Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)

Sample assessment materials (SAMs)

First teaching September 2017

First certification from 2018

Edexcel, BTEC and LCCI qualifications

Edexcel, BTEC and LCCI qualifications are awarded by Pearson, the UK's largest awarding body offering academic and vocational qualifications that are globally recognised and benchmarked. For further information, please visit our qualification website at qualifications.pearson.com. Alternatively, you can get in touch with us using the details on our contact us page at qualifications.pearson.com/contactus

About Pearson

Pearson is the world's leading learning company, with 35,000 employees in more than 70 countries working to help people of all ages to make measurable progress in their lives through learning. We put the learner at the centre of everything we do, because wherever learning flourishes, so do people. Find out more about how we can help you and your learners at qualifications.pearson.com

Original origami artwork: Mark Bolitho
Origami photography: Pearson Education Ltd/Naki Kouyioumtzis

ISBN 978 1 4469 3354 1

Contents

Introduction

The Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

2. Abbreviations

   These are some of the traditional marking abbreviations that may appear in the mark schemes.

   - **bod** benefit of doubt
   - **ft** follow through
   - $\sqrt{}$ this symbol is used for correct ft
   - **cao** correct answer only
   - **cso** correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** ignore subsequent working
   - **awrt** answers which round to
   - **SC:** special case
   - **o.e.** or equivalent (and appropriate)
   - **d…** or **dep** dependent
   - **indep** independent
   - **dp** decimal places
   - **sf** significant figures
   - ***** The answer is printed on the paper or ag- answer given

- or d… The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Pearson Edexcel Level 3 GCE	
Further Mathematics **Advanced Subsidiary** **Paper 1: Core Pure Mathematics**	
Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/01**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions. Write your answers in the spaces provided.

1. $$\mathrm{f}(z) = z^3 + pz^2 + qz - 15$$

where p and q are real constants.

Given that the equation $\mathrm{f}(z) = 0$ has roots

$$\alpha, \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $\mathrm{f}(z) = 0$

(5)

(b) Hence find the value of p.

(2)

Question 1 continued

(Total for Question 1 is 7 marks)

2. The plane Π passes through the point A and is perpendicular to the vector **n**

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where O is the origin,

(a) find a Cartesian equation of Π.

(2)

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is $21.2°$ correct to one decimal place.

(4)

(c) Find the coordinates of the point X.

(4)

Question 2 continued

(Total for Question 2 is 10 marks)

3. Tyler invested a total of £5000 across three different accounts;
a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%

- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

Question 4 continued

(Total for Question 4 is 5 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.

(2)

The hexagon R is transformed to the hexagon S by the transformation represented by the matrix **M**.

Given that the area of hexagon R is 5 square units,

(b) find the area of hexagon S.

(1)

The matrix **M** represents an enlargement, with centre $(0, 0)$ and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about $(0, 0)$.

(c) Find the value of k.

(2)

(d) Find the value of θ.

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

6. (a) Prove by induction that for all positive integers *n*,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

(b) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers *n*,

$$\sum_{r=1}^{n} r(r+6)(r-6) = \frac{1}{4}n(n+1)(n-8)(n+9)$$

(4)

(c) Hence find the value of *n* that satisfies

$$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2$$

(5)

Question 6 continued

(Total for Question 6 is 15 marks)

7. Diagrams not drawn to scale

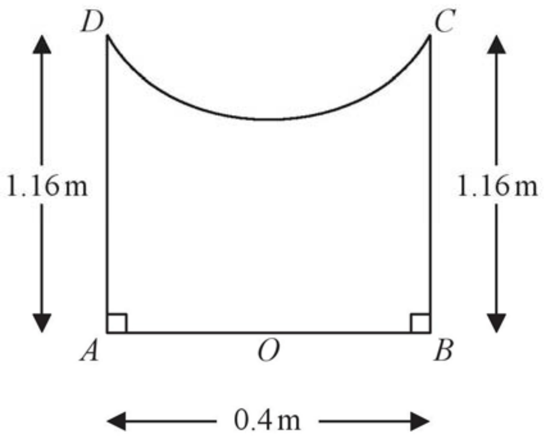


Figure 1

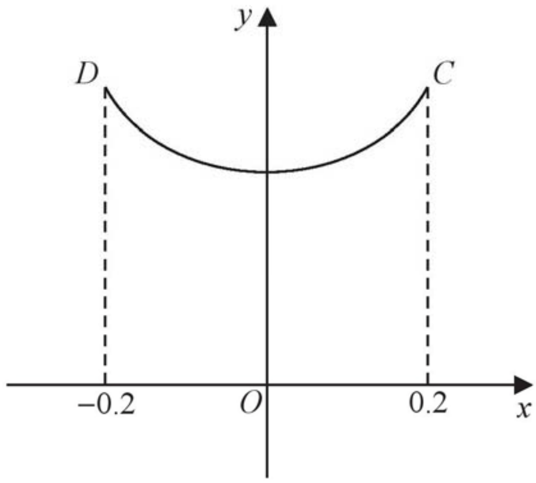


Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where *k* is a constant and where *O* is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of *k*.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was $0.127\ m^3$ correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

Question 7 continued

(Total for Question 7 is 12 marks)

8. (a) Shade on an Argand diagram the set of points

$$\left\{ z \in \mathbb{C} : \left|z - 4\mathrm{i}\right| \leqslant 3 \right\} \cap \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leqslant \frac{\pi}{4} \right\}$$

(6)

The complex number w satisfies

$$\left|w - 4\mathrm{i}\right| = 3$$

(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

Question 8 continued

(Total for Question 8 is 8 marks)

9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish *F* swims from a point *A* to a point *B*.

The octopus is modelled as a fixed particle at the origin *O*.

Fish *F* is modelled as a particle moving in a straight line from *A* to *B*.

Relative to *O*, the coordinates of *A* are $(-3, 1, -7)$ and the coordinates of *B* are $(9, 4, 11)$, where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish *F*.

(7)

(b) Criticise the model in relation to fish *F*.

(1)

(c) Criticise the model in relation to the octopus.

(1)

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 80 MARKS

AS Paper 1: Core Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1. (a)	$\alpha\left(\frac{5}{\alpha}\right)\left(\alpha+\frac{5}{\alpha}-1\right)=15$	M1	1.1b
		A1	1.1b
	$\Rightarrow 5\alpha+\frac{25}{\alpha}-5=15 \Rightarrow \alpha^2-4\alpha+5=0$ $\Rightarrow \alpha=\frac{--4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ or $(\alpha-2)^2-4+5=0 \Rightarrow \alpha=...$	M1	3.1a
	$\Rightarrow \alpha=2\pm\text{i}$	A1	1.1b
	Hence the roots of $\text{f}(z)=0$ are $2+\text{i}, 2-\text{i}$ and 3	A1	2.2a
		(5)	
(b)	$p=-\left(\text{"}(2+\text{i})\text{"}+\text{"}(2-\text{i})\text{"}+\text{"}3\text{"}\right) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ **cso**	A1	1.1b
		(2)	
(b) ALT 1	$\text{f}(z)=(z-3)(z^2-4z+5) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ **cso**	A1	1.1b
		(2)	
	(7 marks)		

Question 1 Notes

1. (a)	M1	Multiplies the three given roots together and sets the result equal to 15 or -15
	A1	Obtains a correct equation in α.
	M1	Forms a quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$
	A1	$\alpha=2\pm\text{i}$
	A1	Deduces the roots are $2+\text{i}, 2-\text{i}$ and 3
(b)	M1	Applies the process of finding $-\sum\left(\text{of their three roots found in part } (a)\right)$ to give $p=...$
	A1	$p=-7$ by correct solution only.
(b) ALT 1	M1	Applies the process expanding $(z-\text{"}3\text{"})(z-(\text{their sum})z+\text{their product})$ in order to find $p=...$
	A1	$p=-7$ by correct solution only.

Question	Scheme	Marks	AOs
2. (a)	$\mathbf{r}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\,\cos\alpha=\text{"}-3+5+6\text{"}$	M1	1.1b
	$\theta=90^\circ-\arccos\left(\dfrac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta=\dfrac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta=21.2^\circ$ (1 dp) * **cso**	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10\Rightarrow\lambda=...$	M1	3.1a
	$\lambda=-\dfrac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\dfrac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}...\\...\\...\end{pmatrix}$	M1	1.1b
	$X(7.5,\ 5.5,\ -3.5)$	A1ft	1.1b
		(4)	
	(10 marks)		

		Question 2 Notes
2. (a)	M1	Attempts to apply the formula **r.n** = **a.n**
	A1	Correct Cartesian notation. e.g. $3x-y+2z=10$ or $-3x+y-2z=-10$
	Note	Do not allow final answer given as $\mathbf{r}\bullet(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=10$, o.e.
(b)	B1	$\overrightarrow{OA}\bullet\mathbf{n}=8$
	M1	An attempt to apply the correct dot product formula between **n** and **d**.
	M1	Depends on previous M mark. Applies the dot product formula to find the angle between Π and l.
	A1*	21.2° **cso**
(c)	M1	Substitutes l into Π and solves the resulting equation to give $\lambda=....$
	A1	$\lambda=-\dfrac{1}{2}$ o.e.
	M1	Depends on previous M mark. Substitutes their λ into l and finds at least one of the coordinates.
	A1ft	$(7.5,\ 5.5,\ -3.5)$ but follow through on their value of λ.

Question	Scheme	Marks	AOs
3.	x = value of savings account, y = value of property bond account, z = value of share dealing account. $x+y+z=5000$	M1	3.1b
	$x+400=y$ $0.015x+0.035y-0.025z=79$ **or** $1.015x+1.035y+0.975z=5079$	A1	1.1b
	Let $\mathbf{A}=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}$ or $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1.015 & 1.035 & 0.975\end{pmatrix}$		
	e.g. $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}5000\\ -400\\ 79\end{pmatrix}$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}^{-1}\begin{pmatrix}5000\\ -400\\ 79\end{pmatrix}=\begin{pmatrix}...\\ ...\\ ...\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1800\\ 2200\\ 1000\end{pmatrix}$	A1	1.1b
	Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account.	A1ft	3.2a
		(7)	
	(7 marks)		
	Question 3 Notes		

3.	M1	Attempts to set up 3 equations with 3 unknowns.
	A1	At least 2 equations are correct with the appropriate variables defined.
	M1	Sets up a matrix equation of the form, e.g. $\begin{pmatrix}... & ... & ...\\ ... & ... & ...\\ ... & ... & ...\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}...\\ ...\\ ...\end{pmatrix}$, where "…" are numerical values.
	A1	Correct matrix equation (or equivalent).
	M1	Depends on previous M mark. Applies $(\text{their } \mathbf{A})^{-1}\begin{pmatrix}5000\\ \text{their "}-400\text{"}\\ \text{their "79"}\end{pmatrix}$ and obtains at least one value of x, y or z.
	A1	Correct answer.
	A1ft	Correct follow through answer in context.
	Note	$\mathbf{A}^{-1}=\begin{pmatrix}0.25 & 0.6 & 10\\ 0.25 & -0.4 & 10\\ 0.5 & -0.2 & -20\end{pmatrix}$ or $\begin{pmatrix}-9.75 & 0.6 & 10\\ -9.75 & -0.4 & 10\\ 20.5 & -0.2 & -20\end{pmatrix}$

Question	Scheme	Marks	AOs
4.	$\{w = x-1 \Rightarrow\}\ x = w+1$	B1	3.1a
	$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$	M1	3.1a
	$w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w + 1) - 8w - 8 + 6 = 0$		
	$w^3 + 6w^2 + w + 2 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
ALT 1	$\alpha + \beta + \gamma = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -8,\ \alpha\beta\gamma = -6$	B1	3.1a
	sum roots $= \alpha - 1 + \beta - 1 + \gamma - 1$	M1	3.1a
	$= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$		
	pair sum $= (\alpha-1)(\beta-1) + (\alpha-1)(\gamma-1) + (\beta-1)(\gamma-1)$		
	$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha+\beta+\gamma) + 3$		
	$= -8 - 2(-3) + 3 = 1$		
	product $= (\alpha-1)(\beta-1)(\gamma-1)$		
	$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha+\beta+\gamma) - 1$		
	$= -6 - (-8) - 3 - 1 = -2$		
	$w^3 + 6w^2 + w + 2 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
	(5 marks)		

Question 4 Notes

4.	B1	Selects the method of making a connection between *x* and *w* by writing $x = w+1$
	M1	Applies the process of substituting their $x = w+1$ into $x^3 + 3x^2 - 8x + 6 = 0$
	M1	Depends on previous M mark. Manipulating their equation into the form $w^3 + pw^2 + qw + r = 0$
	A1	At least two of *p*, *q*, *r* are correct.
	A1	Correct final equation.
ALT 1	B1	Selects the method of giving three correct equations each containing α, β and γ.
	M1	Applies the process of finding sum roots, pair sum and product.
	M1	Depends on previous M mark. Applies $w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their}\ \alpha\beta\gamma = 0$
	A1	At least two of *p*, *q*, *r* are correct.
	A1	Correct final equation.

