

Revision of surds

Surds are numbers with square root signs.

e.g $\sqrt{20}$, $\sqrt{7}$, $3\sqrt{2}$

Surds questions are most likely to be on the NON-CALCULATOR paper. (If you get a surds question on a calculator paper use your casio calculators for things like simplifying surds and rationalising the denominator)

Surds rules

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

e.g. $\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$

$$\sqrt{x} \times \sqrt{x} = x \quad (\sqrt{x})^2$$

e.g. $\sqrt{7} \times \sqrt{7} = 7$

$$\sqrt{x} \times \sqrt{y} = \sqrt{x \times y} = \sqrt{xy}$$

$$\checkmark \quad 3\sqrt{x} + 5\sqrt{x} = 8\sqrt{x}$$

$$\times \quad \sqrt{x} + \sqrt{y} \neq \sqrt{x+y}$$

Simplifying surds

$$\sqrt{20}$$

$$\sqrt{4 \times 5}$$

look for the largest square factor

$$= \sqrt{4} \times \sqrt{5}$$

$$= 2 \times \sqrt{5}$$

$$= 2\sqrt{5}$$

e.g

$$\sqrt{48}$$

$$\sqrt{4 \times 12}$$

$$\sqrt{4} \times \sqrt{12}$$

$$2\sqrt{12}$$

$$2 \times \sqrt{4} \times \sqrt{3}$$

$$2 \times 2 \times \sqrt{3}$$

$$\underline{4\sqrt{3}}$$

Express these surds in the form $a\sqrt{b}$.

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|------|--------------|------|--------------|------|--------------|------|--------------|------|--------------|
| 1). | $\sqrt{8}$ | 2). | $\sqrt{27}$ | 3). | $\sqrt{20}$ | 4). | $\sqrt{32}$ | 5). | $\sqrt{80}$ |
| 6). | $\sqrt{44}$ | 7). | $\sqrt{75}$ | 8). | $\sqrt{72}$ | 9). | $\sqrt{45}$ | 10). | $\sqrt{108}$ |
| 11). | $\sqrt{28}$ | 12). | $\sqrt{125}$ | 13). | $\sqrt{245}$ | 14). | $\sqrt{192}$ | 15). | $\sqrt{405}$ |
| 16). | $\sqrt{112}$ | 17). | $\sqrt{63}$ | 18). | $\sqrt{180}$ | 19). | $\sqrt{99}$ | 20). | $\sqrt{48}$ |

Work out the following. Leave the answer in surd form where appropriate.

- | | | | | | | | |
|------|-----------------------------|------|-----------------------------|------|-----------------------------|------|-----------------------------|
| 1). | $\sqrt{3} \times \sqrt{6}$ | 2). | $\sqrt{6} \times \sqrt{2}$ | 3). | $\sqrt{10} \times \sqrt{5}$ | 4). | $\sqrt{8} \times \sqrt{5}$ |
| 5). | $\sqrt{10} \times \sqrt{2}$ | 6). | $\sqrt{3} \times \sqrt{3}$ | 7). | $\sqrt{2} \times \sqrt{8}$ | 8). | $\sqrt{14} \times \sqrt{2}$ |
| 9). | $\sqrt{2} \times \sqrt{9}$ | 10). | $\sqrt{5} \times \sqrt{15}$ | 11). | $\sqrt{3} \times \sqrt{8}$ | 12). | $\sqrt{5} \times \sqrt{5}$ |
| 13). | $\sqrt{2} \times \sqrt{18}$ | 14). | $\sqrt{6} \times \sqrt{6}$ | 15). | $\sqrt{5} \times \sqrt{30}$ | | |

