**PURE MATHEMATICS**
**A level Practice Papers**

**PAPER Q**
**MARK SCHEME**

| | | |
|---|---|---|
| **1** | Correctly factorises the denominator of the left-hand fraction: $\frac{6}{(2x+5)(2x-1)}+\frac{3x+1}{2x-1}$ | **M1** |
| | Multiplies the right-hand fraction by $\frac{2x+5}{2x+5}$ <br> For example: $\frac{6}{(2x+5)(2x-1)}+\frac{(3x+1)(2x+5)}{(2x-1)(2x+5)}$ is seen. | **M1** |
| | Makes an attempt to distribute the numerator of the right-hand fraction. <br> For example: $\frac{6+6x^2+17x+5}{(2x+5)(2x-1)}$ is seen. | **M1** |
| | Fully simplified answer is seen. <br> Accept either $\frac{6x^2+17x+11}{(2x+5)(2x-1)}$ or $\frac{(6x+11)(x+1)}{(2x+5)(2x-1)}$ | **A1** |
| | **TOTAL: 4 marks** | |

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| **2a** | Uses $a_n=a+(n-1)d$ substituting $a$ = 5 and $d$ = 3 to get $a_n=5+(n-1)3$ | **M1** |
| | Simplifies to state $a_n=3n+2$ | **A1** |
| | | **(2 marks)** |
| **2b** | Use the sum of an arithmetic series to state $\frac{k}{2}\left[10+(k-1)3\right]=948$ | **M1** |
| | States correct final answer $3k^2+7k-1896=0$ | **A1** |
| | | **(2 marks)** |
| | **TOTAL: 4 marks** | |

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| **3a** | Deduces from $3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1 = 0$ that $3\sin\left(\frac{x}{6}\right)^3 = \frac{1}{10}x + 1$ | **M1** |
| | States $\left(\frac{x}{6}\right)^3 = \arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)$ | **M1** |
| | Multiplies by $6^3$ and then takes the cube root: $x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$ | **A1** |
| | | **(3 marks)** |
| **3b** | Attempts to use iterative procedure to find subsequent values. | **M1** |
| | Correctly finds: $x_1 = 4.716$ $x_2 = 4.802$ $x_3 = 4.812$ $x_4 = 4.814$ | **A1** |
| | | **(2 marks)** |
| | **TOTAL: 5 marks** | |

**NOTES: 3b**

Award M1 if finds at least one correct answer.

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| **4a** | Shows that $2\cos 3\theta \approx 2\left(1 - \frac{9\theta^2}{2}\right) = 2 - 9\theta^2$ | **M1** |
| | Shows that $2\cos 3\theta - 1 \approx 1 - 9\theta^2 = (1 - 3\theta)(1 + 3\theta)$ | **M1** |
| | Shows $1 + \sin\theta + \tan 2\theta = 1 + \theta + 2\theta = 1 + 3\theta$ | **M1** |
| | Recognises that $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1} \approx \frac{1+3\theta}{(1-3\theta)(1+3\theta)} = \frac{1}{1-3\theta}$ | **A1** |
| | | **(4 marks)** |
| **4b** | When $\theta$ is small, $\frac{1}{1-3\theta} \approx 1$ | **A1** |
| | | **(1 mark)** |
| | **TOTAL: 5 marks** | |

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| **5a** | Writes out the first $n$ terms of the arithmetic sequence in both ascending and descending form<br>$S = a + (a+d) + (a+2d) + \ldots + (a+(n-1)d)$<br>$S = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \ldots + a$ | **M1** |
| | Attempts to add these two sequences $2S = (2a + (n-1)d) \times n$ | **M1** |
| | States $S = \frac{n}{2}(2a + (n-1)d)$ | **A1** |
| | | **(3 marks)** |
| **5b** | Makes an attempt to find the sum. For example, $S = \frac{200}{2}(2 + 199(2))$ is seen. | **M1** |
| | States correct final answer. $S = 40\,000$ | **A1** |
| | | **(2 marks)** |
| | **TOTAL: 5 marks** | |