Question	Scheme	Marks	AOs
5. (a)	$\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area}(S) = 4(5) = 20$	B1ft	1.2
		(1)	
(c)	$k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$	M1	1.1b
	$= 2$	A1ft	1.1b
		(2)	
(d)	$\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$	M1	1.1b
	$\theta = 60°$ or $\frac{\pi}{3}$	A1	1.1b
		(2)	
	(7 marks)		

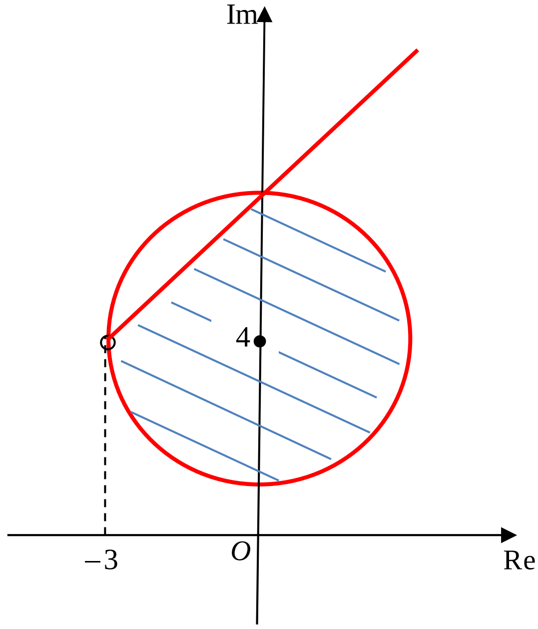
		Question 5 Notes
5. (a)	M1	An attempt to find $\det(\mathbf{M})$.
	A1	$\det(\mathbf{M}) = 4$ **and** reference to zero, e.g. $4 \neq 0$ **and** conclusion.
(b)	B1ft	20 or a correct ft based on their answer to part (a).
(c)	M1	$\sqrt{(\text{their } \det\mathbf{M})}$
	A1ft	2
(d)	M1	**Either** $\cos\theta = \frac{1}{(\text{their } k)}$ **or** $\sin\theta = \frac{\sqrt{3}}{(\text{their } k)}$ **or** $\tan\theta = \sqrt{3}$
	A1	$\theta = 60^{\circ}$ or $\frac{\pi}{3}$. Also accept any value satisfying $360n + 60^{\circ}$, $n \in \mathbb{Z}$, o.e.

Question	Scheme	Marks	AOs
6. (a)	$n=1,\ \sum_{r=1}^{1} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$	B1	2.2a
	Assume general statement is true for $n = k$. So assume $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true.	M1	2.4
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1	2.1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$	A1	1.1b
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$	A1	1.1b
	Then the general result is true for $n = k+1$. As the general result has been shown to be true for $n = 1$, then the general result is true for all $n \in \mathbb{Z}^+$.	A1	2.4
		(6)	
(b)	$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n} (r^3 - 36r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$	M1 A1	2.1 1.1b
	$= \frac{1}{4}n(n+1)\left[n(n+1) - 72\right]$	M1	1.1b
	$= \frac{1}{4}n(n+1)(n-8)(n+9)$ * **cso**	A1*	1.1b
		(4)	
(c)	$\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$	M1	1.1b
	$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$	M1	1.1b
	$3n^2 - 65n - 250 = 0$	A1	1.1b
	$(3n+10)(n-25) = 0$	M1	1.1b
	(As n must be a positive integer,) $n = 25$	A1	2.3
		(5)	
	(15 marks)		

		Question 6 Notes
6. (a)	B1	Checks $n=1$ works for both sides of the general statement.
	M1	Assumes (general result) true for $n=k$.
	M1	Attempts to add $(k+1)$th term to the sum of k terms.
	A1	Correct algebraic work leading to either $\frac{1}{6}(k+1)(2k^2+7k+6)$
		or $\frac{1}{6}(k+2)(2k^2+5k+3)$ or $\frac{1}{6}(2k+3)(k^2+3k+2)$
	A1	Correct algebraic work leading to $\frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$
	A1	**cso** leading to a correct induction statement conveying **all** three underlined points.
(b)	M1	Substitutes at least one of the standard formulae into their expanded expression.
	A1	Correct expression.
	M1	Depends on previous M mark. Attempt to factorise at least $n(n+1)$ having used both standard formulae correctly.
	A1*	Obtains $\frac{1}{4}n(n+1)(n-8)(n+9)$ by **cso**.
(c)	M1	Sets their part (a) answer equal to $\frac{17}{6}n(n+1)(2n+1)$
	M1	Cancels out $n(n+1)$ from both sides of their equation.
	A1	$3n^2 - 65n - 250 = 0$
	M1	A valid method for solving a 3 term quadratic equation.
	A1	**Only one solution** of $n=25$

Question	Scheme		Marks	AOs
7. (a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y = 1 + kx^2 \Rightarrow 1.16 = 1 + k(0.2)^2 \Rightarrow k = ...$		M1	3.3
	$\Rightarrow k = 4$ **cao** $\left\{\text{So } y = 1 + 4x^2\right\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int (y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$= \left\{\frac{\pi}{4}\right\}\int_1^{1.16} (y-1)\mathrm{d}y$	$= \left\{\frac{\pi}{4}\right\}\int_0^{0.16} y\mathrm{d}y$	M1	3.3
	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2} - y\right]_1^{1.16}$	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1 A1	1.1b 1.1b
	$= \frac{\pi}{4}\left(\left(\frac{1.16^2}{2} - 1.16\right) - \left(\frac{1}{2} - 1\right)\right) \{= 0.0032\pi\}$	$= \frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right) - (0)\right) \{= 0.0032\pi\}$		
	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)\ \{= 0.0464\pi\}$		B1	1.1b
	$\text{Volume } = 0.0464\pi - 0.0032\pi\ \{= 0.0432\pi\}$		M1	3.4
	$= 0.1357168026... = 0.136(\text{m}^3)\ (3\text{sf})$		A1	1.1b
			(7)	
(d)	Any one of e.g. The measurements may not be accurate. The inside surface of the bowl may not be smooth. There may be wastage of concrete when making the bird bath.		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason. e.g. $\left[\left(\frac{0.136 - 0.127}{0.127}\right) \times 100 = 7.0866...\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model. **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used.		B1ft	3.5a
			(1)	
	(12 marks)			

		Question 7 Notes
7. (a)	B1	Infers that the maximum depth of the bird bath could be 0.16 (m).
(b)	M1	Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$ and rearranges to give $k = ...$
	A1	$k = 4$ **cao**
(c)	B1ft	Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int (y-1)\,\mathrm{d}y$ or $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$
	M1	Chooses limits that are appropriate to their model.
	M1	Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant.
	A1	Uses their model correctly to give either $y - 1 \to \frac{y^2}{2} - y$ or $y \to \frac{y^2}{2}$
	B1	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π or $\frac{29}{625}\pi$, o.e.
	M1	Depends on **both** previous M marks. Uses the model to find $V_{\text{their cylinder}}$ – their integrated volume.
	A1	0.136 **cao**
(d)	B1	States an acceptable limitation of the model.
(e)	B1ft	Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason.

Question	Scheme	Marks	AOs
8. (a)		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(6)	
(b)	$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
	$= 2.42\ (2\text{dp})$ **cao**	A1	1.1b
		(2)	
	(8 marks)		

		Question 8 Notes
8. (a)	M1	Circle.
	A1	Centre $(0,4)$ and above the real axis.
	M1	Half-line.
	A1	$(-3,4)$ positioned correctly and the half-line intersects the top of the circle on the *y*-axis.
	M1	Depends on **both** previous M marks Shades in a region inside the circle and below the half line.
	A1	**Cso**
	Note	Final A1 mark is dependent on all previous marks being scored in part (a).
(b)	M1	Uses trigonometry to give an expression for an angle in the range $\left(\frac{\pi}{2}, \pi\right)$ or $(90°, 180°)$.
	A1	2.42 **cao**

Question	Scheme	Marks	AOs
9. (a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{= \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{\overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$	M1	1.1b
	$\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$		
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{\overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	M1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish *F*.	A1ft	3.2a
		(7)	
(b)	E.g. Fish *F* may not swim in an exact straight line from *A* to *B*. Fish *F* may hit an obstacle whilst swimming from *A* to *B*. Fish *F* may deviate his path to avoid being caught by the octopus.	B1	3.5b
		(1)	
(c)	E.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed. Octopus may during the fish *F*'s motion move away from its fixed location at *O*.	B1	3.5b
		(1)	
	(9 marks)		

Question	Scheme	Marks	AOs
9. (a) ALT 1	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{ = \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert . \lvert\overrightarrow{AB}\rvert} \right\} = \dfrac{\pm\left(\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right)}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} . \sqrt{(12)^2 + (3)^2 + (18)^2}}$	dM1	1.1b
	$\left\{ \cos\theta = \dfrac{-36 + 3 - 126}{\sqrt{59}.\sqrt{477}} = \dfrac{-159}{\sqrt{59}.\sqrt{477}} \right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2}\sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	
9. (a) ALT 2	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	

		Question 9 Notes
9. (a)	M1	Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**.
	M1	Applies $\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent.
	M1	Depends on previous M mark. Writes down $(\text{their } \overrightarrow{OF} \text{ which is in terms of } \lambda)\bullet(\text{their } \overrightarrow{AB}) = 0.$ Can be implied.
	A1	Lambda is correct. e.g. $\lambda = \frac{1}{3}$ for $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\lambda = 1$ for $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$
	M1	Depends on previous M mark. Complete method for finding $\left\lvert\overrightarrow{OF}\right\rvert$.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
9. (a) ALT 1	M1	Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**.
	M1	Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	M1	Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	A1	θ = awrt 161.4 or awrt 18.6 or $\sin\theta$ = awrt 0.319
	M1	Depends on previous M mark. $(\text{their } OA)\sin(\text{their } \theta)$
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
9. (a) ALT 2	M1	Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**.
	M1	Applies $\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent.
	M1	Depends on previous M mark. Applies Pythagoras by finding $\left\lvert\overrightarrow{OF}\right\rvert^2$, o.e.
	A1	$\left\lvert\overrightarrow{OF}\right\rvert^2 = 477\lambda^2 - 318\lambda + 59$
	M1	Depends on previous M mark. Method of completing the square or differentiating their $\left\lvert\overrightarrow{OF}\right\rvert^2$ w.r.t. λ.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(b)	B1	An acceptable criticism for fish *F*, which is in context with the question.
(c)	B1	An acceptable criticism for the octopus, which is in context with the question.