Answers

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|------------------|------------------|------------------|-------------------|------------------|
| 1). $2\sqrt{2}$ | 2). $3\sqrt{3}$ | 3). $2\sqrt{5}$ | 4). $4\sqrt{2}$ | 5). $4\sqrt{5}$ |
| 6). $2\sqrt{11}$ | 7). $5\sqrt{3}$ | 8). $6\sqrt{2}$ | 9). $3\sqrt{5}$ | 10). $6\sqrt{3}$ |
| 11). $2\sqrt{7}$ | 12). $5\sqrt{5}$ | 13). $7\sqrt{5}$ | 14). $8\sqrt{3}$ | 15). $9\sqrt{5}$ |
| 16). $4\sqrt{7}$ | 17). $3\sqrt{7}$ | 18). $6\sqrt{5}$ | 19). $3\sqrt{11}$ | 20). $4\sqrt{3}$ |

- | | | | | |
|------------------|-----------------|-----------------|------------------|------------------|
| 1). $3\sqrt{2}$ | 2). $2\sqrt{3}$ | 3). $5\sqrt{2}$ | 4). $2\sqrt{10}$ | 5). $2\sqrt{5}$ |
| 6). 3 | 7). 4 | 8). $2\sqrt{7}$ | 9). $3\sqrt{2}$ | 10). $5\sqrt{3}$ |
| 11). $2\sqrt{6}$ | 12). 5 | 13). 6 | 14). 6 | 15). $5\sqrt{6}$ |

Adding and subtracting surds

$$\sqrt{7} + \sqrt{6} \neq \sqrt{13}$$

$$3\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$$

$$3x + 5x = 8x$$

$$\begin{array}{ccc} \sqrt{18} & + & \sqrt{50} \\ \swarrow \searrow & & \swarrow \searrow \\ \sqrt{9} \times \sqrt{2} & & \sqrt{25} \sqrt{2} \end{array}$$

$$3\sqrt{2} + 5\sqrt{2}$$

$$= \underline{8\sqrt{2}}$$

- | | | | | | | | |
|------|---------------------------|------|---------------------------|------|-----------------------------------|------|-------------------------------------|
| 1). | $\sqrt{8} + \sqrt{2}$ | 2). | $\sqrt{20} - \sqrt{5}$ | 3). | $\sqrt{3} + \sqrt{12}$ | 4). | $\sqrt{8} - \sqrt{2}$ |
| 5). | $\sqrt{27} + \sqrt{12}$ | 6). | $\sqrt{125} - \sqrt{20}$ | 7). | $\sqrt{48} + \sqrt{75}$ | 8). | $\sqrt{18} + \sqrt{72}$ |
| 9). | $\sqrt{75} - \sqrt{27}$ | 10). | $\sqrt{80} - \sqrt{20}$ | 11). | $\sqrt{108} - \sqrt{27}$ | 12). | $\sqrt{27} - \sqrt{12}$ |
| 13). | $\sqrt{147} - \sqrt{108}$ | 14). | $\sqrt{48} - \sqrt{27}$ | 15). | $\sqrt{98} + \sqrt{8} + \sqrt{2}$ | 16). | $\sqrt{99} - \sqrt{44} - \sqrt{11}$ |
| 17). | $3\sqrt{2} - \sqrt{18}$ | 18). | $\sqrt{175} - 4\sqrt{7}$ | 19). | $3\sqrt{8} + \sqrt{50}$ | 20). | $5\sqrt{5} + \sqrt{20}$ |
| 21). | $2\sqrt{45} + 3\sqrt{20}$ | 22). | $3\sqrt{32} - 2\sqrt{18}$ | | | | |

$$\begin{aligned}
 1) \quad & \sqrt{8} + \sqrt{2} \\
 & \quad \wedge \\
 & \sqrt{4} \times \sqrt{2} + \sqrt{2} \\
 & 2\sqrt{2} + \sqrt{2} \\
 & = 3\sqrt{2}
 \end{aligned}$$

