

4723 Core Mathematics 3

<p>1 (i) Show correct process for composition of functions</p> <p>Obtain $(-3$ and hence) -23</p>	<p>M1 numerical or algebraic; the right way round</p> <p>A1 2</p>
<p>(ii) <u>Either</u>: State or imply $x^3 + 4 = 12$</p> <p>Attempt solution of equation involving x^3</p> <p>Obtain 2</p> <p><u>Or</u>: Attempt expression for f^{-1}</p> <p>Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$</p> <p>Obtain 2</p>	<p>B1</p> <p>M1 as far as $x = \dots$</p> <p>A1 3 and no other value</p> <p>M1 involving x or y; involving cube root</p> <p>A1</p> <p>A1 (3) and no other value</p>
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<p>2 (i) Obtain correct first iterate 2.864</p> <p>Carry out correct iteration process</p> <p>Obtain 2.877</p> <p style="text-align: center;">$[3 \rightarrow 2.864327 \rightarrow 2.878042 \rightarrow 2.876661 \rightarrow 2.876800]$</p>	<p>B1 or greater accuracy 2.864327...; condone 2 dp here and in working</p> <p>M1 to find at least 3 iterates in all</p> <p>A1 3 after at least 4 steps; answer required to exactly 3 dp</p>
<p>(ii) State or imply $x = \sqrt[3]{31 - \frac{5}{2}x}$</p> <p>Attempt rearrangement of equation in x</p> <p>Obtain equation $2x^3 + 5x - 62 = 0$</p>	<p>B1</p> <p>M1 involving cubing and grouping non-zero terms on LHS</p> <p>A1 3 or equiv with integers</p>
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<p>3 (a) State correct equation involving $\cos \frac{1}{2}\alpha$</p> <p>Attempt to find value of α</p> <p>Obtain 151</p>	<p>B1 such as $\cos \frac{1}{2}\alpha = \frac{1}{4}$ or $\frac{1}{\cos \frac{1}{2}\alpha} = 4$</p> <p>or ...</p> <p>M1 using correct order for the steps</p> <p>A1 3 or greater accuracy; and no other values between 0 and 180</p>
<p>(b) State or imply $\cot \beta = \frac{1}{\tan \beta}$</p> <p>Rearrange to the form $\tan \beta = k$</p> <p>Obtain 69.3</p> <p>Obtain 111</p>	<p>B1</p> <p>M1 or equiv involving $\sin \beta$ only or $\cos \beta$ only; allow missing \pm</p> <p>A1</p> <p>A1 4 or greater accuracy; and no others between 0 and 180</p>
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<p>4 (i) Obtain derivative of form $kh^5(h^6 + 16)^n$</p> <p>Obtain correct $3h^5(h^6 + 16)^{-\frac{1}{2}}$</p> <p>Substitute to obtain 10.7</p>	<p>M1 any constant k; any $n < \frac{1}{2}$; allow if -4 term retained</p> <p>A1 or (unsimplified) equiv; no -4 now</p> <p>A1 3 or greater accuracy or exact equiv</p>
<p>(ii) Attempt multn or divn using 8 and answer from (i) M1</p> <p>Attempt 8 divided by answer from (i)</p> <p>Obtain 0.75</p>	<p>M1</p> <p>A1 $\sqrt{3}$ or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)</p>

