A2 Practice Paper C

1. $\frac{18x^2-98x+78}{(x-4)^2(3x+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{3x+1}, \quad x>4$

$18x^2-98x+78 = A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)$

$x=4, \quad 18(4)^2-98(4)+78 = B(13)$

$-26 = 13B$

$\underline{-2 = B}$

$x=-\frac{1}{3}, \quad 18\left(-\frac{1}{3}\right)^2-98\left(-\frac{1}{3}\right)+78 = C\left(-4\tfrac{1}{3}\right)\left(-4\tfrac{1}{3}\right)$

$2+\frac{98}{3}+78 = C\frac{169}{9}$

$\frac{338}{3} = \frac{169}{9}C$

$\underline{6 = C}$

$18x^2 = 3Ax^2 + Cx^2$

$18 = 3A + C$

$18 = 3A + 6$

$12 = 3A$

$\underline{4 = A}$

$\frac{4}{x-4} - \frac{2}{(x-4)^2} + \frac{6}{3x+1}$

2. $4^x = 2xy \rightarrow$ Product rule.

$4^x \ln 4 = 2x\frac{dy}{dx} + 2y$

$4^x \ln 4 - 2y = 2x\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{4^x \ln 4 - 2y}{2x} \qquad x=2 \quad y=4$

$\frac{dy}{dx} = \frac{4^2 \ln 4 - 8}{4} = \underline{4\ln 4 - 2}$

3a) $\cos 7x + \cos 3x = 2\cos 5x \cos 2x$

$\cos(5x + 2x) = \cos 5x \cos 2x - \sin 5x \sin 2x$

$\cos(5x - 2x) = \cos 5x \cos 2x + \sin 5x \sin 2x$

$= \underline{2\cos 5x \cos 2x}$.

b) $\int (\cos 5x \cos 2x)\, dx$

$\int \frac{1}{2}(\cos 7x + \cos 3x)\, dx$

$\frac{1}{2}\int (\cos 7x + \cos 3x)\, dx$.

$\frac{1}{2}\left[\frac{1}{7}\sin 7x + \frac{1}{3}\cos 3x\right] + c$

$\frac{1}{14}\sin 7x + \frac{1}{6}\cos 3x + c$.

4a) $T(t) = T_R + (90 - T_R)e^{-1/20\, t}$

$t = 0,\ T(t) = T_R + (90 - T_R)e^0$

$T(t) = 90$

As T_R terms will always cancel at $t = 0$, therefore the room temperature does not influence the initial coffee temperature.

b) $T(t) = 20 + (90 - 20)e^{-1/10}$

$= 20 + 70e^{-1/10}$

$= \underline{62 \cdot 457°C}$

5. Proof by contradiction:
Assumption: if n is odd, n^3+1 is odd

$2k+1 \rightarrow$ odd.

$(2k+1)^3+1$
$(2k+1)(2k+1)(2k+1)+1$
$(2k+1)(4k^2+4k+1)+1$
$8k^3+8k^2+2k+4k^2+4k+1+1$
$8k^3+12k^2+16k+2.$
$2(4k^3+6k^2+3k+1)$ is even

This contradicts the assumption that there exists a number n such that n is odd and n^3+1 is also odd, so if n is odd, then n^3+1 is even

6. $x = \sec^2 t + 1$ $\qquad$ $y = 2\sin t$

$x = \dfrac{1}{\cos^2 t} + 1$ $\qquad$ $\dfrac{y}{2} = \sin t.$

$x - 1 = \dfrac{1}{\cos^2 t}.$ $\qquad$ $\dfrac{y^2}{4} = \sin^2 t.$

$\cos^2 t = \dfrac{1}{x-1}$

$\sin^2 t + \cos^2 t = 1$

$\dfrac{y^2 \times (x-1)}{4 \times (x-1)} + \dfrac{1 \times 4}{x-1 \times 4} = 1$

$\dfrac{y^2(x-1)+4}{4(x-1)} = 1$

$y^2(x-1)+4 = 4(x-1)$

$y^2(x-1) = 4x-4-4$

$y = \sqrt{\dfrac{4x-8}{x-1}} \rightarrow$ must be positive $\therefore$ $x<1$ or $x>2.$

7 a) $\underbrace{1 - 4x}_{\times -4x} \underbrace{+ 16x^2}_{\times -4x} \underbrace{- 64x^3}_{\times -4x} + \dots$ convergent.