**NOTES:** **5a** Do not award full marks for an incomplete proof.

**5a** Do award second method mark if student indicates that $(2a + (n - 1)d$ appears $n$ times.

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| **6** | Makes an attempt to set up a long division. For example: $x+6\overline{)x^3+8x^2-9x+12}$ is seen. | **M1** |
| | $$\begin{array}{r} x^2+2x-21 \\ x+6\overline{)x^3+8x^2-9x+12} \\ \underline{x^3+6x^2} \\ 2x^2-9x \\ \underline{2x^2+12x} \\ -21x+12 \\ \underline{-21x-126} \\ 138 \end{array}$$<br>Award 1 accuracy mark for each of the following:<br>$x^2$ seen, $2x$ seen, $-21$ seen.<br>For the final accuracy mark<br>either $D = 138$ or $\frac{138}{x+6}$<br>or the remainder is 138 must be seen. | **A4** |
| | **TOTAL: 5 marks** | |

**NOTES:**

This question can be solved by first writing $(Ax^2 + Bx + C)(x+6) + D \equiv x^3 + 8x^2 - 9x + 12$ and then solving for $A$, $B$, $C$ and $D$. Award 1 mark for the setting up the problem as described. Then award 1 mark for each correct coefficient found. For example:

Equating the coefficients of $x^3$: $A = 1$

Equating the coefficients of $x^2$: $6 + B = 8$, so $B = 2$

Equating the coefficients of $x$: $12 + C = -9$, so $C = -21$

Equating the constant terms: $-126 + D = 12$, so $D = 138$

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| **7** | Begins the proof by assuming the opposite is true.<br>'Assumption: there is a finite amount of prime numbers.' | **B1** |
| | Considers what having a finite amount of prime numbers means by making an attempt to list them: Let all the prime numbers exist be $p_1, p_2, p_3, \ldots p_n$ | **M1** |
| | Consider a new number that is one greater than the product of all the existing prime numbers:<br>Let $N = (p_1 \times p_2 \times p_3 \times \ldots \times p_n) + 1$ | **M1** |
| | Understands the implication of this new number is that division by any of the existing prime numbers will leave a remainder of 1. So none of the existing prime numbers is a factor of *N*. | **M1** |
| | Concludes that either *N* is prime or *N* has a prime factor that is not currently listed. | **B1** |
| | Recognises that either way this leads to a contradiction, and therefore there is an infinite number of prime numbers. | **B1** |
| | **TOTAL: 6 marks** | |

**NOTES:** If *N* is prime, it is a new prime number separate to the finite list of prime numbers, $p_1, p_2, p_3, \ldots p_n$.

If *N* is divisible by a previously unknown prime number, that prime number is also separate to the finite list of prime numbers.

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| 8 | Makes an attempt to differentiate $y = \ln 3x$ using the chain rule, or otherwise. | **M1** |
| | Differentiates $y = \ln 3x - e^{-2x}$ to obtain $\frac{dy}{dx} = \frac{1}{x} + 2e^{-2x}$ | **A1** |
| | Evaluates $\frac{dy}{dx}$ at $x = 1$ $\frac{dy}{dx} = 1 + \frac{2}{e^2} = \frac{e^2 + 2}{e^2}$ | **A1** |
| | Evaluates $y = \ln 3x - e^{-2x}$ at $x = 1$ $y = \ln 3 - e^{-2} = \ln 3 - \frac{1}{e^2}$ | **M1** |
| | Attempts to substitute values into $y - y_1 = m(x - x_1)$ E.g. $y - \ln 3 + \frac{1}{e^2} = \left(\frac{e^2 + 2}{e^2}\right)(x - 1)$ is seen. | **M1 ft** |
| | Shows logical progression to simplify algebra, arriving at: $y = \left(\frac{e^2 + 2}{e^2}\right)x - \left(\frac{e^2 + 3}{e^2}\right) + \ln 3$ | **A1** |
| | **TOTAL: 6 marks** | |