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2A: Further Pure Mathematics 1 and Further Pure Mathematics 2

Sample assessment material for first teaching September 2017	Paper Reference(s)
Time: 1 hour 40 minutes	**8FM0/2A**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate to the nearest £ the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

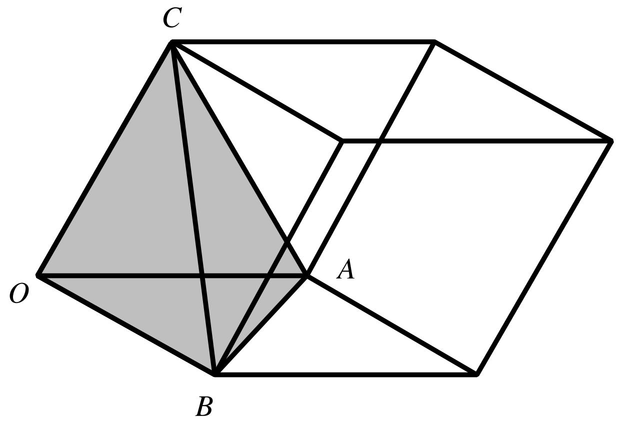


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

....

....

....

....

....

....

....

....

....

Question 4 continued

(Total for Question 4 is 10 marks)

5.

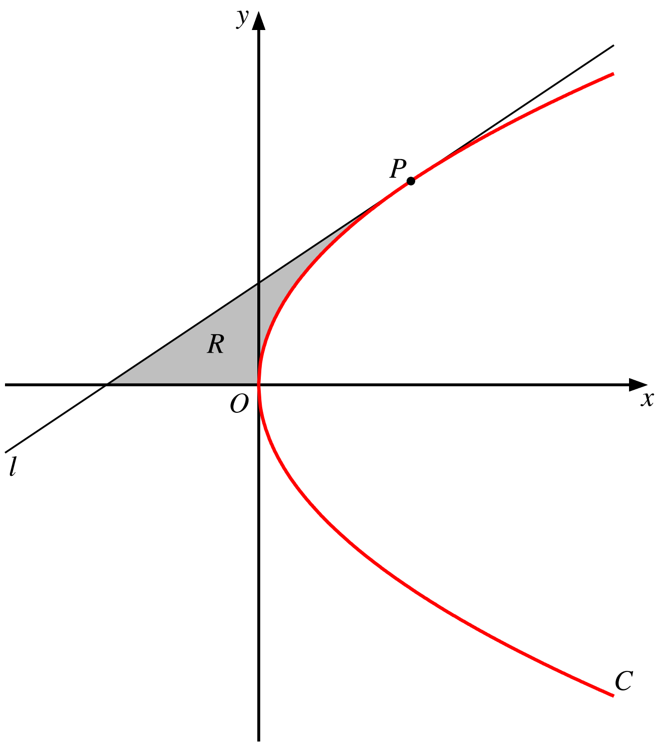


Diagram not drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided

6. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix **A**.

(2)

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

Question 6 continued

(Total for Question 6 is 5 marks)

7. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

(ii) Use the Euclidean algorithm to find integers a and b such that

$$27a + 31b = 1$$

(4)

Question 7 continued

(Total for Question 7 is 6 marks)

8. A curve C is described by the equation

$$|z - 9 + 12\text{i}| = 2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \leqslant \text{Re}(w) \leqslant b$$

(2)

Question 8 continued

(Total for Question 8 is 8 marks)

9. The operation * is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y - xy \pmod 7$

*	0	2	3	4	5	6
0						
2		0				
3						5
4						
5		4				
6						

(a) (i) Complete the Cayley table shown above

(ii) Show that S is a group under the operation *

(You may assume the associative law is satisfied.)

(6)

(b) Show that the element 4 has order 3

(2)

(c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

Question 9 continued

(Total for Question 9 is 11 marks)

10. A population of deer on a large estate is assumed to increase by 10% during each year due to natural causes.

The population is controlled by removing a constant number, Q, of the deer from the estate at the end of each year.

At the start of the first year there are 5000 deer on the estate.

Let P_n be the population of deer at the end of year n.

(a) Explain, in the context of the problem, the reason that the deer population is modelled by the recurrence relation

$$P_n = 1.1P_{n-1} - Q, \qquad P_0 = 5000, \qquad n \in \mathbb{Z}^+$$

(3)

(b) Prove by induction that $P_n = (1.1)^n(5000 - 10Q) + 10Q, \qquad n \geqslant 0$

(5)

(c) Explain how the long term behaviour of this population varies for different values of Q.

(2)

(Total for Question 10 is 10 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option A

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1. (a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
		(6 marks)	

Notes

1. (a)

M1 Uses $\sec x = \dfrac{1}{\cos x}$ and the t-substitutions for both cos x and tan x to obtain an expression in terms of t.

M1 Sorts out the sec x term, and puts over a common denominator of $1-t^2$

A1* Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer.

(b)

M1 Uses the t-substitution for sin x in both numerator and denominator.

M1 Multiples through by $1+t^2$ in numerator and denominator.

A1* Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result.

Question	Scheme	Marks	AOs
2.	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes

B1	Identifies the correct initial conditions and requirement for h.
M1	Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0.
M1	Applies the approximation formula with their values.
A1ft	3.5 or exact equivalent. Follow through their step value.
M1	Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5
A1	Applies the approximation and interprets the result to give £396.

Question	Scheme	Marks	AOs
3.	$\dfrac{1}{x} < \dfrac{x}{x+2}$		
	$\dfrac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\dfrac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \dfrac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R}: x < -2\} \cup \{x \in \mathbb{R}: -1 < x < 0\} \cup \{x \in \mathbb{R}: x > 2\}$	M1	2.2a
		A1	2.5
		(6)	
			(6 marks)

Notes

M1	Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side.
M1	Factorise numerator or find roots of numerator or factorise resulting inequation into 4 factors.
A1	At least 2 correct critical values found.
A1	Exactly 4 correct critical values.
M1	Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means.
A1	Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set. E.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R}: x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$.

Question	Scheme	Marks	AOs
4. (a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
(a) **ALT 1**	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB}.\mathbf{AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB}.\mathbf{AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=...$	M1	3.1a
	Volume of glass = $\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
4. (b) ALT	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB=\frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert=\frac{1}{2}\times 2\times\sqrt{10}=\sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+\mathbf{2k}-p(-\mathbf{j}+\mathbf{3k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})\Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+\mathbf{3k}\rvert=\frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V=5\times$ Volume of tetrahedron $=5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$=\frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete. Measurements in m may not be accurate. The surface of the concrete tetrahedron may not be smooth. Pockets of air may form when the concrete is being poured.	B1	3.2b
		(1)	
		(10 marks)	

Notes

4. (a) Accept use of column vectors throughout.

M1 Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*. Any correct method for the triangle area is fine.

M1 Finds **AB** and **AC** or any other appropriate pair of vectors to use in the vector product and attempts to use them.

M1 Correct procedure for the vector product with at least 1 correct term.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(a) ALT

M1 As main method.

M1 Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides. May use different sides to those shown.

M1 Correct full method to find the area of the triangle using their two sides.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(b)

M1 Attempts volume of concrete by finding volume of tetrahedron with appropriate method.

M1 Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted.

A1 Correct value for the volume of concrete.

M1 Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped.

A1 $\frac{25}{3}$ only, ignore reference to units.

Notes	
4. (b) ALT	
M1	Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$
A1	Finds (using that **OA** and **OB** are perpendicular), area of $AOB=\sqrt{10}$.
M1	Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get height of tetrahedron. $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert=\frac{1}{2}\sqrt{10}\right]$
M1	Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference).
A1	$\frac{25}{3}$ only, ignore reference to units.
(c)	
B1	Any acceptable reason in context.

Question	Scheme	Marks	AOs
5. (a)	$y^2=(8p)^2=64p^2$ and $16x=16(4p^2)=64p^2$ $\Rightarrow$ $P(4p^2,8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2=16x$ gives $a=4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x}=16$ so $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{y}$	M1	2.2a
	$l:\ y-8p=\left(\frac{8}{8p}\right)\left(x-4p^2\right)$	M1	1.1b
	leading to $py=x+4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4,\frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3}=-4+4p^2$	M1	3.1a
	$6p^2-5p-6=0\Rightarrow(2p-3)(3p+2)=0\Rightarrow p=\ldots$	M1	1.1b
	$p=\frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0)=x+4\left(\frac{3}{2}\right)^2\Rightarrow x=\ldots$	M1	2.1
	$x=-9$	A1	1.1b
	$p=\frac{3}{2}\Rightarrow P(9,12)\Rightarrow$ Area$(R)=\frac{1}{2}(9--9)(12)-\int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x=\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}(+c)$ or $\frac{8}{3}x^{\frac{3}{2}}(+c)$	M1	1.1b
		A1	1.1b
	Area$(R)=\frac{1}{2}(18)(12)-\frac{8}{3}\left(9^{\frac{3}{2}}-0\right)=108-72=36$ *	A1*	1.1b
		(8)	

(c) ALT 1	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) dy$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
5. (c) ALT 2	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow$ Area$(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) dx$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$ $(+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Notes

5. (a)

B1 Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C.

(b)

M1 Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it.

M1 Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c.

A1* Obtains $py = x + 4p^2$ by **cso**.

Notes Continued	
5. (c)	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}} \mathrm{d}x$
M1	Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}},$ where $\lambda, \mu \neq 0$
A1	Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36
(c) ALT 1	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$
A1	Finds *l* as $x = \frac{3}{2}y - 9$
M1	Fully correct method for finding the area of *R*. i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right) \mathrm{d}y$
M1	Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$,where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Notes Continued	
5. (c) ALT 2	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}y$
M1	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Further Pure Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6(a)	Consider $\det\begin{pmatrix} 3-\lambda & 1 \\ 6 & 4-\lambda \end{pmatrix} = (3-\lambda)(4-\lambda)-6$	M1	1.1b
	So $\lambda^2 - 7\lambda + 6 = 0$ is characteristic equation	A1	1.1b
		(2)	
(b)	So $\mathbf{A}^2 = 7\mathbf{A} - 6\mathbf{I}$	B1ft	1.1b
	Multiplies both sides of their equation by **A** so $\mathbf{A}^3 = 7\mathbf{A}^2 - 6\mathbf{A}$	M1	3.1a
	Uses $\mathbf{A}^3 = 7(7\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$ So $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$ *	A1*cso	1.1b
		(3)	
			(5 marks)

Notes:

(a)
M1: Complete method to find characteristic equation
A1: Obtains a correct three term quadratic equation – may use variable other than λ

(b)
B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with **A** and constant term with constant multiple of identity matrix, **I**
M1: Multiplies equation by **A**
A1*: Replaces $\mathbf{A}^2$ by linear expression in **A** and achieves printed answer with no errors

Question	Scheme	Marks	AOs
7(i)	Adding digits 8 + 1 + 8 + 4 = 21 which is divisible by 3 (or continues to add digits giving 2+1=3 which is divisible by 3) so concludes that 8184 is divisible by 3	M1	1.1b
	8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6	A1	1.1b
		(2)	
(ii)	Starts Euclidean algorithm $31=27\times1+4$ and $27 = 4 \times 6 + 3$	M1	1.2
	$4 = 3 \times 1 + 1$ (so hcf = 1)	A1	1.1b
	So $1 = 4 - 3 \times 1 = 4 - (27 - 4 \times 6) \times 1 = 4 \times 7 - 27 \times 1$	M1	1.1b
	$(31 - 27 \times 1) \times 7 - 27 \times 1 = 31 \times 7 - 27 \times 8$ $a = -8$ and $b = 7$	A1cso	1.1b
		(4)	
			(6 marks)

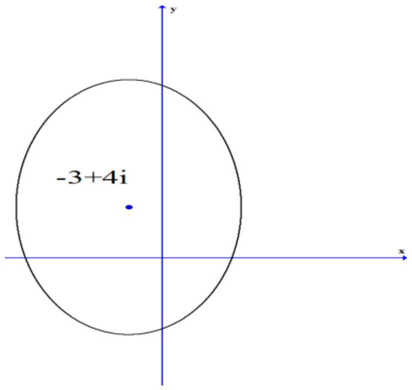
Notes:

