed on the variable?
- What is the inverse operation?

Example: Solve $x + 2 = -5$



What is the variable?	x
What operation is performed on the variable?	$+ 2$
What is the inverse operation?	-2

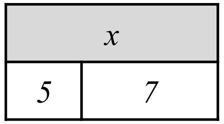


$x + 2 = -5$

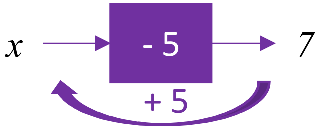
$-2 \qquad -2$

$\underline{\boldsymbol{x = -7}}$

Example: Solve $x - 5 = 7$



What is the variable?	x
What operation is performed on the variable?	$- 5$
What is the inverse operation?	$+ 5$



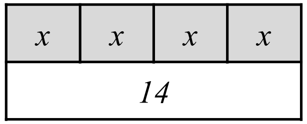
$x - 5 = 7$

$+5 \qquad +5$

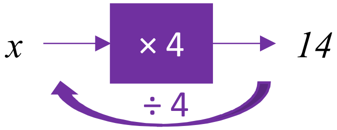
$\underline{\boldsymbol{x = 12}}$

Always indicate your steps (inverse operations).
Write your steps one under another, preferably aligning the equal symbols.

Example: Solve $4x = 14$



What is the variable?	x
What operation is performed on the variable?	$\times 4$
What is the inverse operation?	$\div 4$

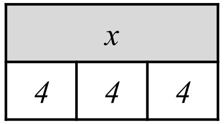


$4x = 14$

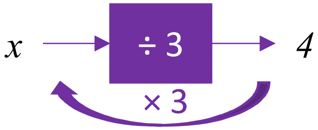
$\div 4 \qquad \div 4$

$\underline{\boldsymbol{x = 3.5}}$

Example: Solve $\frac{x}{3} = 4$



What is the variable?	x
What operation is performed on the variable?	$\div 3$
What is the inverse operation?	$\times 3$



$\frac{x}{3} = 4$

$\times 3 \qquad \times 3$

$\underline{\boldsymbol{x = 12}}$

Year 7 Mathematics Knowledge Organiser – Unit 7: Solving equations

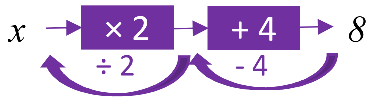
SOLVING 2-STEP EQUATIONS

Solving a two-step equation involves working backwards concerning the order of operations, using inverse operations.

You can always imagine equation as function machine or bar model to help you understand what is happening with variable.

Example: Solve $2x + 4 = 8$

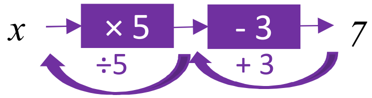
x	x	4
8		



$$\begin{array}{rcl} 2x + 4 &=& 8 \\ -4 && -4 \\ 2x &=& 4 \\ \div 2 && \div 2 \\ \underline{x} &\underline{=}& \underline{2} \end{array}$$

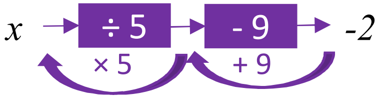
Example: Solve $5x – 3 = 7$

x	x	x	x	x
3	7			



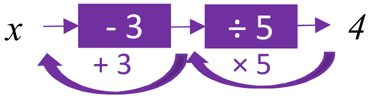
$$\begin{array}{rcl} 5x – 3 &=& 7 \\ +3 && +3 \\ 5x &=& 10 \\ \div 5 && \div 5 \\ \underline{x} &\underline{=}& \underline{2} \end{array}$$

Example: Solve $\frac{x}{5} - 9 = -2$



$$\begin{array}{rcl} \frac{x}{5} – 9 &=& -2 \\ +9 && +9 \\ \frac{x}{5} &=& 7 \\ \times 5 && \times 5 \\ \underline{x} &\underline{=}& \underline{35} \end{array}$$

Example: Solve $\frac{x-3}{5} = 4$



$$\begin{array}{rcl} \frac{x-3}{5} &=& 4 \\ \times 5 && \times 5 \\ x – 3 &=& 20 \\ +3 && +3 \\ \underline{x} &\underline{=}& \underline{23} \end{array}$$

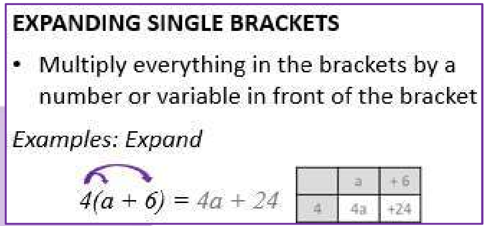
SOLVING EQUATIONS WITH BRACKETS

Expand the brackets and solve as 2-step equation.