**NOTES:** Award ft marks for a correct attempt to substitute into the formula using incorrect values.

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| 9a | Clearly states that $\int \frac{6}{x}\,\mathrm{d}x = 6\ln x$ | **A1** |
| | Makes an attempt to integrate the remaining two terms.<br>Raising a power by 1 would constitute an attempt. | **M1** |
| | States the fully correct answer $6\ln x - \frac{3}{x} - 2x^{\frac{7}{2}} + C$ | **A1** |
| | | **(3 marks)** |
| 9b | Makes an attempt to substitute the limits into the expression.<br>For example, $\left(6\ln 9 - \frac{3}{9} - 2(2187)\right) - \left(6\ln 4 - \frac{3}{4} - 2(128)\right)$ is seen. | **M1** |
| | Begins to simplify this expression. For example, $6\ln\frac{9}{4} + \frac{5}{12} - 4118$ is seen. | **M1** |
| | States the fully correct answer $-\frac{49\,411}{12} + 6\ln\frac{9}{4}$ or states $m = -\frac{49\,411}{12}$, $n$ = 6 and $p = \frac{9}{4}$<br>Also accept $-\frac{49\,411}{12} + 12\ln\frac{3}{2}$ or equivalent. | **A1** |
| | | **(3 marks)** |
| | **TOTAL: 6 marks** | |

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| 10a | Correctly states $\cos(5x+2x) \equiv \cos 5x\cos 2x - \sin 5x\sin 2x$ | **M1** |
| | Correctly states $\cos(5x-2x) \equiv \cos 5x\cos(-2x) - \sin 5x\sin(-2x)$<br>or states $\cos(5x-2x) \equiv \cos 5x\cos(2x) + \sin 5x\sin(2x)$ | **M1** |
| | Adds the two above expressions and states $\cos 7x + \cos 3x \equiv 2\cos 5x\cos 2x$ | **A1** |
| | | **(3 marks)** |
| 10b | States that $\int(\cos 5x\cos 2x)\,\mathrm{d}x = \frac{1}{2}\int(\cos 7x + \cos 3x)\,\mathrm{d}x$ | **M1** |
| | Makes an attempt to integrate. Changing cos to sin constitutes an attempt. | **M1** |
| | Correctly states the final answer $\frac{1}{14}\sin 7x + \frac{1}{6}\sin 3x + C$ o.e. | **A1** |
| | | **(3 marks)** |
| | **TOTAL: 6 marks** | |

**NOTES: 10b**

Student does not need to state '+C' to be awarded the first method mark.
Must be stated in the final answer.

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| Understands that integration is required to solve the problem.<br><br>For example, writes $\int_{\frac{\pi}{2}}^{\pi}\left(x\sin^2 x\right)\mathrm{d}x$ | **M1** |
| Uses the trigonometric identity $\cos 2x \equiv 1-2\sin^2 x$<br><br>to rewrite $\int_{\frac{\pi}{2}}^{\pi} x\sin^2 x\,\mathrm{d}x$ as $\int_{\frac{\pi}{2}}^{\pi}\left(\frac{1}{2}x-\frac{1}{2}x\cos 2x\right)\mathrm{d}x$ o.e. | **M1** |
| Shows $\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}x\,\mathrm{d}x=\left[\frac{1}{4}x^2\right]_{\frac{\pi}{2}}^{\pi}$ | **A1** |
| Demonstrates an understanding of the need to find $\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}x\cos 2x\,\mathrm{d}x$ using integration by parts.<br><br>For example, $u=x, \frac{\mathrm{d}u}{\mathrm{d}x}=1$<br><br>$\frac{\mathrm{d}v}{\mathrm{d}x}=\cos 2x, v=\frac{1}{2}\sin 2x$ o.e. is seen. | **M1** |
| States fully correct integral<br><br>$\int_{\frac{\pi}{2}}^{\pi}\left(\frac{1}{2}x-\frac{1}{2}x\cos 2x\right)\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{4}x\sin 2x-\frac{1}{8}\cos 2x\right]_{\frac{\pi}{2}}^{\pi}$ | **A1** |
| Makes an attempt to substitute the limits<br><br>$\left(\frac{\pi^2}{4}-\frac{1}{4}(0)-\frac{1}{8}(1)\right)-\left(\frac{\pi^2}{16}-\frac{1}{4}(0)-\frac{1}{8}(-1)\right)$ | **M1** |
| States fully correct answer:<br><br>either $\frac{3\pi^2}{16}-\frac{1}{4}$ or $\frac{3\pi^2-4}{16}$ o.e. | **A1** |
| **TOTAL: 7 marks** | |