(i)

M1: Explains divisibility by 3 rule in context of this number by adding digits
A1: Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6

(ii)

M1: Uses Euclidean algorithm showing two stages
A1: Completes the algorithm. Does not need to state that hcf = 1
M1: Starts reversal process, doing two stages and simplifying
A1cso: Correct completion, giving clear answer following complete solution

Question	Scheme	Marks	AOs
8(a)	$(x-9)^2+(y+12)^2=4[x^2+y^2]$	M1	2.1
	$3x^2+3y^2+18x-24y-225=0$ which is the equation of a circle	A1*	2.2a
	As $x^2+y^2+6x-8y-75=0$ so $(x+3)^2+(y-4)^2=10^2$	M1	1.1b
	Giving centre at (-3, 4) and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from **their** – 3 –10 to their –3 + 10	M1	3.1a
	So $-13 \le \text{Re}(w) \le 7$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Obtains an equation in terms of *x* and *y* using the given information

A1: Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle

M1: Completes the square for their equation to find centre and radius

A1ft:

(b)

M1: Draws a circle with centre and radius as given from **their** equation

A1: Correct circle drawn, as above, with centre at – 3 + 4i and passing through all four quadrants

(c)

M1: Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using " their – 3 –10" to "their –3 + 10"

A1ft: Correctly obtains the correct answer for their centre and radius

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="6">9(a)</td><td>(i)
<table>
<tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>2</td><td>2</td><td>0</td><td></td><td></td><td>4</td><td></td></tr>
<tr><td>3</td><td>3</td><td></td><td></td><td></td><td></td><td>5</td></tr>
<tr><td>4</td><td>4</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>5</td><td>4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>6</td><td>6</td><td></td><td>5</td><td></td><td></td><td></td></tr>
</table></td><td>M1</td><td>1.1b</td></tr>
<tr><td><table>
<tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>2</td><td>2</td><td>0</td><td>6</td><td>5</td><td>4</td><td>3</td></tr>
<tr><td>3</td><td>3</td><td>6</td><td>4</td><td>2</td><td>0</td><td>5</td></tr>
<tr><td>4</td><td>4</td><td>5</td><td>2</td><td>6</td><td>3</td><td>0</td></tr>
<tr><td>5</td><td>5</td><td>4</td><td>0</td><td>3</td><td>6</td><td>2</td></tr>
<tr><td>6</td><td>6</td><td>3</td><td>5</td><td>0</td><td>2</td><td>4</td></tr>
</table></td><td>M1 A1</td><td>1.1b
1.1b</td></tr>
<tr><td>(ii) Identity is zero and there is closure as shown above</td><td>M1</td><td>2.1</td></tr>
<tr><td>3 and 5 are inverses, 4 and 6 are inverses, 2 is self inverse,
0 is identity so is self inverse</td><td>M1</td><td>2.5</td></tr>
<tr><td>Associative law may be assumed so S forms a group</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(6)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td>4*4*4 = 4* (4 * 4) = 4 * 6 or 4*4*4 = (4* 4) * 4 = 6* 4</td><td>M1</td><td>2.1</td></tr>
<tr><td>= 0 (the identity) so 4 has order 3</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="3">(c)</td><td>3 and 5 each have order 6 so either generates the group</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Either $3^1 = 3,\ 3^2 = 4,\ 3^3 = 2,\ 3^4 = 6,\ 3^5 = 5,\ 3^6 = 0$
Or $5^1 = 5,\ 5^2 = 6,\ 5^3 = 2,\ 5^4 = 4,\ 5^5 = 3,\ 5^6 = 0$</td><td>A1, A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(11 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a)(i)
M1: Begins completing the table – obtaining correct first row and first column and using symmetry
M1: Mostly correct – three rows or three columns correct (so demonstrates understanding of using *
A1: Completely correct</td></tr>
<tr><td colspan="4">(a)(ii)
M1: States closure and identifies the identity as zero
M1: Finds inverses for each element</td></tr>
</table>

A1: States that associative law is satisfied and so all axioms satisfied and *S* is a group
(b) **M1:** Clearly begins process to find 4*4*4 reaching 6*4 or 4*6 with clear explanation **A1:** Gives answer as zero, states identity and deduces that order is 3
(c) **M1:** Finds either 3 or 5 or both **A1:** Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself) **A1:** Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)

Question	Scheme	Marks	AOs
10(a)	P_{n-1} is the population at the end of year n – 1 and this is increased by 10% by the end of year n, so is multiplied by 110% = 1.1 to give $1.1\times P_{n-1}$ as new population by natural causes	B1	3.3
	Q is subtracted from $1.1\times P_{n-1}$ as Q is the number of deer removed from the estate	B1	3.4
	So $P_n = 1.1P_{n-1} - Q,\quad P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$	B1	1.1b
		(3)	
(b)	Let n = 0, then $P_0 = (5000-10Q)(1.1)^0 + 10Q = 5000$ so result is true when n = 0	B1	2.1
	Assume result is true for $n = k$, $P_k = (1.1)^k(5000-10Q)+10Q$, then **as** $P_{k+1} = 1.1P_k - Q$, so $P_{k+1} = ...$	M1	2.4
	$P_{k+1} = 1.1\times 1.1^k(5000-10Q) + 1.1\times 10Q - Q$	A1	1.1b
	So $P_{k+1} = (5000-10Q)(1.1)^{k+1} + 10Q,$	A1	1.1b
	Implies result holds for $n = k + 1$ and so by induction $P_n = (5000-10Q)(1.1)^n + 10Q,$ is true for all integer n	B1	2.2a
		(5)	
(c)	For $Q < 500$ the population of deer will grow, for $Q > 500$ the population of deer will fall	B1	3.4
	For $Q = 500$ the population of deer remains steady at 5000,	B1	3.4
		(2)	
			(10 marks)

Notes:

(a)
B1: Need to see 10% increase linked to multiplication by scale factor 1.1
B1: Needs to explain that subtraction of Q indicates the removal of Q deer from population
B1: Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q,\quad P_0 = 5000$ being the initial number of deer

(b)
B1: Begins proof by induction by considering n = 0
M1: Assumes result is true for $n = k$ and uses iterative formula to consider $n = k + 1$
A1: Correct algebraic statement
B1: Correct statement for k + 1 in required form
B1: Completes the inductive argument

(c)
B1: Consideration of both possible ranges of values for Q as listed in the scheme
B1: Gives the condition for the steady state

Pearson Edexcel Level 3 GCE	
# Further Mathematics **Advanced Subsidiary** **Paper 2: Further Mathematics options** **Option 2B: Further Pure Mathematics 1 and Further Statistics 1**	
Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2B**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate to the nearest £ the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

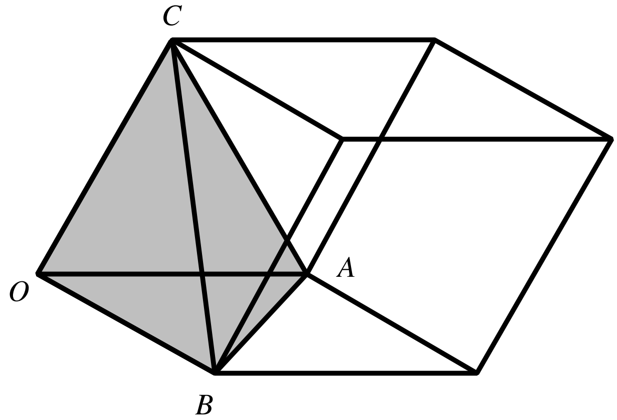


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

....

....

....

....

....

....

....

....

....

Question 4 continued

(Total for Question 4 is 10 marks)

5.

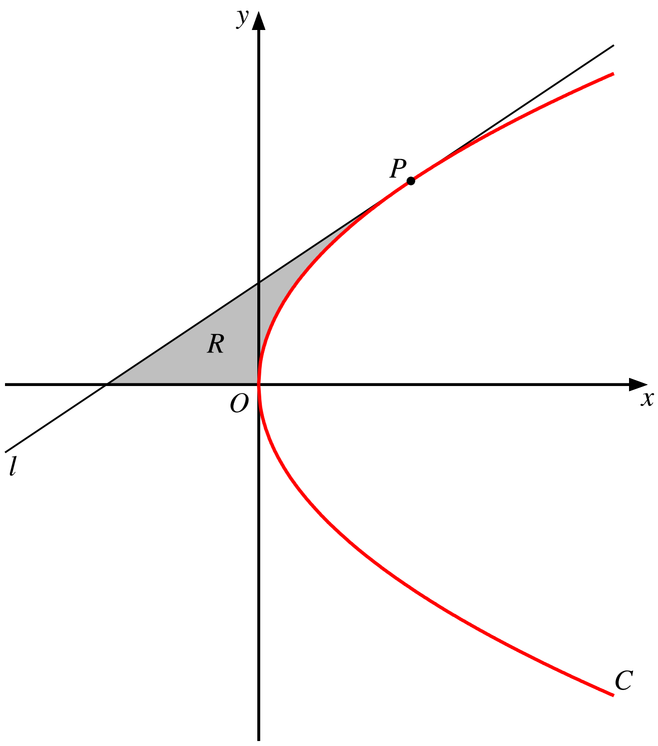


Diagram not drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.
(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3
(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9
(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.
(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.
(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 11 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option B

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1. (a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
		(6 marks)	

Notes

1. (a)

M1 Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*.

M1 Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1* Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer.

(b)

M1 Uses the *t*-substitution for sin *x* in both numerator and denominator.

M1 Multiples through by $1+t^2$ in numerator and denominator.

A1* Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result.

Question	Scheme	Marks	AOs
2.	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step *h* required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes

B1	Identifies the correct initial conditions and requirement for *h*.
M1	Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0.
M1	Applies the approximation formula with their values.
A1ft	3.5 or exact equivalent. Follow through their step value.
M1	Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5
A1	Applies the approximation and interprets the result to give £396.

Question	Scheme	Marks	AOs
3.	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1	2.2a
		A1	2.5
		(6)	
		(6 marks)	

Notes

M1	Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side.
M1	Factorise numerator or find roots of numerator or factorise resulting inequation into 4 factors.
A1	At least 2 correct critical values found.
A1	Exactly 4 correct critical values.
M1	Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means.
A1	Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set. E.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$.

Question	Scheme	Marks	AOs
4. (a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
(a) ALT 1	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
4. (b) **ALT**	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB=\frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert=\frac{1}{2}\times2\times\sqrt{10}=\sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+\mathbf{2k}-p(-\mathbf{j}+\mathbf{3k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})\Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+\mathbf{3k}\rvert=\frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V=5\times$ Volume of tetrahedron $=5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$=\frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete. Measurements in m may not be accurate. The surface of the concrete tetrahedron may not be smooth. Pockets of air may form when the concrete is being poured.	B1	3.2b
		(1)	
			(10 marks)

Notes

4. (a) Accept use of column vectors throughout.

M1 Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*. Any correct method for the triangle area is fine.

M1 Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them.

M1 Correct procedure for the vector product with at least 1 correct term.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(a) ALT

M1 As main method.

M1 Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides. May use different sides to those shown.

M1 Correct full method to find the area of the triangle using their two sides.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(b)

M1 Attempts volume of concrete by finding volume of tetrahedron with appropriate method.

M1 Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted.

A1 Correct value for the volume of concrete.

M1 Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped.

A1 $\frac{25}{3}$ only, ignore reference to units.

Notes	
4. (b) ALT	
M1	Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$
A1	Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$.
M1	Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k} - p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j}+\mathbf{k})$ to get height of tetrahedron. $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}\right]$
M1	Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference).
A1	$\frac{25}{3}$ only, ignore reference to units.
(c)	
B1	Any acceptable reason in context.

Question	Scheme	Marks	AOs
5. (a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow$ $P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\,dx$	M1	2.1
	$\int 4x^{\frac{1}{2}}\,dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

(c) ALT 1	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) \mathrm{d}y$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) \mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
5. (c) ALT 2	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = \ldots$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \ldots$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Notes

5. (a)

B1 — Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C.