Example: Solve $3(2x – 1) = 13$

1. Expand brackets
2. Solve as 2-step equation

$$\begin{array}{rcl} 3(2x – 1) &=& 13 \\ 6x – 3 &=& 13 \\ +3 && +3 \\ 6x &=& 16 \\ \div 6 && \div 6 \\ x = \frac{16}{6} &=& 2\frac{2}{3} \end{array}$$



EXPANDING SINGLE BRACKETS

- Multiply everything in the brackets by a number or variable in front of the bracket

Examples: Expand

$4(a + 6) = 4a + 24$

	a	+ 6
4	4a	+24

If needed, leave your answers as fractions rather than decimals (never leave your answer as recurring decimal). **Check your answer** by substitution.

SOLVING EQUATIONS WITH UNKNOWN ON BOTH SIDES

Rearrange all the variables onto one side of the equation (the one with more variables) and all numbers onto the other side.

Example: Solve $5x + 3 = 15 + 2x$

x	x	x	x	x	3
x	x	15			

$$\begin{array}{rcl} 5x + 3 &=& 15 + 2x \\ -2x && -2x \\ 3x + 3 &=& 15 \\ -3 && -3 \\ 3x &=& 12 \\ \div 3 && \div 3 \\ \underline{x} &\underline{=}& \underline{4} \end{array}$$

Example: Solve $22 – 3x = 2 + 2x$

22		
3x	2x	2

$$\begin{array}{rcl} 22 – 3x &=& 2 + 2x \\ +3x && +3x \\ 22 &=& 2 + 5x \\ -2 && -2 \\ 20 &=& 5x \\ \div 5 && \div 5 \\ 4 = x & \text{or} & \underline{x = 4} \end{array}$$

Year 7 Mathematics Knowledge Organiser – Unit 7: Solving equations

FORM AND SOLVE EQUATIONS

Example: I think of a number. When I add 3 and multiply the result by 2, the answer is 11. Form the equation. $2(x + 3) = 11$

Example: After tennis training, Andy collects twice as many tennis balls as Roger and five more than Maria. They collect 35 tennis balls in total. How many tennis balls does Andy collect?

Let x be the number of balls Roger collects. Then
Andy collects $2x$ balls and
Maria collects $2x – 5$.

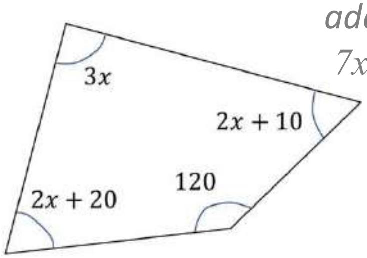
Total balls collected: $x + 2x + 2x – 5 = 35$

$5x – 5 = 35$

$x = 8$

So Andy collected $2 \times x = 2 \times 8 = 16$ balls.

Example: Find x



Angles in the quadrilateral add up to 360°.

$7x + 150 = 360$

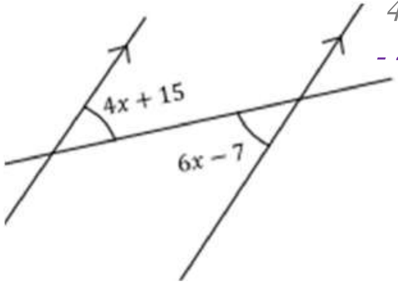
$-150 \qquad -150$

$7x = 210$

$\div 7 \qquad \div 7$

$x = 30°$

Example: Find x



Alternate angles are equal.

$4x + 15 = 6x – 7$

$-4x \qquad -4x$

$15 = 2x – 7$

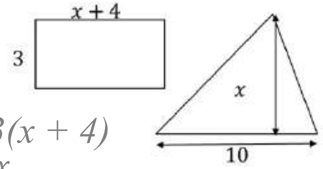
$+7 \qquad +7$

$22 = 2x$

$\div 2 \qquad \div 2$

$x = 11$

Example: The rectangle and the triangle have the same area. Determine the width of the rectangle.