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| **12a** Writes $\left(\sin 3\theta + \cos 3\theta\right)^2 \equiv \left(\sin 3\theta + \cos 3\theta\right)\left(\sin 3\theta + \cos 3\theta\right)$ <br> $\equiv \sin^2 3\theta + 2\sin 3\theta \cos 3\theta + \cos^2 3\theta$ | **M1** |
| Uses $\sin^2 3\theta + \cos^2 3\theta \equiv 1$ and $2\sin 3\theta \cos 3\theta \equiv \sin 6\theta$ to write: $\left(\sin 3\theta + \cos 3\theta\right)^2 \equiv 1 + \sin 6\theta$ <br> Award one mark for each correct use of a trigonometric identity. | **A2** |
| | **(3 marks)** |
| **12b** States that: <br> $1 + \sin 6\theta = \frac{2+\sqrt{2}}{2}$ | **B1** |
| Simplifies this to write: <br> $\sin 6\theta = \frac{\sqrt{2}}{2}$ | **M1** |
| Correctly finds $6\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ <br> Additional answers might be seen, but not necessary in order to award the mark. | **M1** |
| States $\theta = \frac{\pi}{24}, \frac{3\pi}{24}$ <br> Note that $\theta \neq \frac{9\pi}{24}, \frac{11\pi}{24}$ <br> For these values $3\theta$ lies in the third quadrant, therefore $\sin 3\theta$ and $\cos 3\theta$ are both negative and cannot be equal to a positive surd. | **A1** |
| | **(4 marks)** |
| **TOTAL: 7 marks** | |

**NOTES: 12b**

Award all 4 marks if correct final answer is seen, even if some of the $6\theta$ angles are missing in the preceding step.

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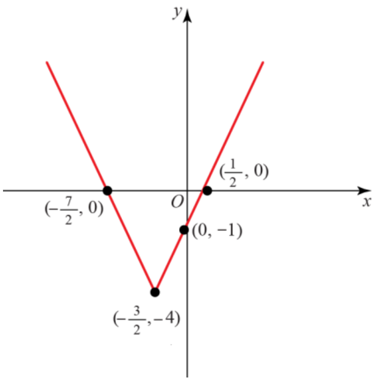
| 13a <br>Graph has a distinct V-shape. | **M1** |
| Labels vertex $\left(-\frac{3}{2}, -4\right)$ | **A1** |
| Finds intercept with the *y*-axis. | **M1** |
| Makes attempt to find *x*-intercept, for example states that $\lvert 2x+3\rvert - 4 = 0$ | **M1** |
| Successfully finds both *x*-intercepts. | **A1** |
| | **(5 marks)** |
| 13b Recognises that there are two solutions.<br>For example, writing $2x+3=-\frac{1}{4}x+2$ and $-(2x+3)=-\frac{1}{4}x+2$ | **M1** |
| Makes an attempt to solve both questions for *x*, by manipulating the algebra. | **M1** |
| Correctly states $x = -\frac{4}{9}$ or $x = -\frac{20}{7}$. Must state both answers. | **A1** |
| Makes an attempt to substitute to find *y*. | **M1** |
| Correctly finds *y* and states both sets of coordinates correctly $\left(-\frac{4}{9}, -\frac{17}{9}\right)$ and $\left(-\frac{20}{7}, -\frac{9}{7}\right)$ | **A1** |
| | **(5 marks)** |
| **TOTAL: 10 marks** | |