(b)

M1 — Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it.

M1 — Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c.

A1* — Obtains $py = x + 4p^2$ by **cso**.

Notes Continued	
5. (c)	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*.
	i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,dx$
M1	Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$
A1	Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36
(c) ALT 1	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$
A1	Finds *l* as $x = \frac{3}{2}y - 9$
M1	Fully correct method for finding the area of *R*.
	i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)dy$
M1	Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Notes Continued	
5. (c) ALT 2	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their } \left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}y$
M1	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Further Statistics 1 Mark Scheme (Section B)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">6(a)</td><td>H_0: There is no association between language and gender.</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>$\frac{54 \times 85}{150} = 30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="4">(c)</td><td>
<table>
<tr><td colspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td></td><td></td><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \ldots + \frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>awrt <u>3.6/3.7</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td>(a)</td><td colspan="3">B1 for correct hypothesis in context</td></tr>
<tr><td>(b)</td><td colspan="3">B1* for a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td>(c)</td><td colspan="3">M1 for attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1 for applying $\sum \frac{(O-E)^2}{E}$
A1 awrt 3.6 or 3.7</td></tr>
<tr><td>(d)</td><td colspan="3">M1 for using degrees of freedom to set up a χ^2 model critical value
A1 for correct comparison and conclusion</td></tr>
<tr><td>(e)</td><td colspan="3">B1 for correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
7(a)	$-4 = 2 - 5\mathrm{E}(X)$	M1	3.1a
	$\mathrm{E}(X) = 1.2$		
	$-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ [1]		
	$\mathrm{P}(Y \geqslant -3) = 0.45$ gives $\mathrm{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\mathrm{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45$ [2]		
	$2a + b + 2c = 1$ [3]	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ or	M1	1.1b
	e.g. [3] − [2] $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into [1] $\Rightarrow a = 0.1$ etc		
	$a = 0.1$ $b = 0.3$ $c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\mathrm{Var}(Y) = 75 - (-4)^2$ or 59	M1	1.1a
	[$\mathrm{Var}(Y) = 5^2\mathrm{Var}(X)$ implies] $\mathrm{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\mathrm{P}(Y > X) = \mathrm{P}(2 - 5X > X) \rightarrow \mathrm{P}(X < \frac{1}{3})$	M1	3.1a
	$\mathrm{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes

(a)	1st M1 for using given information to find an expression for E(X) i.e. use of $\mathrm{E}(Y) = 2 - 5\mathrm{E}(X)$ 2nd M1 for use of $\sum x\mathrm{P}(X = x) =$ '1.2' 3rd M1 for use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \mathrm{P}(X = x) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values	1st M1 for using given information to find the probability distribution for Y leading to an expression for E(Y) 2nd M1 for use of $\sum y\mathrm{P}(Y = y) = -4$ 3rd M1 for use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \mathrm{P}(Y = y) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values
(b)	M1 for use of $\mathrm{Var}(Y) = \mathrm{E}(Y^2) - [\mathrm{E}(Y)]^2$ (may be implied by a correct answer) A1 for use of $\mathrm{Var}(aX) = a^2\,\mathrm{Var}(X)$ to reach 2.36 or exact equivalent	
(c)	M1 for rearranging to the form $\mathrm{P}(X < k)$ A1ft '0.1' + '025' (provided their a and c and their a+c are all probabilities)	M1 for comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ A1ft '0.1' + '025' (provided their a and c and their a+c are all probabilities)

Question	Scheme	Marks	AOs
8(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) = $\text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)(i)	Mean = np = **<u>2.4</u>**	B1	1.1b
(d)(ii)	Variance = $np(1 - p) = 2.3904$ awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)(i)	$[D \sim \text{Po}(2.4)$ $\text{P}(D \leq 4)]$		
	$= 0.9041\ldots$ awrt **<u>0.904</u>**	B1	1.1b
(e)(ii)	Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes

(a)	M1 for $\text{P}(X=2)\times\text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) A1 awrt 0.0544
(b)	M1 for combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1 awrt 0.205
(c)	M1 for setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1 awrt 0.648
(d)(i) **(d)(ii)**	B1 for 2.4 B1 for awrt 2.39
(e)(i) **(e)(ii)**	B1 for 0.904 B1 for a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
9(a)(i)	P(X = 1) = 0.34523… awrt **0.345**	B1	1.1b
(a)(ii)	P($X \leqslant$ 4) = 0.98575… awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times 10)+1\times 16+2\times 7+3\times 4+4\times 2+(5\times 0)+6\times 1}{40}=1.4^*$	B1*cso	1.1b
		(1)	
(c)	r = 40 × ‘0.34523…’ s = 40 × ‘1 – 0.986…’	M1	3.4
	r = **13.81** s = **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+...+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since 1.10< $\chi^2_{2,(0.05)}=5.991$).The number of mortgages approved each week follows a Poisson distribution.	A1	3.5a
		(6)	
			(11 marks)

Notes	
(a)(i) **(a)(ii)**	B1 awrt 0.345 B1 awrt 0.986
(b)	B1* for a fully correct calculation leading to given answer with no errors seen
(c)	M1 for attempt at r or s (may be implied by correct answers) A1ft for both values correct (follow through their answers to part (a))
(d)	1st B1 for both hypotheses correct (lambda should not be defined so correct use of the model) 1st M1 for understanding the need to combine cells before calculating the test statistic (may be implied) 2nd M1 for attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$ 1st A1 awrt 1.1 2nd B1 for realising that there are 2 degrees of freedom leading to a critical value of $\chi_2{}^2(0.05)=5.991$ 2nd A1 concluding that a Poisson model is suitable for the number of mortgages approved each week

Pearson Edexcel Level 3 GCE	
Further Mathematics **Advanced Subsidiary** **Paper 2: Further Mathematics options** **Option 2C: Further Pure Mathematics 1 and Further Mechanics 1**	
Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2C**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate to the nearest £ the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

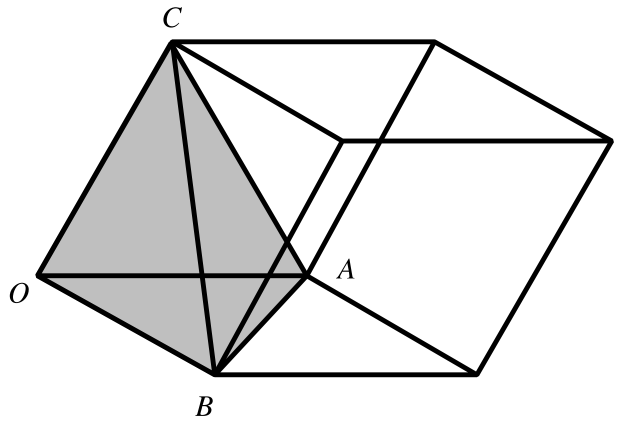


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

....

....

....

....

....

....

....

....

....

Question 4 continued

(Total for Question 4 is 10 marks)

5.

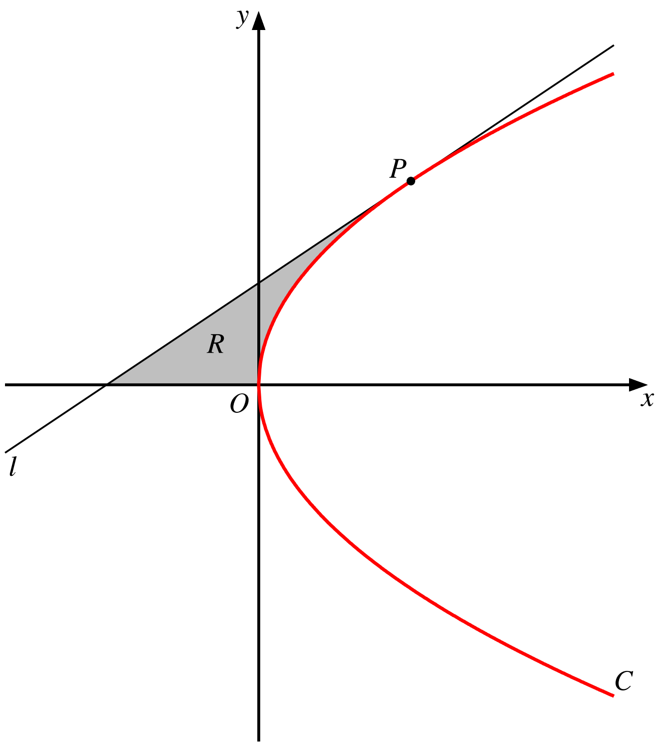


Diagram not drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{dy}{dx} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B
Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5
(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.
(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.
(1)

Question 6 continued

(Total for Question 6 is 10 marks)

7. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of $25\,\mathrm{m\,s^{-1}}$. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 7 continued

(Total for Question 7 is 6 marks)

8. [*In this question use g =10 m s*$^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of 4 $\mathrm{m\,s^{-1}}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\mathrm{m\,s^{-1}}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\mathrm{m\,s^{-1}}$.

(5)

Question 8 continued

(Total for Question 8 is 8 marks)

9. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option C

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1. (a)	$\sec x - \tan x = \dfrac{1}{\frac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes

1. (a)

M1 Uses $\sec x = \dfrac{1}{\cos x}$ and the t-substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of t.

M1 Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$

A1* Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer.

(b)

M1 Uses the t-substitution for $\sin x$ in both numerator and denominator.

M1 Multiples through by $1+t^2$ in numerator and denominator.

A1* Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result.

Question	Scheme	Marks	AOs
2.	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step *h* required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = ...$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963...$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes

B1 Identifies the correct initial conditions and requirement for *h*.

M1 Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0.

M1 Applies the approximation formula with their values.

A1ft 3.5 or exact equivalent. Follow through their step value.

M1 Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1 Applies the approximation and interprets the result to give £396.

Question	Scheme	Marks	AOs
3.	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1	2.2a
		A1	2.5
		(6)	
			(6 marks)

Notes

M1	Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side.
M1	Factorise numerator or find roots of numerator or factorise resulting inequation into 4 factors.
A1	At least 2 correct critical values found.
A1	Exactly 4 correct critical values.
M1	Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means.
A1	Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set. E.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$.

Question	Scheme	Marks	AOs
4. (a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
(a) ALT 1	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=...$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
4. (b) ALT	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB=\frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert=\frac{1}{2}\times 2\times\sqrt{10}=\sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+\mathbf{2k}-p(-\mathbf{j}+\mathbf{3k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})\Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+\mathbf{3k}\rvert=\frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $=5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$=\frac{25}{3}\left(\mathrm{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete. Measurements in m may not be accurate. The surface of the concrete tetrahedron may not be smooth. Pockets of air may form when the concrete is being poured.	B1	3.2b
		(1)	
			(10 marks)

Notes

4. (a) Accept use of column vectors throughout.

M1 Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*. Any correct method for the triangle area is fine.

M1 Finds **AB** and **AC** or any other appropriate pair of vectors to use in the vector product and attempts to use them.

M1 Correct procedure for the vector product with at least 1 correct term.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(a) ALT

M1 As main method.

M1 Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides. May use different sides to those shown.

M1 Correct full method to find the area of the triangle using their two sides.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(b)

M1 Attempts volume of concrete by finding volume of tetrahedron with appropriate method.

M1 Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted.

A1 Correct value for the volume of concrete.

M1 Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped.

A1 $\frac{25}{3}$ only, ignore reference to units.

Notes	
4. (b) ALT	
M1	Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$
A1	Finds (using that **OA** and **OB** are perpendicular), area of $AOB=\sqrt{10}$.
M1	Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get height of tetrahedron. $\left[(\mu=\lambda=)\ p=\frac{1}{2},\text{ so } h=\frac{1}{2}\left\|-\mathbf{j}+3\mathbf{k}\right\|=\frac{1}{2}\sqrt{10}\right]$
M1	Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference).
A1	$\frac{25}{3}$ only, ignore reference to units.
(c)	
B1	Any acceptable reason in context.