Answers

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|-------------------|------------------|-----------------|-------------------|-------------------|
| 1). $3\sqrt{2}$ | 2). $\sqrt{5}$ | 3). $3\sqrt{3}$ | 4). $\sqrt{2}$ | 5). $5\sqrt{3}$ |
| 6). $3\sqrt{5}$ | 7). $9\sqrt{3}$ | 8). $9\sqrt{2}$ | 9). $2\sqrt{3}$ | 10). $2\sqrt{5}$ |
| 11). $3\sqrt{3}$ | 12). $\sqrt{3}$ | 13). $\sqrt{3}$ | 14). $\sqrt{3}$ | 15). $10\sqrt{2}$ |
| 16). 0 | 17). 0 | 18). $\sqrt{7}$ | 19). $11\sqrt{2}$ | 20). $7\sqrt{5}$ |
| 21). $12\sqrt{5}$ | 22). $6\sqrt{2}$ | | | |

Surds and powers

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|------|-----------------|------|-----------------|------|-----------------|------|-----------------|------|-----------------|
| 1). | $(\sqrt{2})^3$ | 2). | $(\sqrt{3})^3$ | 3). | $(\sqrt{2})^5$ | 4). | $(\sqrt{3})^4$ | 5). | $(\sqrt{5})^5$ |
| 6). | $(3\sqrt{2})^2$ | 7). | $(2\sqrt{7})^2$ | 8). | $(2\sqrt{3})^3$ | 9). | $(2\sqrt{2})^3$ | 10). | $(2\sqrt{3})^2$ |
| 11). | $(2\sqrt{5})^2$ | 12). | $(5\sqrt{3})^2$ | 13). | $(2\sqrt{5})^3$ | 14). | $(3\sqrt{6})^2$ | 15). | $(3\sqrt{5})^3$ |

Answers

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|-----------------|-----------------|-------------------|------------------|--------------------|
| 1). $2\sqrt{2}$ | 2). $3\sqrt{3}$ | 3). $4\sqrt{2}$ | 4). 9 | 5). $25\sqrt{5}$ |
| 6). 18 | 7). 28 | 8). $24\sqrt{3}$ | 9). $16\sqrt{2}$ | 10). 12 |
| 11). 20 | 12). 75 | 13). $40\sqrt{5}$ | 14). 54 | 15). $135\sqrt{5}$ |

Cancelling surds down

$$\frac{6\sqrt{3}}{2\sqrt{12}}$$

$$\frac{3\sqrt{3}}{\sqrt{12}}$$

$$\frac{\cancel{3}\sqrt{\cancel{3}}}{2\sqrt{\cancel{3}}}$$

$$= \frac{3}{2}$$



$$\frac{\cancel{3}\sqrt{\cancel{3}}}{\sqrt{4 \times \cancel{3}}}$$

$$\begin{array}{c} \sqrt{12} \\ \diagdown \quad \diagup \\ \sqrt{4} \times \sqrt{3} \\ 2\sqrt{3} \end{array}$$

$$\frac{3\sqrt{3}}{\sqrt{12}}$$

$$= \frac{3}{1} \sqrt{\frac{3}{12}}$$

$$= 3 \sqrt{\frac{1}{4}}$$

$$= \frac{3}{2}$$

$$\frac{\sqrt{3}}{\sqrt{12}} = \sqrt{\frac{3}{12}}$$

- 1). $6\sqrt{15} \div 2\sqrt{3}$ 2). $14\sqrt{3} \div 7\sqrt{3}$ 3). $8\sqrt{6} \div 2\sqrt{3}$ 4). $20\sqrt{15} \div 4\sqrt{5}$
 5). $10\sqrt{2} \div 2\sqrt{2}$ 6). $15\sqrt{7} \div 3\sqrt{7}$ 7). $10\sqrt{30} \div 5\sqrt{5}$ 8). $18\sqrt{32} \div 3\sqrt{8}$
 9). $27\sqrt{24} \div 3\sqrt{8}$ 10). $24\sqrt{28} \div 3\sqrt{2}$ 11). $2\sqrt{27} \div 2\sqrt{3}$ 12). $21\sqrt{3} \div 3\sqrt{3}$
 13). $32\sqrt{35} \div 4\sqrt{5}$ 14). $10\sqrt{24} \div 2\sqrt{3}$ 15). $8\sqrt{32} \div 4\sqrt{2}$ 16). $4\sqrt{48} \div 4\sqrt{3}$
 17). $2\sqrt{10} \div 2\sqrt{2}$ 18). $12\sqrt{28} \div 3\sqrt{7}$ 19). $15\sqrt{30} \div 5\sqrt{5}$ 20). $30\sqrt{150} \div 5\sqrt{6}$