Area of rectangle = $3(x + 4)$

Area of triangle = $\frac{10x}{2}$

$3(x + 4) = \frac{10x}{2}$

$3x + 12 = 5x$

$-3x \qquad -3x$

$12 = 2x$

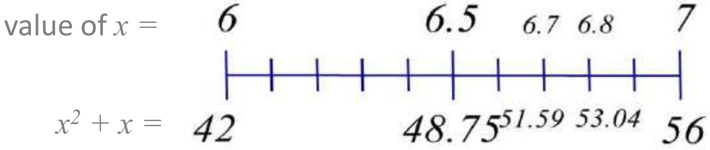
$\div 2 \qquad \div 2$

$x = 6$

Width of the rectangle = $x + 4 = 6 + 4 = 10$

TRIAL AND IMPROVEMENT

Example: Solve $x^2 + x = 53$ *correct to 1 decimal place*



- First find the two whole number values of x that will give the closest answers to 53: one will be a bit too small, the other will be a bit too big.

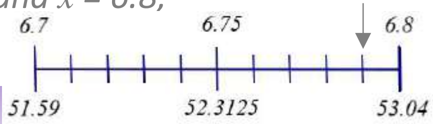
when $x = 6$ $\quad x^2 + x = 6^2 + 6 = 42$ $\quad$ too small
when $x = 7$ $\quad x^2 + x = 7^2 + 7 = 56$ $\quad$ too big
- answer is between 6 and 7

- Then ZOOM in between these two numbers to find the **two** numbers with one decimal place that give the closest answers to 53: again one will be a bit too small, the other will be a bit too big.

when $x = 6.5$ $\quad x^2 + x = 6.5^2 + 6.5 = 48.75$ $\quad$ too small
when $x = 6.8$ $\quad x^2 + x = 6.8^2 + 6.8 = 53.04$ $\quad$ too big
when $x = 6.7$ $\quad x^2 + x = 6.7^2 + 6.7 = 51.59$ $\quad$ too small
- answer is between 6.7 and 6.8

- To work out which one of those to numbers is the BEST answer you need to ZOOM in again to HALFWAY between them, then you will be able to see which one was closest to 53. (This last step is the most commonly missed out part because sometimes it seems unnecessary, but it is important.)

when $x = 6.75$ $\quad x^2 + x = 6.75^2 + 6.75 = 52.3125$
- the answer 53 is to the right side from 52.3125 on the number line, between $x = 6.75$ and $x = 6.8$,
so the answer is $x = 6.8$



Year 7 Mathematics Knowledge Organiser – Unit 8: Multiplicative reasoning

Keywords

Length	is the distance of something measured.
Capacity	how much something can hold (generally liquid), commonly known as VOLUME
Mass	is a measure of how much matter is in an object, commonly known as WEIGHT.
The Imperial system	is a system of weights and measures originally developed in England. It consists of many different units of measurement, which are named differently and have different conversion factors.
The Metric system	is a system of measurement that uses the **meter**, **litre**, and **gram** as base units of length (distance), capacity (volume), and weight (mass) respectively.

CONVERTING BETWEEN IMPERIAL AND METRIC UNITS

LENGTH

1 inch ≈ 2.5 cm

1 foot ≈ 30 cm

1 yard ≈ 0.9 metre

1 mile ≈ 1.6 km

Example: Estimate a length of half a metre in inches?

Half a metre is 50 cm.

1 inch ≈ 2.5 cm.

50 ÷ 2.5 ≈ 20inches

inch	*1*	*2*	*20*
cm	*2.5*	*5*	*50*

×2 ×10

CAPACITY / VOLUME

1 pint ≈ 0.6 litres (l)

1 litre (l) ≈ 1.75 pints (pt)

1 gallon ≈ 4.5 litres.

Example: How many litres are in 8 gallons of petrol?

1 gallon ≈ 4.5 litres

8 gallons = 8 × 4.5 = 36 l

gallon	*1*	*2*	*8*
litre	*4.5*	*9*	*36*

×2 ×4

MASS / WEIGHT

1 kilogram (kg) ≈ 2.2 pounds (lb).

1 pound (lb) ≈ half a kilogram.

1 stone (st) ≈ 6.5 kilograms (kg).