Question	Scheme	Marks	AOs
5. (a)	$y^2=(8p)^2=64p^2$ and $16x=16(4p^2)=64p^2$ $\Rightarrow$ $P(4p^2,8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2=16x$ gives $a=4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x}=16$ so $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{y}$	M1	2.2a
	$l:\ y-8p=\left(\frac{8}{8p}\right)\left(x-4p^2\right)$	M1	1.1b
	leading to $py=x+4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4,\frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3}=-4+4p^2$	M1	3.1a
	$6p^2-5p-6=0 \Rightarrow (2p-3)(3p+2)=0 \Rightarrow p=\ldots$	M1	1.1b
	$p=\frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0)=x+4\left(\frac{3}{2}\right)^2 \Rightarrow x=\ldots$	M1	2.1
	$x=-9$	A1	1.1b
	$p=\frac{3}{2}\Rightarrow P(9,12)\Rightarrow$ Area$(R)=\frac{1}{2}(9--9)(12)-\int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x=\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R)=\frac{1}{2}(18)(12)-\frac{8}{3}\left(9^{\frac{3}{2}}-0\right)=108-72=36$ *	A1*	1.1b
		(8)	

(c) ALT 1	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow$ Area$(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) dy$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
5. (c) ALT 2	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int\left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Notes

5. (a)

B1 Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that *P* lies on *C*.

(b)

M1 Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it.

M1 Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*.
Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c*.

A1* Obtains $py = x + 4p^2$ by **cso**.

Notes Continued	
5. (c)	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}} dx$
M1	Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$
A1	Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36
(c) ALT 1	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$
A1	Finds *l* as $x = \frac{3}{2}y - 9$
M1	Fully correct method for finding the area of *R*. i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right) dy$
M1	Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Notes Continued	
5. (c) ALT 2	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their } \left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}y$
M1	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Further Mechanics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g$ ´ $2.4 = 4.8g$ => $v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g$ ´ 0.6 => $u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1 *	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\ (1.2g) - 2gh$ => $h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one.	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)

M1: for a complete method to find v

A1: for a correct value (may be numerical)

M1: for a complete method to find u

A1: for a correct value (may be numerical)

M1: for finding <u>both</u> v and u and use of Newton's Law of Restitution

A1*: for the given answer

(b)

M1: for use of Newton's Law of Restitution to find rebound speed

M1: for a complete method to find h

A1: for 0.15 (m) oe

(c)

B1: for a clear description including reference to a quarter

Question	Scheme	Marks	AOs
7(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	$= 58.25 = 58$ (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1 **ft**	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)

M1: for a difference in KE and PE

A1: for a correct expression

A1: for either 58 (2SF) or 58.3(3SF)

(b)

M1: for use of work-energy principle

A1ft: for either 2.9 (2SF) or 2.91 (3SF) follow through on their answer to (a)

(c)

B1: for variable resistance oe

Question	Scheme	Marks	AOs
8(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force ´ Speed = 30 ´ 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force ´ Speed = 70 ´ 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
		(8 marks)	

Notes:

(a)

B1: for force = 30 seen

M1: for use of $P = Fv$

A1ft: for 120 (W), follow through on their '30'

(b)

M1: for resolving parallel to the slope with correct no. of terms and 60g resolved

A1: for a correct equation

A1: for $F = 70$

M1: for use of $P = Fv$

A1ft: for 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
9(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = \frac{-u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

(a)

M1: for use of CLM, with correct no. of terms, condone sign errors

A1: for a correct equation

M1: for use of Newton's Law of Restitution, with e on the correct side

A1: for a correct equation

M1: for setting *two* equations and solving their equations for v

A1: for a correct expression for v

(b)	
M1:	for use of an appropriate inequality
A1:	for a complete range of values of e
(c)	
M1:	for solving their equations for w
A1:	for the given answer
(d)	
M1:	for substituting $e = 5/9$ into their v and w
A1:	for correct expressions for v and w
M1:	for use of Newton's Law of Restitution, with e on the correct side
M1:	for use of appropriate inequality
A1:	for a correct inequality
A1:	for a correct range

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2D: Further Pure Mathematics 1 and Decision Maths 1

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2D**

You must have:
Decision Mathematics question insert
Mathematical Formulae and Statistical Tables
Calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for each question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate to the nearest £ the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

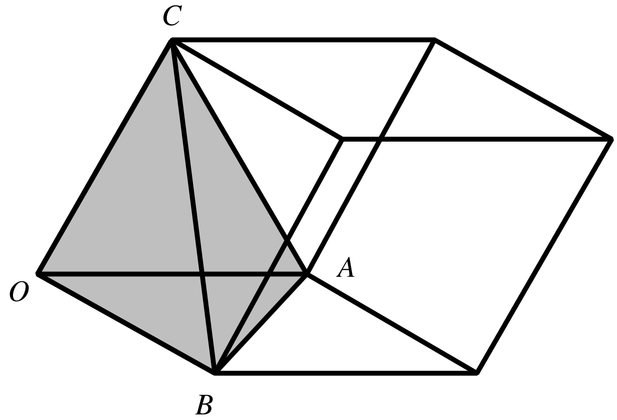


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

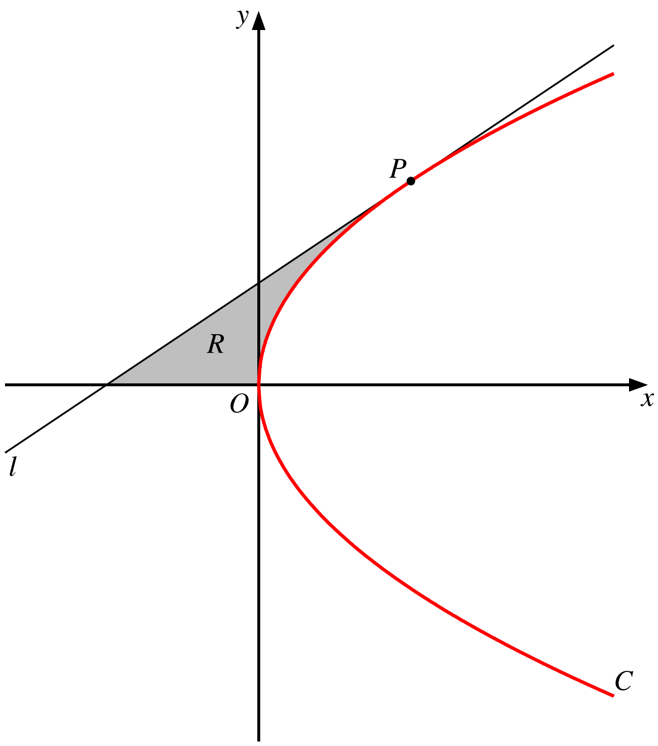


Diagram not drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

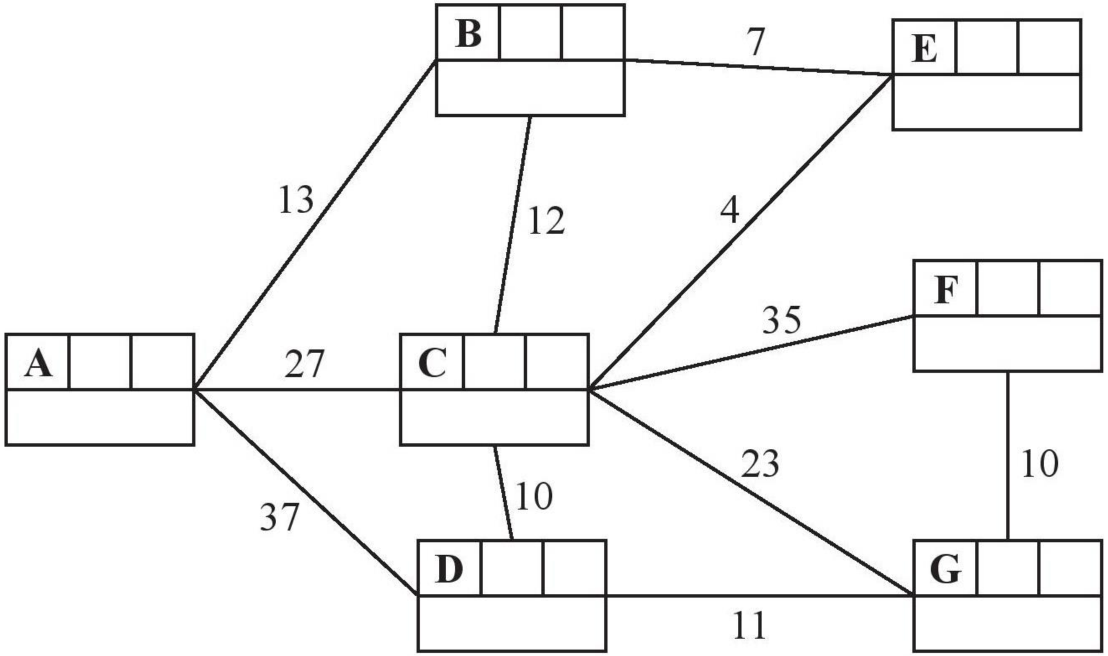
Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B. The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics question insert.

6.



Key:

Vertex	Order of labelling	Final value
Working value		

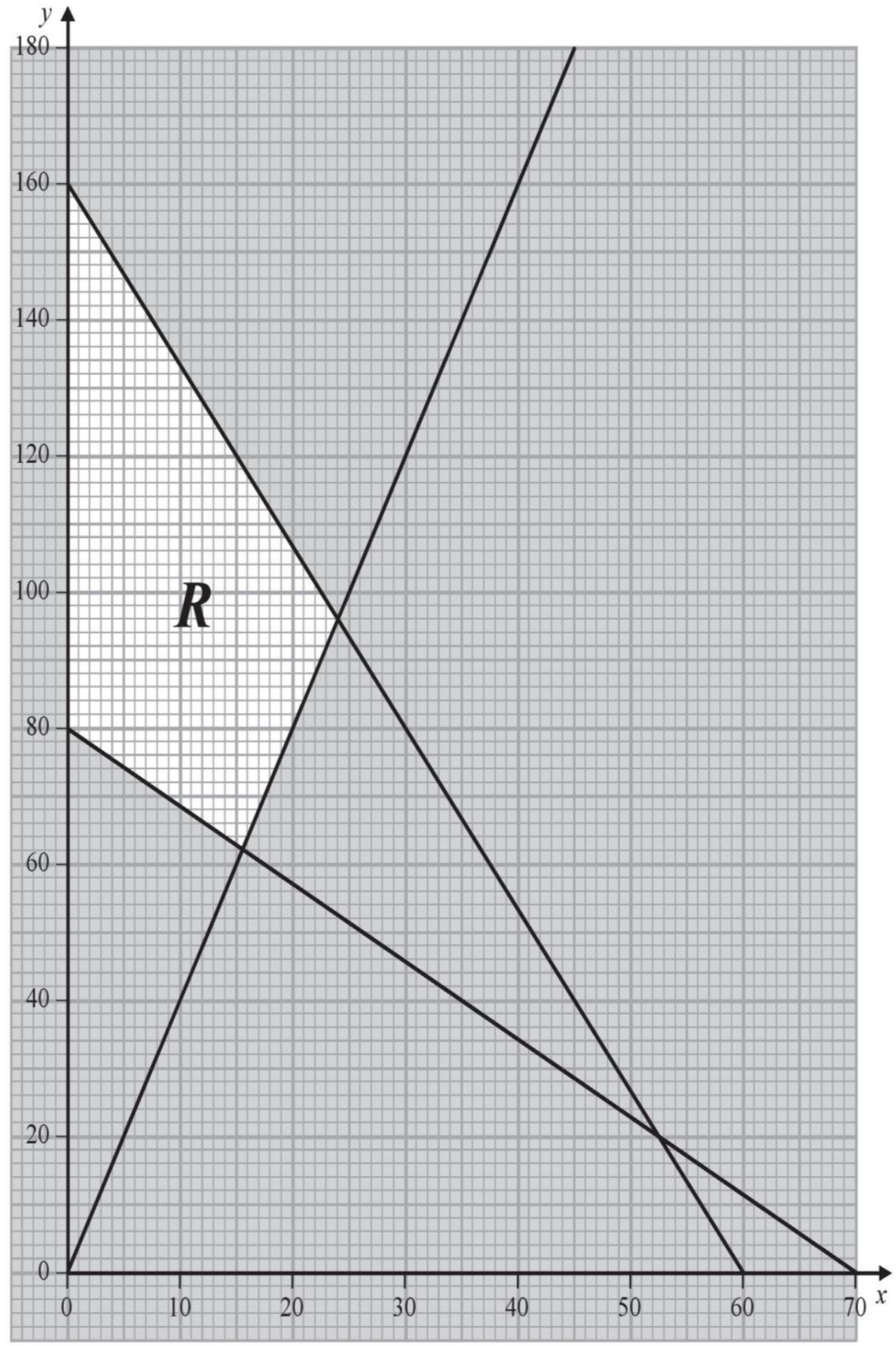
Shortest path: ....