$$1) \quad \frac{6\sqrt{15}}{2\sqrt{3}} \qquad 8) \quad \frac{18\sqrt{32}}{3\sqrt{8}} = 6\sqrt{4}$$

$$= 6 \times 2$$

$$= 12$$

$$5\sqrt{8}$$

$$5 \times 2\sqrt{2}$$

$$= 10\sqrt{2}$$

Answers

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|-----------------|-----------------|------------------|-------------------|-------------------|
| 1). $3\sqrt{5}$ | 2). 2 | 3). $4\sqrt{2}$ | 4). $5\sqrt{3}$ | 5). 5 |
| 6). 5 | 7). $2\sqrt{6}$ | 8). 12 | 9). $9\sqrt{3}$ | 10). $8\sqrt{14}$ |
| 11). 3 | 12). 7 | 13). $8\sqrt{7}$ | 14). $10\sqrt{2}$ | 15). 8 |
| 16). 4 | 17). $\sqrt{5}$ | 18). 8 | 19). $3\sqrt{6}$ | 20). 30 |

Rationalising the denominator of a fraction.

$$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$\sqrt{x} \times \sqrt{x} = x$$

$$\frac{(2 + \sqrt{5})}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{2\sqrt{5} + 5}{5}$$

Rationalise.

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|------|-------------------------------|------|------------------------------|------|-------------------------------|------|-------------------------------|------|-------------------------------|
| 1). | $\frac{3}{\sqrt{3}}$ | 2). | $\frac{10}{\sqrt{5}}$ | 3). | $\frac{21}{\sqrt{7}}$ | 4). | $\frac{8}{\sqrt{2}}$ | 5). | $\frac{24}{\sqrt{6}}$ |
| 6). | $\frac{1}{\sqrt{3}}$ | 7). | $\frac{1}{\sqrt{2}}$ | 8). | $\frac{1}{\sqrt{5}}$ | 9). | $\frac{2}{\sqrt{3}}$ | 10). | $\frac{9}{\sqrt{15}}$ |
| 11). | $\frac{21}{\sqrt{6}}$ | 12). | $\frac{8}{\sqrt{18}}$ | 13). | $\frac{2}{\sqrt{5}}$ | 14). | $\frac{9}{\sqrt{6}}$ | 15). | $\frac{30}{\sqrt{75}}$ |
| 16). | $\frac{\sqrt{12}}{\sqrt{50}}$ | 17). | $\frac{\sqrt{12}}{\sqrt{3}}$ | 18). | $\frac{3\sqrt{2}}{\sqrt{10}}$ | 19). | $\frac{3\sqrt{7}}{\sqrt{21}}$ | 20). | $\frac{4\sqrt{5}}{\sqrt{20}}$ |

Answers

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|--------------------|--------------------|--------------------|--------------------|---------------------|
| 1). $\sqrt{3}$ | 2). $2\sqrt{5}$ | 3). $3\sqrt{7}$ | 4). $4\sqrt{2}$ | 5). $4\sqrt{6}$ |
| 6). $\sqrt{3/3}$ | 7). $\sqrt{2/2}$ | 8). $\sqrt{5/5}$ | 9). $2\sqrt{3/3}$ | 10). $3\sqrt{15/5}$ |
| 11). $7\sqrt{6/2}$ | 12). $4\sqrt{2/3}$ | 13). $2\sqrt{5/5}$ | 14). $3\sqrt{6/2}$ | 15). $2\sqrt{3}$ |
| 16). $\sqrt{6/5}$ | 17). 2 | 18). $3\sqrt{5/5}$ | 19). $\sqrt{3}$ | 20). 2 |