5 (a)	Obtain integral of form $k(3x + 7)^{10}$	M1	any constant k
	Obtain (unsimplified) $\frac{1}{10} \times \frac{1}{3} (3x + 7)^{10}$	A1	or equiv
	Obtain (simplified) $\frac{1}{30} (3x + 7)^{10} + c$	A1 3	
(b)	State $\int \pi \left(\frac{1}{2\sqrt{x}}\right)^2 dx$	B1	or equiv involving x ; condone no dx
	Integrate to obtain $k \ln x$	M1	any constant k involving π or not; or equiv such as $k \ln 4x$ or $k \ln 2x$
	Obtain $\frac{1}{4}\pi \ln x$ or $\frac{1}{4} \ln x$ or $\frac{1}{4}\pi \ln 4x$ or $\frac{1}{4} \ln 4x$	A1	
	Show use of the $\log a - \log b$ property	M1	not dependent on earlier marks
	Obtain $\frac{1}{4}\pi \ln 2$	A1 5	or similarly simplified equiv
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6 (i)	<u>Either:</u> Refer to translation and reflection State translation by 1 in negative x -direction	B1 B1	in either order; allow clear equivs or equiv but now using correct terminology
	State reflection in x -axis	B1 3	using correct terminology
	<u>Or:</u> Refer to translation and reflection State reflection in y -axis State translation by 1 in positive x -direction	B1 B1 B1 (3)	in either order; allow clear equivs with order reflection then translation clearly intended
(ii)	Show sketch with attempt at reflection of 'negative' part in x -axis Show (more or less) correct sketch	M1 A1 2	and curve for $0 < x < 1$ unchanged with correct curvature
(iii)	Attempt correct process for finding at least one value	M1	as far as $x = \dots$; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$
	Obtain $1 - \frac{1}{2}\sqrt{3}$	A1	or exact equiv
	Obtain $1 + \frac{1}{2}\sqrt{3}$	A1 3	or exact equiv; give A1A0 if extra incorrect solution(s) provided
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7 (i)	Attempt use of product rule for $x e^{2x}$	M1	obtaining $\dots + \dots$
	Obtain $e^{2x} + 2x e^{2x}$	A1	or equiv; maybe within QR attempt
	Attempt use of quotient rule	M1	with or without product rule
	Obtain unsimplified $\frac{(x+k)(e^{2x} + 2xe^{2x}) - xe^{2x}}{(x+k)^2}$	A1	
	Obtain $\frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$	A1 5	AG; necessary detail required
(ii)	Attempt use of discriminant	M1	or equiv
	Obtain $4k^2 - 8k = 0$ or equiv and hence $k = 2$	A1	
	Attempt solution of $2x^2 + 2kx + k = 0$	M1	using their numerical value of k or solving in terms of k using correct formula
	Obtain $x = -1$	A1	
	Obtain $-e^{-2}$	A1 5	or exact equiv

8 (i)	State or imply $h = 1$ Attempt calculation involving attempts at y values Obtain $a(1 + 4 \times 2 + 2 \times 4 + 4 \times 8 + 2 \times 16 + 4 \times 32 + 64)$ Obtain 91	B1 M1 A1 4	addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values any constant a
(ii)	State $e^{x \ln 2}$ or $k = \ln 2$ Integrate e^{kx} to obtain $\frac{1}{k} e^{kx}$ Obtain $\frac{1}{\ln 2} (e^{6 \ln 2} - e^0)$ Simplify to obtain $\frac{63}{\ln 2}$	B1 M1 A1 A1 4	allow decimal equiv such as $e^{0.69x}$ any constant k or in terms of general k or exact equiv allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii) Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	M1 A1 2	provided $\ln 2$ involved other than in power of e AG ; necessary correct detail required
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9 (i)	State at least one of $\cos \theta \cos 60 - \sin \theta \sin 60$ and $\cos \theta \cos 30 - \sin \theta \sin 30$ Attempt complete multiplication of identities of form $\pm \cos \cos \pm \sin \sin$ Use $\cos^2 \theta + \sin^2 \theta = 1$ and $2 \sin \theta \cos \theta = \sin 2\theta$ Obtain $\sqrt{3} - 2 \sin 2\theta$	B1 M1 M1 A1 4	with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved AG ; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side Obtain $\sqrt{3} - \sqrt{2}$	M1 A1 2	or exact equiv
(iii)	Obtain 10.7 Attempt correct process to find two angles Obtain 79.3	B1 M1 A1 3	or greater accuracy; allow ± 0.1 from values of 2θ between 0 and 180 or greater accuracy and no others between 0 and 90; allow ± 0.1
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are -1 and 1 Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$ Obtain complete correct solution	M1 A1 A1 3	condoning decimal equivs, $\leq \geq$ signs now with exact values and unambiguously stated