NAME:

Date to be handed in:

MARK (out of 100):

PAPER P

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1 The first 3 terms of a geometric sequence are $k+2, 4k, 2k^2,\ k>0$

Find the value of k. **(4 marks)**

---

2 The volume of a sphere V cm^3 is related to its radius r cm by the formula $V=\frac{4}{3}\pi r^3$

The surface area of the sphere is also related to the radius by the formula $S=4\pi r^2$

Given that the rate of decrease in surface area, in cm^2 s^{-1}, is $\frac{dS}{dt}=-12$ find the rate decrease of volume $\frac{dV}{dt}$

(4 marks)

---

3 A curve C has parametric equations $x=\sec^2 t+1,\ y=2\sin t, -\frac{\pi}{4}\leqslant t\leqslant\frac{\pi}{4}$

Show that a cartesian equation of C is $y=\sqrt{\frac{8-4x}{1-x}}$ for a suitable domain which should be stated.

(4 marks)

---

4 Show that $\frac{6(x+7)}{(5x-1)(2x+5)}$ can be written in the form $\frac{A}{5x-1}+\frac{B}{2x+5}$

Find the values of the constants A and B. **(5 marks)**

---

5 A curve C has equation $4^x=2xy$ for $x>0$

Find the exact value of $\frac{dy}{dx}$ at the point C with coordinates (2, 4). **(5 marks)**

---

6 Find the values of the constants A, B, C, D and E in the following identity:

$$5x^4-4x^3+17x^2-5x+7\equiv(Ax^2+Bx+C)(x^2+2)+Dx+E$$ **(5 marks)**

---

7 Given that in the expansion of $\frac{1}{(1+ax)^2}$ the coefficient of the x^2 term is 75 find:

a the possible values of a **(4 marks)**

b the corresponding coefficients of the x^3 term. **(2 marks)**

---

8 A sequence is given by $x_1=4$, $x_{n+1}=px_n-9$ where p is an integer.

a Show that $x_3=4p^2-9p-9$ **(2 marks)**

b Given that $x_3=46$, find the value of p. **(3 marks)**

c Hence find the value of x_5. **(1 mark)**

---

9 The coordinates of A and B are $(-1, 7, k)$ and $(4, 1, 10)$ respectively.

a Given that the distance from A to B is $5\sqrt{5}$ units, find the possible values of the constant k. **(3 marks)**

b For the larger value of k, find the unit vector in the direction of $\overrightarrow{OA}$. **(3 marks)**

---

10 The functions f and g are defined by $f(x) = e^{2x} + 4$, $x \in \mathbb{R}$ and $g(x) = \ln(x+1)$, $x \in \mathbb{R}, x > -1$

a Find fg(x) and state its range. **(4 marks)**

b Solve fg(x) = 85 **(3 marks)**

---

11 $f(x) = x^4 - 8x^2 + 2$

a Show that the equation f(x) = 0 can be written as $x = \sqrt{ax^4 + b}, x > 0$, where a and b are constants to be found. **(2 marks)**

b Let $x_0 = 1.5$. Use the iteration formula $x_{n+1} = \sqrt{ax_n^4 + b}$, together with your values of a and b from part **a**, to find, to 4 decimal places, the values of x_1, x_2, x_3 and x_4. **(2 marks)**

A root of f(x) = 0 is α.

c By choosing a suitable interval, prove that $\alpha = -2.782$ to 3 decimal places. **(3 marks)**

---

12 **a** Show that $\tan^4 x \equiv \sec^2 x \tan^2 x + 1 - \sec^2 x$ **(4 marks)**

b Hence find the exact value of $\int_0^{\frac{\pi}{4}} \tan^4 x \, dx$ **(5 marks)**

---

13 **a** Use proof by contradiction to show that if n^2 is an even integer then n is also an even integer. **(4 marks)**

b Prove that $\sqrt{2}$ is irrational. **(6 marks)**

---

14 The value of a computer, V, decreases over time, t, measured in years.
The rate of decrease of the value is proportional to the remaining value.

a Given that the initial value of the computer is V_0, show that $V = V_0 e^{-kt}$ **(4 marks)**

After 10 years the value of the computer is $\frac{1}{5}V_0$

b Find the exact value of k. **(3 marks)**

c How old is the computer when its value is only 5% of its original value?
Give your answer to 3 significant figures. **(3 marks)**

---

15 **a** Express $5\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \pi$
Write R in surd form and give the value of α correct to 4 decimal places. **(4 marks)**

The temperature of a kiln, $T°C$, used to make pottery can be modelled by the equation $T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant 72$ where x is the time in hours since the pottery was placed in the kiln.

b Calculate the maximum value of T predicted by this model and the value of x, to 2 decimal places, when this maximum first occurs. **(4 marks)**

c Calculate the times during the first 24 hours when the temperature is predicted, by this model, to be exactly 1097 °C. **(4 marks)**

(TOTAL: 100 MARKS)