Length of shortest path: ....

Question 6 continued

(Total for Question 6 is 12 marks)

7.



....

....

....

....

(Total for Question 7 is 12 marks)

8. (a) and (b)

(Total for Question 8 is 7 marks)

9.

(Total for Question 9 is 8 marks)

10.

(Total for Question 10 is 6 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR THE PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE	
Further Mathematics **Advance Subsidiary** **Paper 2: Further Mathematics options** **Option 2D: Section B Decision Mathematics**	
Sample assessment material for first teaching September 2017 **Questions 6 - 10**	Paper Reference(s) **8FM0/2D**
Do not return this document with the question paper.	

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6.

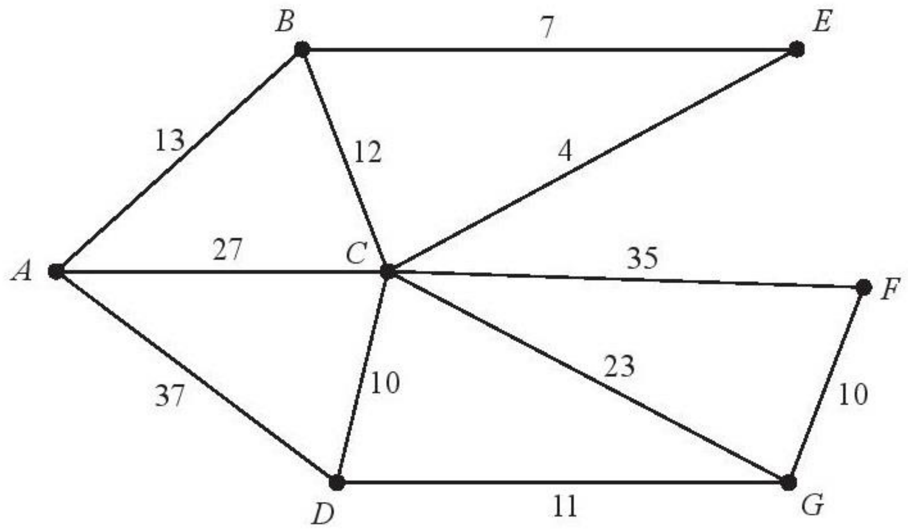


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 6 is 12 marks)

7.

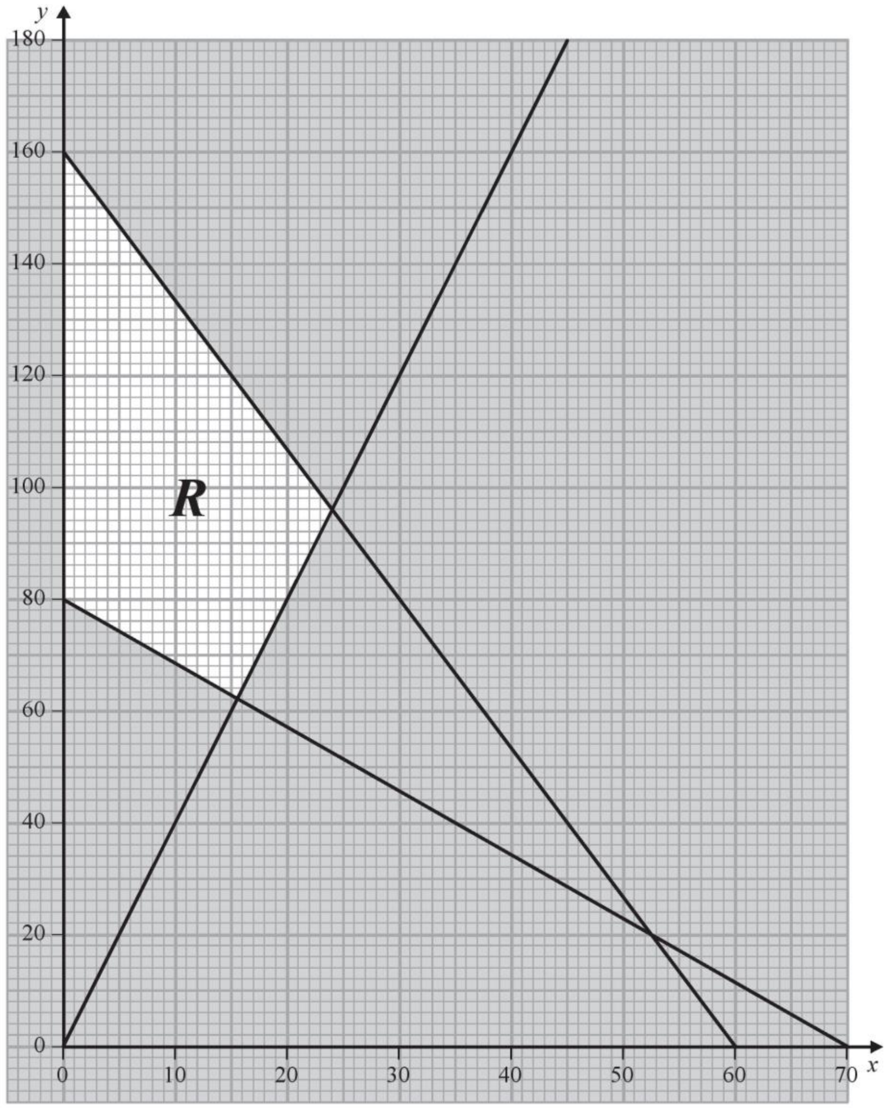


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 7 is 7 marks)

8.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) i) Show that the project can be completed in 21 days, showing your working.
ii) Identify the critical activities.

(4)

(Total for Question 8 is 7 marks)

9. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1,\ 2^x,\ x + 1,\ 2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 9 is 9 marks)

10. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 10 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option D

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1. (a)	$\sec x - \tan x = \dfrac{1}{\frac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes

1. (a)

M1 Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*.

M1 Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1* Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer.

(b)

M1 Uses the *t*-substitution for sin *x* in both numerator and denominator.

M1 Multiples through by $1+t^2$ in numerator and denominator.

A1* Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result.

Question	Scheme	Marks	AOs
2.	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step *h* required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
		(6 marks)	

Notes

B1	Identifies the correct initial conditions and requirement for *h*.
M1	Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0.
M1	Applies the approximation formula with their values.
A1ft	3.5 or exact equivalent. Follow through their step value.
M1	Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5
A1	Applies the approximation and interprets the result to give £396.

Question	Scheme	Marks	AOs
3.	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1	2.2a
		A1	2.5
		(6)	
			(6 marks)

Notes

M1	Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side.
M1	Factorise numerator or find roots of numerator or factorise resulting inequation into 4 factors.
A1	At least 2 correct critical values found.
A1	Exactly 4 correct critical values.
M1	Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means.
A1	Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set. E.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$.

Question	Scheme	Marks	AOs
4. (a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
(a) ALT 1	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}\left(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB})\right)$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=...$	M1	3.1a
	Volume of glass = $\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
4. (b) ALT	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB=\frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert=\frac{1}{2}\times 2\times\sqrt{10}=\sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+\mathbf{2k}-p(-\mathbf{j}+\mathbf{3k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})\Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+\mathbf{3k}\rvert=\frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V=5\times$ Volume of tetrahedron $=5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$=\frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete. Measurements in m may not be accurate. The surface of the concrete tetrahedron may not be smooth. Pockets of air may form when the concrete is being poured.	B1	3.2b
		(1)	
			(10 marks)

Notes

4. (a) Accept use of column vectors throughout.

M1 Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*. Any correct method for the triangle area is fine.

M1 Finds **AB** and **AC** or any other appropriate pair of vectors to use in the vector product and attempts to use them.

M1 Correct procedure for the vector product with at least 1 correct term.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(a) ALT

M1 As main method.

M1 Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides. May use different sides to those shown.

M1 Correct full method to find the area of the triangle using their two sides.

A1 $\frac{1}{2}\sqrt{35}$ or exact equivalent.

(b)

M1 Attempts volume of concrete by finding volume of tetrahedron with appropriate method.

M1 Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted.

A1 Correct value for the volume of concrete.

M1 Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped.

A1 $\frac{25}{3}$ only, ignore reference to units.

Notes	
4. (b) ALT	
M1	Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$
A1	Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$.
M1	Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get height of tetrahedron. $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\left\|-\mathbf{j}+3\mathbf{k}\right\|=\frac{1}{2}\sqrt{10}\right]$
M1	Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference).
A1	$\frac{25}{3}$ only, ignore reference to units.
(c)	
B1	Any acceptable reason in context.

Question	Scheme	Marks	AOs
5. (a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow$ $P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

(c) ALT 1	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12}\left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$	M1	2.1
	$\int\left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right)\mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
5. (c) ALT 2	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = \ldots$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \ldots$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow$ Area$(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Notes

5. (a)

B1 Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C.

(b)

M1 Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it.

M1 Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p.

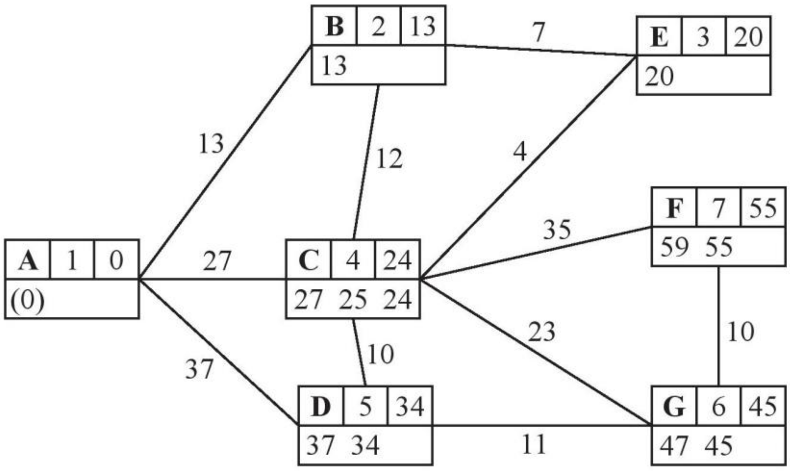
Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c.

A1* Obtains $py = x + 4p^2$ by **cso**.