Example: Estimate a mass of 7 pounds (lb) in kilograms (kg).

1 pound (lb) ≈ half a kilogram.

7 lb = 0.5 × 7 = 3.5 kg

pound	*1*	*2*	*6*	*7*
kg	*0.5*	*1*	*3*	*3.5*

×2 ×3

To convert from pounds to kg: **divide** by 2.2.

÷ 2.2

1kg ≈ 2.2 pounds

× 2.2

To convert from kg to pounds: **multiply** by 2.2.

CONVERTING METRIC UNITS

LENGTH - basic unit is metre (m)

1 cm = 10 mm

1 m = 100 cm

1 km = 1000 m

Example: Convert 3 m to millimetres.

3 m = 300 cm = 3000 mm

× 100 × 10

CAPACITY - basic unit is litre (l)

1 l = 1000 ml

Example: Convert 5700 ml to litres.

5700 ml = 5.7 l

÷ 1000

metre	gram	litre
1m = 1000 mm	1g = 1000mg	1l = 1000 ml
1 km = 1000 m	1 kg = 1000 g	

MASS - basic unit is gram (g)

1 g = 1000 mg

1 kg = 1000 g

1 t = 1000 kg

Example: Convert 8700 mg to kilograms.

8700 mg = 8.7 g = 0.0087 kg

÷ 1000 ÷ 1000

Prefix	Meaning	Length	Capacity	Mass
kilo-	thousand (1,000)	kilometre		kilogram
hecto-	hundred (100)			
deka-	ten (10)			dekagram
BASE UNIT	**ones (1)**	**metre**	**litre**	**gram**
deci-	tenths (0.1)		decilitre	
centi-	hundredths (0.01)	centimetre		
milli-	thousandths (0.001)	millimetre	millilitre	milligram

Year 7 Mathematics Knowledge Organiser – Unit 8: Multiplicative reasoning

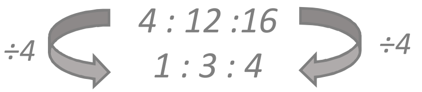
Keywords

A **ratio** shows the relative sizes of two or more values.

SIMPLIFYING RATIO

To simplify ratio, divide all parts of the ratio by the highest common factor.

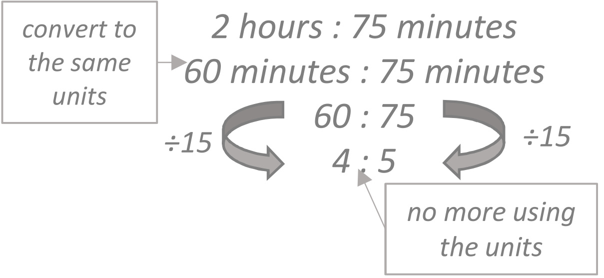
Example: Simplify ratio 4 : 12 : 16.



÷4 *4 : 12 :16* ÷4

1 : 3 : 4

Example: Simplify ratio 2 hours : 75 minutes.



2 hours : 75 minutes

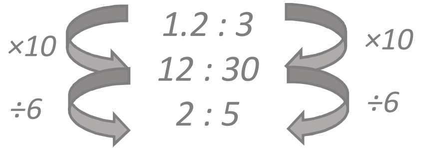
convert to the same units → *60 minutes : 75 minutes*

÷15 *60 : 75* ÷15

4 : 5 ← no more using the units

Example: Simplify ratio 1.2 : 3.

1. *Convert to whole numbers.*
2. *Simplify.*



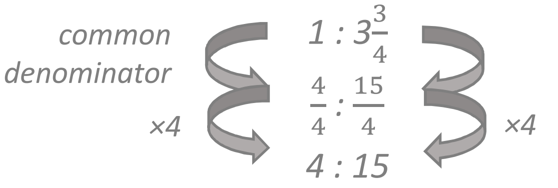
×10 *1.2 : 3* ×10

12 : 30

÷6 *2 : 5* ÷6

Example: Simplify ratio 1 : $3\frac{3}{4}$.

1. *Convert to fractions with common denominator.*
2. *Simplify.*



common denominator *1 : $3\frac{3}{4}$*

$\frac{4}{4}$: $\frac{15}{4}$

×4 *4 : 15* ×4

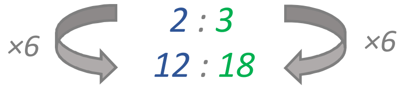
SHARING IN THE RATIO

A ratio can also be used to share a quantity into parts.