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| 14a | Demonstrates an attempt to find the vectors $\overrightarrow{KL}$ , $\overrightarrow{LM}$ and $\overrightarrow{KM}$ | **M1** |
| | Finds $\overrightarrow{KL}=(3,0,-6)$ , $\overrightarrow{LM}=(2,5,4)$ and $\overrightarrow{KM}=(5,5,-2)$ | **A1** |
| | Demonstrates an attempt to find $\lvert\overrightarrow{KL}\rvert$ , $\lvert\overrightarrow{LM}\rvert$ and $\lvert\overrightarrow{KM}\rvert$ | **M1** |
| | Finds $\lvert\overrightarrow{KL}\rvert=\sqrt{(3)^2+(0)^2+(-6)^2}=\sqrt{45}$<br>Finds $\lvert\overrightarrow{LM}\rvert=\sqrt{(2)^2+(5)^2+(4)^2}=\sqrt{45}$<br>Finds $\lvert\overrightarrow{KM}\rvert=\sqrt{(5)^2+(5)^2+(-2)^2}=\sqrt{54}$ | **A1** |
| | Demonstrates an understanding of the need to use the Law of Cosines.<br>Either $c^2=a^2+b^2-2ab\times\cos C$ (or variation) is seen, or attempt to substitute into formula is made $\left(\sqrt{54}\right)^2=\left(\sqrt{45}\right)^2+\left(\sqrt{45}\right)^2-2\left(\sqrt{45}\right)\left(\sqrt{45}\right)\cos\theta$ | **M1 ft** |
| | Makes an attempt to simplify the above equation. For example, $-36=-90\cos\theta$ is seen. | **M1 ft** |
| | Shows a logical progression to state $\theta=66.4°$ | **B1** |
| | | **(7 marks)** |
| 14b | States or implies that $\Delta KLM$ is isosceles. | **M1** |
| | Makes an attempt to find the missing angles $\angle LKM=\angle LMK=\dfrac{180-66.421...}{2}$ | **M1** |
| | States $\angle LKM=\angle LMK=56.789...°$ . Accept awrt 56.8° | **A1** |
| | | **(3 marks)** |
| | **TOTAL: 10 marks** | |

**NOTES: 14b**

Award ft marks for a correct answer to part **a** using their incorrect answer from earlier in part **a**.

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|---|---|---|
| 15a | Shows or implies that if $y = 0$, $t = 1$ | **M1** |
| | Finds the coordinates of $P$. $t=1 \Rightarrow x=3$ $P(3,0)$ | **A1** |
| | | **(2 marks)** |
| 15b | Attempts to find a cartesian equation of the curve.<br>For example, $t = x - 2$ is substituted into $y=\dfrac{t-1}{t+2}$ | **M1** |
| | Correctly finds the cartesian equation of the curve $y=\dfrac{x-3}{x}$<br>Accept any equivalent answer. For example, $y=1-\dfrac{3}{x}$ | **A1** |
| | | **(2 marks)** |
| 15c | Finds $\dfrac{dy}{dx}=3x^{-2}=\dfrac{3}{x^2}$ | **M1** |
| | Substitutes $t=-1$ to find $x = 1$ and $\dfrac{dy}{dx}=\dfrac{3}{(1)^2}=3$ | **M1** |
| | Finds the gradient of the normal $m_N=-\dfrac{1}{3}$ | **M1** |
| | Substitutes $t=-1$ to find $x = 1$ and $y=-2$ | **A1** |
| | Makes an attempt to find the equation of the normal. For example, $y+2=-\dfrac{1}{3}(x-1)$ is seen. | **M1** |
| | States fully correct answer $x+3y+5=0$ | **A1** |
| | | **(6 marks)** |
| 15d | Substitutes $x=t+2$ and $y=\dfrac{t-1}{t+2}$ into $x+3y+5=0$ obtaining $t+2+3\left(\dfrac{t-1}{t+2}\right)+5=0$ | **M1 ft** |
| | Manipulates and simplifies this equation to obtain $t^2+12t+11=0$ | **M1 ft** |
| | Factorises and solves to find $t=-1$ or $t=-11$ | **M1 ft** |
| | Substitutes $t=-11$ to find $x=-9$ and $y=\dfrac{4}{3}$, i.e. $\left(-9,\dfrac{4}{3}\right)$ | **A1 ft** |
| | | **(4 marks)** |
| | **TOTAL: 14 marks** | |

**NOTES: 15c** Award ft marks for correct answer using incorrect values from part **b**.

**(TOTAL: 100 MARKS)**