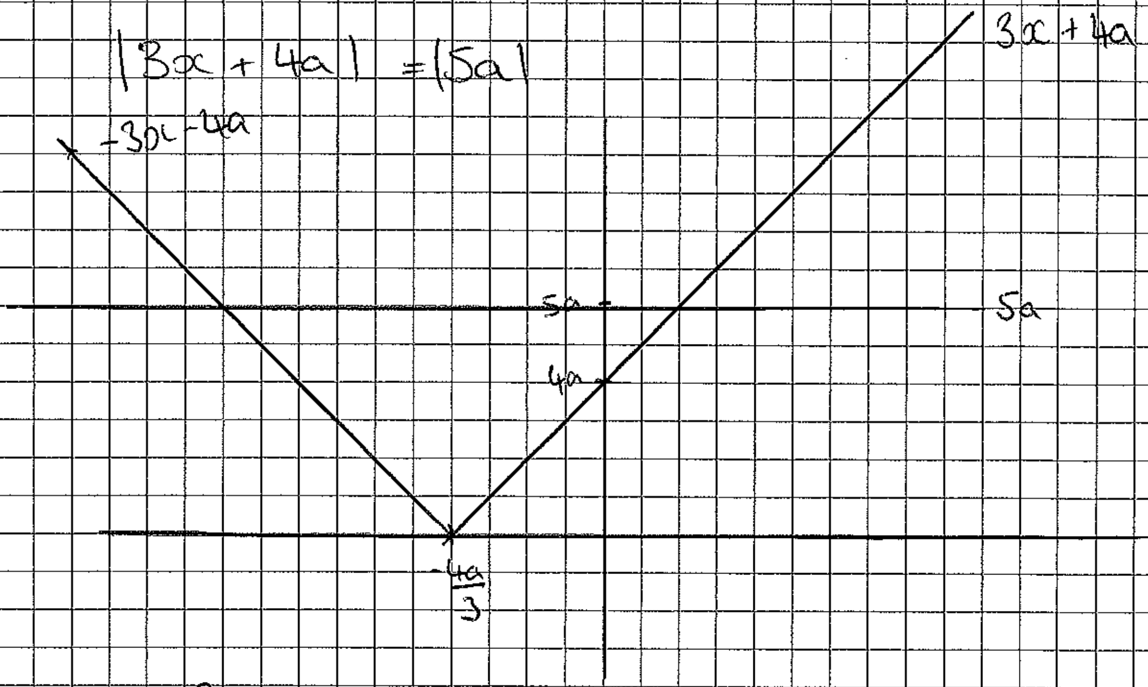
C3 - Jan 11

1) $|3x + 4a| = |5a|$



$3x + 4a = 5a$

$3x = a$

$x = \frac{a}{3}$

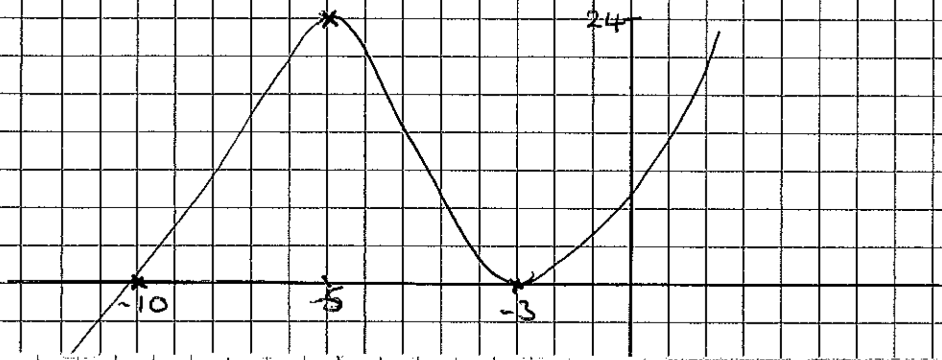
$-3x - 4a = 5a$

$-3x = 9a$

$x = -3a.$

2) $y = -4f(x+3)$

translation -3 in x
stretch s.f 4 in y
reflection in x.