Example: Share 30 sweets into a ratio of 2 : 3.

Total amount of parts = 2 + 3 = 5.

If 30 sweets are shared equally in 5 parts, each part contains 30 ÷ 5 = 6 sweets.

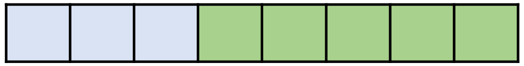


×6 *2 : 3* ×6

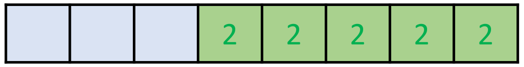
12 : 18

Example: Alison and Henry shared some money in a ratio of 3 : 5. Henry got £10 as a result. How much money did they share?

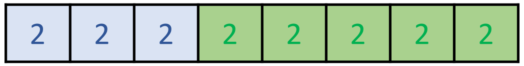
Total amount of parts = 3 + 5 = 8.



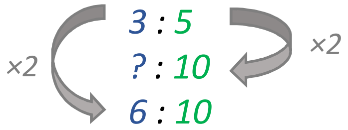
If Henry received £10, this amount is shared in 5 parts. One part is worth 10 ÷ 5 = £2.



Therefore Alison received 3 × £2 = £6.



They shared £10 + £6 = £2 × 8 = £16.



×2 *3 : 5* ×2

? : 10

6 : 10

RELATIONSHIP BETWEEN RATIO AND PROPORTION

RATIO is a **part to part comparison**.

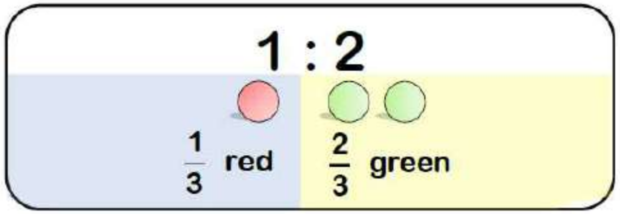
PROPORTION is a **part to whole comparison.**

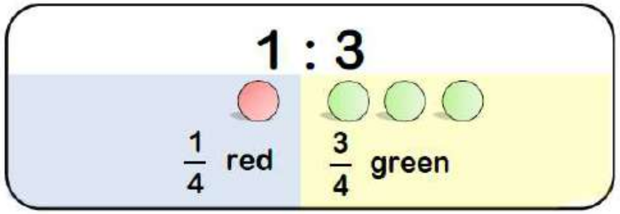
Proportions can be expressed as fractions or percentages.

Example: To make the colour I need for my painting I mix the blue and red paint in the ratio 3 : 7. What fraction is blue paint? What percentage is red paint?

Blue paint is represented by 3 parts out of 10, therefore the proportion of blue paint is $\frac{3}{10}$.

The rest of the paint is red = $\frac{7}{10}$, so the red paint makes 70% of the whole paint.



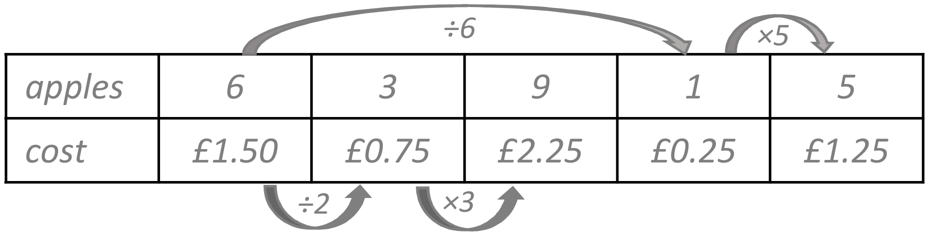


Year 7 Mathematics Knowledge Organiser – Unit 8: Multiplicative reasoning

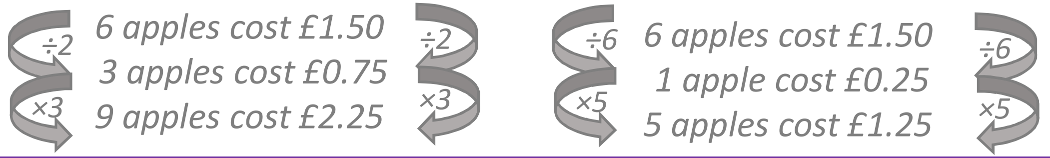
SOLVING SIMPLE DIRECT PROPORTION WORD PROBLEMS

Two quantities are said to be in DIRECT PROPORTION if they increase or decrease at the same rate. That is, if the ratio between the two quantities is always the same.