$r = -4x$

$|r| < 1$

$|-4x| < 1$

$|x| < \frac{1}{4}$

b) $\sum_{r=1}^{\infty} (-4x)^{r-1} = 4$

$a = 1$

$r = -4x$

$S_\infty = \frac{a}{1-r} = 4$

$= \frac{1}{1 - -4x} = 4$

$\frac{1}{1+4x} = 4$

$1 = 4(1+4x)$

$1 = 4 + 16x$

$-3 = 16x$

$\frac{-3}{16} = x$

8 a) $f(x) = 2 - 3\sin^3 x - \cos x$

$f(1 \cdot 9) = -0 \cdot 2188$

$f(2 \cdot 0) = +0 \cdot 1606$

Change of sign and continuous function in the interval $[1.9, 2.0]$ there is a root.

b) Newton Raphson : $x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = -9 \sin^2 x \cos x + \sin x$

8b) continued

$$x_n - \frac{2 - 3\sin^3 x - \cos x}{-9\sin^2 x \cos x + \sin x}$$

$f(1.95) = -0.0348$
$f'(1.95) = 3.8040$

$$x_1 = 1.95 - \frac{-0.0348}{3.8040}$$
$= \underline{1.959}$ (3dp)

9. $(b-a)i - 2abcj + 2k = 10i - 96j + (7a+5b)k$

$b - a = 10 \times 5$ $\quad 2abc = 96 \quad$ $2 = 7a + 5b$.
$5b - 5a = 50$
$-\ 5b + 7a = 2$
$-12a = 48$
$\underline{a = -4}$

$b - a = 10$
$b - -4 = 10$
$b + 4 = 10$
$\underline{b = 6}$.

$2abc = 96$
$2(-4)(6)c = 96$
$-48c = 96$
$\underline{c = -2}$.

10. Proof by contradiction:
Assumption: There exists a positive integer solution to $x^2 - y^2 = 1$.
$(x-y)(x+y) = 1$
$x - y = 1$
$-\ x + y = 1$
$-2y = 0$
$y = 0$
$x = 1$

Therefore are not both positive integers and there does not exist positive integers x and y such that $x^2 - y^2 = 1$.

11. $g(x) = x^2 - 8x + 7$

$$y = x^2 - 8x + 7$$
$$y = (x-4)^2 - 16 + 7$$
$$y = (x-4)^2 - 9$$
$$x = (y-4)^2 - 9$$
$$x + 9 = (y-4)^2$$
$$\sqrt{x+9} = y - 4$$
$$\sqrt{x+9} + 4 = y$$

$$g'(x) = \sqrt{x+9} + 4$$

Domain $x > -9$
Range $y > 4$.

12. $$\frac{4x^2 + x - 23}{(x-3)(4-x)(x+5)} = \frac{A}{x-3} + \frac{B}{4-x} + \frac{C}{x+5}$$

$$4x^2 + x - 23 = A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x)$$

$x = 4$, $4(4)^2 + (4) - 23 = B(1)(9)$
$$45 = 9B$$
$$\underline{5 = B}$$

$x = -5$, $4(-5)^2 + (-5) - 23 = C(-8)(9)$
$$72 = -72C$$
$$\underline{-1 = C}$$

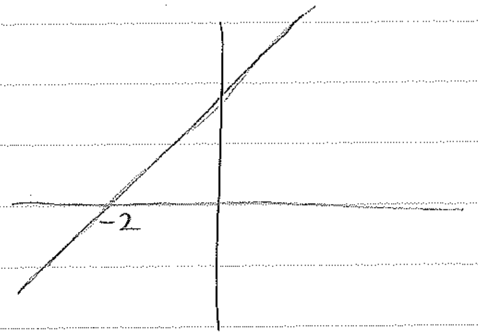
$x = 3$, $4(3)^2 + (3) - 23 = A(1)(8)$
$$16 = 8A$$
$$\underline{2 = A}$$

$$\frac{2}{x-3} + \frac{5}{4-x} - \frac{1}{x+5}$$

13. a) $y = x^3 + 6x^2 - 12x + 6.$

$\frac{dy}{dx} = 3x^2 + 12x - 12.$

$\frac{d^2y}{dx^2} = 6x + 12.$



$y = 6x + 12$
$x = 0, \ y = 12.$
$y = 0, \ x = -2.$

For $-5 \leq x \leq -3$ y is negative for $\frac{d^2y}{dx^2}$
therefore C is concave.

b) $\frac{d^2y}{dx^2} = 0$

$6x + 12 = 0$
$6x = -12$
$x = \frac{-12}{6}$
$x = -2.$

$y = x^3 + 6x^2 - 12x + 6.$
$y = (-2)^3 + 6(-2)^2 - 12(-2) + 6$
$y = 46$

Point of inflection at $(-2, 46)$.