3) $\frac{dr}{dt} = 12 \text{ cm h}^{-1}$

$SA = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$ when $r = 150$

$\frac{dA}{dr} = 1200\pi$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$\frac{dA}{dt} = 12 \times 1200\pi$

$= 14400\pi \text{ cm}^2/\text{h} = 45000 \text{ cm}^2/\text{h}.$

4) $24\sin\theta + 7\cos\theta$

$R = \sqrt{24^2 + 7^2} = 25$

$\alpha = \tan^{-1}\left(\frac{7}{24}\right) = 16.3$

$24\sin\theta + 7\cos\theta = 25\sin(\theta + 16.3)$

ii) $25\sin(\theta + 16.3) = 12$

$\sin(\theta + 16.3) = 0.48$

$\theta + 16.3 = 28.68, 151.3$

$\theta = 135.1^\circ$ or 12.4°

5) $\int_1^a \frac{6}{\sqrt{3x-2}} = 16 = \int_1^a 6(3x-2)^{-1/2}$ [7]

$= \left[\frac{6(3x-2)^{1/2}}{3/2}\right]_1^a = 16$

$= \left[4\sqrt{3x-2}\right]_1^a = 16$

$\Rightarrow 4\sqrt{3a-2} - 4 = 16$

$\sqrt{3a-2} = 5$

$3a - 2 = 25$

$\underline{a = 9}$ [7]

ii) $\pi \int_1^9 \left(\frac{36}{3x-2}\right) dx = \pi \int_1^9 36(3x-2)^{-1}$

$= \left[\frac{36\pi}{3} \ln(3x-2)\right]_1^9 = \left[12\pi \ln(3x-2)\right]_1^9$

$12\pi \ln 25$

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6) $y = \dfrac{3x+4}{x^3-4x^2+2}$

$\dfrac{dy}{dx} = \dfrac{3(x^3-4x^2+2) - (3x+4)(3x^2-8x)}{(x^3-4x^2+2)^2}$

$= \dfrac{3(x^3-4x^2+2) - (9x^3+12x^2-24x^2-32x)}{(x^3-4x^2+2)^2}$

$= \dfrac{3(x^3-4x^2+2) - 9x^3 + 12x^2 + 32x}{(x^3-4x^2+2)^2}$

$\dfrac{dy}{dx} = \dfrac{-6x^3 + 32x + 6}{(x^3-4x^2+2)^2}$

st pt when $\dfrac{dy}{dx} = 0$

$0 = -6x^3 + 32x + 6$

$6x^3 = 32x + 6$

$x^3 = \dfrac{32x}{6} + 1$

$x^3 = \dfrac{16x}{3} + 1$

$x = \sqrt[3]{\dfrac{16x}{3} + 1}$

ii) $x_1 = 2.4$

$x_2 = 2.39$

$x_3 = 2.398$

$x_4 = 2.3980$

$x = 2.3979$

$= 2.398$ (3dp)

$y = -1.552$

7 i) $f(x) = \ln x \qquad g(x) = x^2 + 8$

$fg(x) = \ln(x^2 + 8)$

$\ln(x^2 + 8) = 8$

$x^2 + 8 = e^8$

$x^2 = e^8 - 8$

$x = \pm\sqrt{e^8 - 8}$

$= \sqrt{8}$

ii) $\ln x$ has an inverse as it is one to one.

$y = \ln x$

$e^y = x$

$f^{-1}(x) = e^x \qquad x \in \mathbb{R}$

iii) $gf(x) = (\ln x)^2 + 8$

~~gf'(x~~

$g'f(x) = 2(\ln x) \times \frac{1}{x}$

$= \frac{2\ln x}{x}$

At $x = e^3$

$g'f(x) = \frac{6}{e^3}$

iv) $fg(x) = \ln(x^2 + 8)$

-4 -2 0 2 4

$y_0 = \ln 24$

$y_1 = \ln 12$

$y_2 = \ln 8$

$y_3 = \ln 12$

$y_4 = \ln 24$

$\int_{-4}^{4} = \frac{2}{3}\left(\ln 24 + \ln 24 + 4(\ln 12 + \ln 12) + 2\ln 8\right)$

$= 20.3$ (3sf).