Example: The cost of 6 apples is £1.50. What is the cost for 9 apples? What would be the cost of 5 apples?



apples	*6*	*3*	*9*	*1*	*5*
cost	*£1.50*	*£0.75*	*£2.25*	*£0.25*	*£1.25*



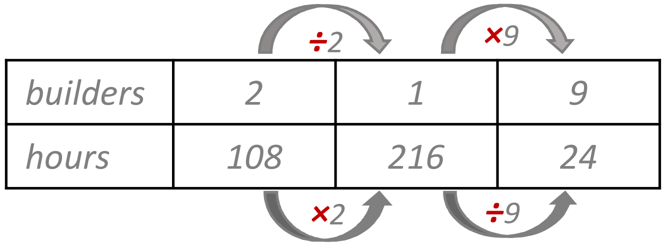
6 apples cost £1.50
3 apples cost £0.75
9 apples cost £2.25

6 apples cost £1.50
1 apple cost £0.25
5 apples cost £1.25

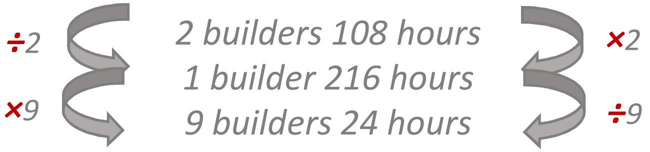
SOLVING SIMPLE INVERSE PROPORTION WORD PROBLEMS

Two quantities are said to be INVERSELY PROPORTIONAL if, as one quantity increases, the other quantity decreases at the same rate.

Example: I want to build a wall in my garden. 2 builders would do it in 108 hours. How long would it take 9 builders to build the wall?



builders	*2*	*1*	*9*
hours	*108*	*216*	*24*

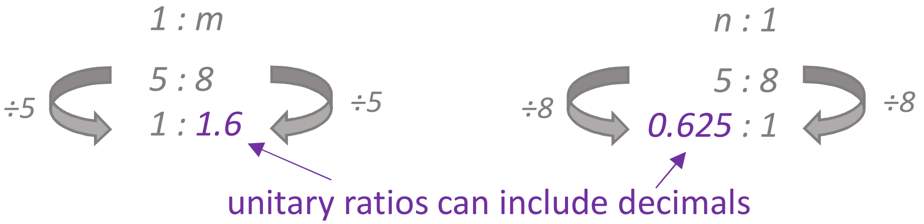


2 builders 108 hours
1 builder 216 hours
9 builders 24 hours

USING UNITARY METHOD

Using unitary method means find the value of 1 part.

Example: Write the ratio 5 : 8 in the form 1 : m and n : 1.



1 : m
5 : 8
1 : 1.6

n : 1
5 : 8
0.625 : 1

unitary ratios can include decimals

BEST BUYS

We can compare the value of products using the UNITARY METHOD, either through

- the cost per unit amount, or
- the amount per unit cost.

Example: Kayden buys an ice cream from Mr Whipcream. He gets 200ml of ice cream for £1.50. Lilly buys an ice cream from Iced Delights. She gets 300ml of ice cream for £2.40.
Who gets better value for money?

<u>Cost per unit amount</u> method:
Mr Whipcream: 1.50 ÷ 200 = £0.0075 per ml →
£1.50 : 200 ml
£0.0075 : 1 ml

Iced Delights: 2.40 ÷ 300 = £0.008 per ml →
£2.40 : 300 ml
£0.008 : 1 ml

Mr Whipcream is cheaper per ml, therefore better value for money.

<u>Amount per unit cost</u> method:
Mr Whipcream: 200 ÷ 1.50 = 133.33 ml per £ →
£1.50 : 200 ml
£1 : 133.33 ml

Iced Delights: 300 ÷ 2.40 = 125 ml per £ →
£2.40 : 300ml
£1 : 125 ml

Mr Whipcream gives more ice cream per £, therefore better value for money.