14. $\int_{\frac{\pi}{12}}^{\pi/8} \sin 4x (1 - \cos 4x)^3 \, dx$

$x = \frac{\pi}{8}, \ u = 1$

$x = \pi/_{12}, \ u = \frac{1}{2}$

$u = 1 - \cos 4x$

$\frac{du}{dx} = 4 \sin 4x$

$dx = \frac{du}{4 \sin 4x}$

$\int_{\frac{\pi}{12}}^{\pi/8} \cancel{\sin 4x} \, (u)^3 \frac{du}{4 \cancel{\sin 4x}}$

$\frac{1}{4} \left[\frac{1}{4} u^4 \right]_{\frac{1}{2}}^{1}$

$= \frac{1}{4} \left[\left(\frac{1}{4} \right) - \left(\frac{1}{4} \left(\frac{1}{2} \right)^4 \right) \right]$

$= \frac{1}{4} \left[\frac{1}{4} - \frac{1}{64} \right]$

$= \frac{15}{256}.$

15. a) $\dfrac{4x^2 - 4x - 9}{2x^2 - x - 1}$

$$\begin{array}{r} 2 \\ 2x^2 - x - 1 \overline{\smash{)}\, 4x^2 - 4x - 9} \\ \underline{4x^2 - 2x - 2} \\ -2x - 7 \end{array}$$

$A = 2.$

$-2x - 7 = B(x-1) + C(2x+1)$

$x = 1, \quad -2(1) - 7 = C(3)$

$-9 = 3C$

$\underline{-3 = C}$

$x = -\frac{1}{2}, \quad -2(-\frac{1}{2}) - 7 = B(-\frac{1}{2} - 1)$

$-6 = -\frac{3}{2} B$

$\underline{4 = B}$

$2 + \dfrac{4}{2x+1} - \dfrac{3}{x-1}$

b) $2 + 4(2x+1)^{-1} - 3(x-1)^{-1}$

$4(1+2x)^{-1} = 4\left[1 + (-1)(2x) + \dfrac{(-1)(-2)}{1 \times 2}(2x)^2 + \dots\right]$

$= 4 - 8x + 16x^2 \dots$

$\dfrac{-3}{x-1} = \dfrac{3}{1-x} = 3(1-x)^{-1}$

15b) continued...

$$3(1-x)^{-1} = 3\left[1 + (-1)(-x) + \frac{(-1)(-2)}{1\times 2}(-x)^2\right]$$

$$= 3 + 3x + 3x^2 + \ldots$$

$$2 + 4 - 8x + 16x^2 + 3 + 3x + 3x^2$$

$$= 9 - 5x + 19x^2$$

c) $|2x| < 1$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$ $\therefore$ Expansion not valid for $x = \frac{3}{4}$.

16a) $r = 40$

$$\frac{dh}{dt} = \frac{dv}{dt} \times \frac{dh}{dv}$$

$V = \pi r^2 h$

$V = \pi \times 40^2 \times h$

$V = 1600\pi h$

$\frac{dv}{dh} = 1600\pi$

$\frac{dh}{dv} = \frac{1}{1600\pi}$

$$\frac{dv}{dt} = 4000\pi - 50\pi h$$

$$\frac{dh}{dt} = (4000\pi - 50\pi h) \times \frac{1}{1600\pi}$$

$$\frac{dh}{dt} = \frac{4000\pi}{1600\pi} - \frac{50\pi h}{1600\pi}$$

16a) continued....

$$1600\pi \frac{dh}{dt} = 4000\pi - 50\pi h$$

$\div 10\pi$ $\qquad$ $\div 10\pi$ $\qquad$ $\div 10\pi$

$$160 \frac{dh}{dt} = 400 - 5h$$

b) $t = 0, \quad h = 50m$
$t = ?, \quad h = 60m.$

$$160 \frac{dh}{dt} = 400 - 5h.$$

$$\int \frac{1}{400-5h}\, dh = \int \frac{1}{160}\, dt$$

$$-\frac{1}{5} \ln(400-5h) = \frac{t}{160} + c.$$

$t = 0, \ h = 50$

$$-\frac{1}{5} \ln(400 - 250) = c.$$

$$-\frac{1}{5} \ln(150) = c.$$

$$-\frac{1}{5} \ln(400-5h) = \frac{t}{160} - \frac{1}{5} \ln(150).$$

$h = 60,$

$$-\frac{1}{5} \ln(400-300) = \frac{t}{160} - \frac{1}{5} \ln(150)$$

16b) continued...

$$-\frac{1}{5}\ln(100) = \frac{t}{160} - \frac{1}{5}\ln(150)$$

$$\frac{1}{5}\ln(150) - \frac{1}{5}\ln(100) = \frac{t}{160}.$$

$$\frac{1}{5}\ln\left(\frac{150}{100}\right) = \frac{t}{160}.$$

$$\frac{1}{5}\ln\left(\frac{3}{2}\right) = \frac{t}{160}.$$

$$\frac{160}{5}\ln\left(\frac{3}{2}\right) = t.$$

$$32\ln\left(\frac{3}{2}\right) = t$$