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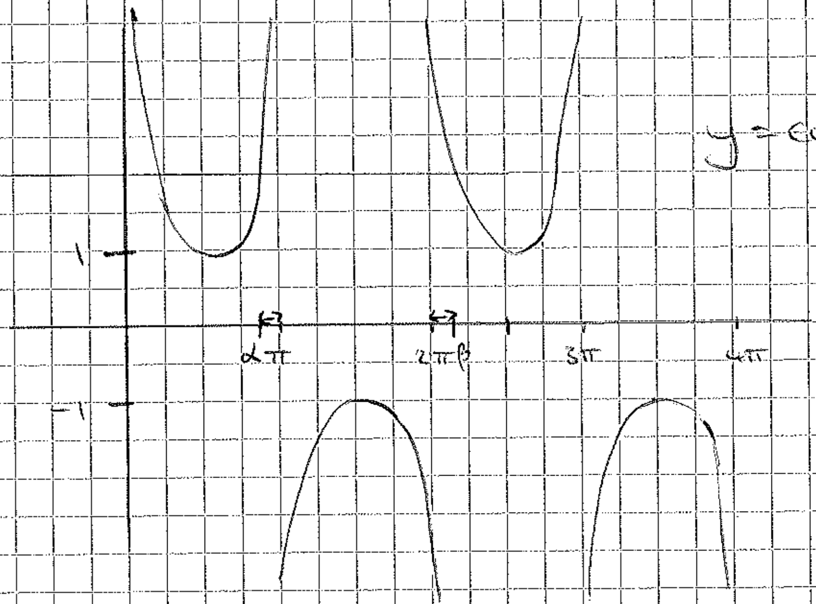
8)



$\pi - \alpha = \cancel{2\pi + \beta}\ \beta - 2\pi$

$3\pi - \alpha = \beta$

b) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

ii) $\cot \phi = 4 = \dfrac{1}{\tan\phi}$ $\quad \tan\phi = \dfrac{1}{4}$

$\cot 2\phi = \dfrac{1 - \tan^2 \cancel{2}\phi}{2\tan \cancel{2}\phi}$

$\tan 4\phi = \dfrac{2\tan 2\phi}{1 - \tan^2 2\phi}$

So $\tan 2\phi = \dfrac{2/4}{1 - 1/16}$

$= \dfrac{8}{15}$

$\cot 2\phi = \dfrac{15}{8}$

$\tan 4\phi = \dfrac{16/15}{1 - \left(\frac{8}{15}\right)^2} = \dfrac{240}{161}$

So $\tan\phi \cot 2\phi \tan 4\phi$

$= \dfrac{1}{4} \times \dfrac{15}{8} \times \dfrac{240}{161}$

$= \dfrac{225}{322}$

9 i) $f(x) = e^{2x} - 3e^{-2x}$

a) $f'(x) = 2e^{2x} + 6e^{-2x}$

e^{2x} and e^{-2x} are positive for all x

b) $f''(x) = 4e^{2x} - 12e^{-2x}$

$= 4(e^{2x} - 3e^{-2x})$

$= 4f(x)$

$e^{2x} - 3e^{-2x} > 0$

$e^{2x} > 3e^{-2x}$

$\frac{e^{2x}}{e^{-2x}} > 3$

$e^{4x} > 3$

$4x > \ln 3$

$x > \frac{1}{4}\ln 3$

ii) $g(x) = e^{2x} + ke^{-2x}$

$g'(x) = 2e^{2x} - 2ke^{-2x}$

$0 = \cancel{2}e^{2x} - \cancel{2}ke^{-2x}$

$ke^{-2x} = e^{2x}$

$k = e^{4x}$

$x = \frac{1}{4}\ln k$

when $x = \frac{1}{4}\ln k$

$g(x) = e^{\frac{1}{2}\ln k} + ke^{-\frac{1}{2}\ln k}$

$e^{\ln k^{1/2}} + ke^{\ln k^{-1/2}}$

$= k^{1/2} + k^{1/2} = 2\sqrt{k}$

So range is $g(x) \geq 2\sqrt{k}$