Notes Continued	
5. (c)	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,dx$
M1	Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$
A1	Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36
(c) ALT 1	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$
A1	Finds *l* as $x = \frac{3}{2}y - 9$
M1	Fully correct method for finding the area of *R*. i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)dy$
M1	Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Notes Continued	
5. (c) ALT 2	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1	Finds that *l* cuts the *x*-axis at $x = -9$
M1	Fully correct method for finding the area of *R*. i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}y$
M1	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Decision Mathematics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)

M1: for a larger number replaced by a smaller one in the working values boxes at C, D, F or G

A1: for all values correct (and in correct order) at A, B, C and D

A1: for all values correct (and in correct order) at E, F & G

A1: for the correct path

A1ft: for 55 or ft their final value at F

(b)

M1: for 3 correct pairings of the four odd nodes (A,B, D & G)

A1: at least two pairings and totals correct

A2: all three pairings and totals correct

A3ft: selecting their shortest pairing, and stating that these arcs should be repeated

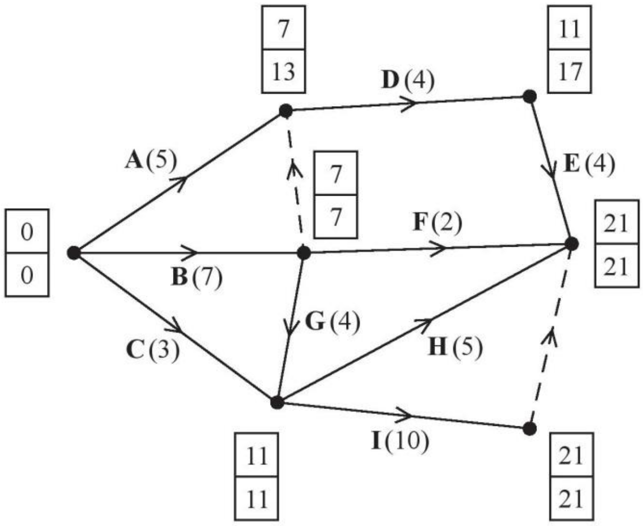
(c)	
B1ft:	for 213 or 189 + their shortest repeat
M1:	for translating the information in the question in to an equation involving x, 2x and 34
A1:	for a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
7	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15±1, 63±2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1:	Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C
M1:	Solving simultaneous equations
A1:	cao
M1: testing	recognition that outcome from this model is non-integer and integer solutions are required – two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$
M1:	testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C
A1:	cao – deducing from tests which integer solution is both valid and optimal
A1ft:	interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">8(a)(b)</td><td>



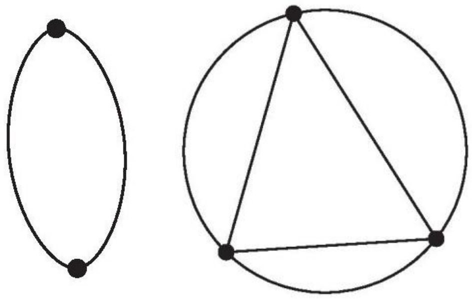
</td><td>M1
A1
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td>The number(s) at the end of activity E indicate this project can be completed in 21 days

Critical activities: B, G, I</td><td>M1
A1
A1 ft
A1</td><td>2.1
1.1b
2.2a
1.1b</td></tr>
<tr><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4" align="right">(7 marks)</td></tr>
</table>

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: all boxes completed, number generally increasing L to R (condone one “rogue”)
A1: all values cao
A1ft: deduction that result in diagram indicates that project can be completed in 21 days (or ft their repeated value at end of E)
A1: critical activities correct

Question	Scheme	Marks	AOs
9(a)	E.g. a graph cannot contain an odd number of odd nodes E.g. number of arcs $=\dfrac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1 A1	2.4 2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)
B1: explanation referring to need for an even number of odd nodes oe

(b)
M1: forming an equation involving the orders of the 5 odd nodes and $2(18)$
M1: simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: construct an argument involving the order of the 5 nodes
A1: explanation considering the number of odd nodes
A1: deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: interprets mathematical language to construct a disconnected graph
A1: deduce a correct graph

Question	Scheme	Marks	AOs
10	Minimise $(C =)\ 25x + 35y$	B1	3.3
	Subject to: $(500x + 800y \geqslant 150\ 000 \Rightarrow)\ 5x + 8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: a correct objective function + minimise

B1: translate information in to a correct inequality

M1: for translating the information given into the LHS inequality

M1: for translating the information given in to the RHS inequality

A1: Simplifying to the correct inequalities

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2E: Further Statistics 1 and Further Mechanics 1

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2E**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A
Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.
(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6
(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.
(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.
(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.
(1)

....

....

....

....

....

....

....

....

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	–1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 5 continued

(Total for Question 5 is 10 marks)

6. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of $25\,\mathrm{m\,s^{-1}}$. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 6 continued

(Total for Question 6 is 6 marks)

7. [*In this question use g =10 m s*$^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\ \text{m s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m s}^{-1}$.

(5)

Question 7 continued

(Total for Question 7 is 8 marks)

8. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2E

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">1(a)</td><td>H_0: There is no association between language and gender.</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>$\frac{54 \times 85}{150} = 30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(c)</td><td>
<table>
<tr><td colspan="2" rowspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>

$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \ldots + \frac{(15-19.83)^2}{19.83}$

awrt <u>3.6/3.7</u>
</td><td>M1
M1
A1</td><td>2.1
1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td>(a)</td><td colspan="3">B1 for correct hypothesis in context</td></tr>
<tr><td>(b)</td><td colspan="3">B1* for a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td>(c)</td><td colspan="3">M1 for attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1 for applying $\sum \frac{(O-E)^2}{E}$
A1 awrt 3.6 or 3.7</td></tr>
<tr><td>(d)</td><td colspan="3">M1 for using degrees of freedom to set up a χ^2 model critical value
A1 for correct comparison and conclusion</td></tr>
<tr><td>(e)</td><td colspan="3">B1 for correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ [1]		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45$ [2]		
	$2a + b + 2c = 1$ [3]	M1	1.1b
	$\begin{pmatrix}1&2&2\\2&0&1\\2&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1.2\\0.45\\1\end{pmatrix}\Rightarrow\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1&2&-2\\2&2&-3\\-2&-3&4\end{pmatrix}\begin{pmatrix}1.2\\0.45\\1\end{pmatrix}$ or e.g. [3] – [2] $\Rightarrow b + c = 0.55$ sub. $2(b+c)$ into [1] $\Rightarrow a = 0.1$ etc	M1	1.1b
	$a = 0.1$ $b = 0.3$ $c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ or 59	M1	1.1a
	[$\text{Var}(Y) = 5^2\text{Var}(X)$ implies] $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \to \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes

(a)	1st M1 for using given information to find an expression for E(X) i.e. use of E(Y) = 2 – 5E(X) 2nd M1 for use of $\sum x\text{P}(X = x) =$ ‘1.2’ 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \text{P}(X = x) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values	1st M1 for using given information to find the probability distribution for Y leading to an expression for E(Y) 2nd M1 for use of $\sum y\text{P}(Y = y) = -4$ 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \text{P}(Y = y) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values
(b)	M1 for use of $\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$ (may be implied by a correct answer) A1 for use of $\text{Var}(aX) = a^2\,\text{Var}(X)$ to reach 2.36 or exact equivalent	
(c)	M1 for rearranging to the form $\text{P}(X < k)$ A1ft ‘0.1’ + ‘025’ (provided their a and c and their $a+c$ are all probabilities)	M1 for comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ A1ft ‘0.1’ + ‘025’ (provided their a and c and their $a+c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	X ~ Po(2.6) Y ~ Po(1.2)		
	P(each hire 2 in 1 hour) = P(X=2) × P(Y=2) = 0.25104…× 0.21685…	M1	3.3
	= 0.05444… awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow$ W ~ Po(3.8)	M1	3.4
	P(W = 3) = 0.20458…. awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	T ~ Po((2.6+1.2)×2)	M1	3.3
	P(T < 9) = 0.64819… awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)(i)	Mean = np = **<u>2.4</u>**	B1	1.1b
(d)(ii)	Variance = $np(1 – p)$ = 2.3904 awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)(i)	[D ~ Po(2.4) P($D \leq 4$)]		
	= 0.9041… awrt **<u>0.904</u>**	B1	1.1b
(e)(ii)	Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes

(a)	M1 for P(X=2)×P(Y=2) from X ~ Po(2.6) and Y ~ Po(1.2) i.e. correct models (may be implied by correct answer) A1 awrt 0.0544
(b)	M1 for combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1 awrt 0.205
(c)	M1 for setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1 awrt 0.648
(d)(i) **(d)(ii)**	B1 for 2.4 B1 for awrt 2.39
(e)(i) **(e)(ii)**	B1 for 0.904 B1 for a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)(i)	$P(X = 1) = 0.34523\ldots$ awrt **0.345**	B1	1.1b
(a)(ii)	$P(X \leqslant 4) = 0.98575\ldots$ awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40} = 1.4$ *	B1*cso	1.1b
		(1)	
(c)	$r = 40 \times$ '0.34523…' $s = 40 \times$ '1 – 0.986…'	M1	3.4
	$r =$ **13.81** $s =$ **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(10-9.86)^2}{9.86} + \ldots + \frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10 < \chi^2_{2,(0.05)} = 5.991$).The number of mortgages approved each week follows a Poisson distribution.	A1	3.5a
		(6)	
			(11 marks)

Notes	
(a)(i) **(a)(ii)**	B1 awrt 0.345 B1 awrt 0.986
(b)	B1* for a fully correct calculation leading to given answer with no errors seen
(c)	M1 for attempt at r or s (may be implied by correct answers) A1ft for both values correct (follow through their answers to part (a))
(d)	1st B1 for both hypotheses correct (lambda should not be defined so correct use of the model) 1st M1 for understanding the need to combine cells before calculating the test statistic (may be implied) 2nd M1 for attempt to find the test statistic using $\chi^2 = \sum \frac{(O-E)^2}{E}$ 1st A1 awrt 1.1 2nd B1 for realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^{\ 2}(0.05) = 5.991$ 2nd A1 concluding that a Poisson model is suitable for the number of mortgages approved each week

Further Mechanics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g$ ´ $2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g$ ´ $0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1 *	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one.	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)

M1: for a complete method to find v

A1: for a correct value (may be numerical)

M1: for a complete method to find u

A1: for a correct value (may be numerical)

M1: for finding both v and u and use of Newton's Law of Restitution

A1*: for the given answer

(b)

M1: for use of Newton's Law of Restitution to find rebound speed

M1: for a complete method to find h

A1: for 0.15 (m) oe

(c)

B1: for a clear description including reference to a quarter

Question	Scheme	Marks	AOs
6(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, 20 R = 58.25	M1	3.3
	R = 2.9125 = 2.9 or 2.91	A1 **ft**	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)

M1: for a difference in KE and PE

A1: for a correct expression

A1: for either 58 (2SF) or 58.3(3SF)

(b)

M1: for use of work-energy principle

A1ft: for either 2.9 (2SF) or 2.91 (3SF) follow through on their answer to (a)

(c)

B1: for variable resistance oe

Question	Scheme	Marks	AOs
7(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force ´ Speed = 30 ´ 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force ´ Speed = 70 ´ 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
		(8 marks)	

Notes:

(a)

B1: for force = 30 seen

M1: for use of $P = Fv$

A1ft: for 120 (W), follow through on their '30'

(b)

M1: for resolving parallel to the slope with correct no. of terms and 60g resolved

A1: for a correct equation

A1: for $F = 70$

M1: for use of $P = Fv$

A1ft: for 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
8(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = \frac{-u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

(a)

M1: for use of CLM, with correct no. of terms, condone sign errors

A1: for a correct equation

M1: for use of Newton's Law of Restitution, with e on the correct side

A1: for a correct equation

M1: for setting *two* equations and solving their equations for v

A1: for a correct expression for v

(b) **M1:** for use of an appropriate inequality **A1:** for a complete range of values of *e*
(c) **M1:** for solving their equations for *w* **A1:** for the given answer
(d) **M1:** for substituting $e = 5/9$ into their *v* and *w* **A1:** for correct expressions for *v* and *w* **M1:** for use of Newton's Law of Restitution, with *e* on the correct side **M1:** for use of appropriate inequality **A1:** for a correct inequality **A1:** for a correct range

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2F: Further Statistics 1 and Decision Mathematics 1

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2F**

You must have:
Decision Mathematics question insert
Mathematical Formulae and Statistical Tables
Calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	–1	0	1	2	3
$\mathrm{P}(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $\mathrm{E}(Y) = -4$ and $\mathrm{P}(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $\mathrm{E}(Y^2) = 75$

(b) find the exact value of $\mathrm{Var}(X)$

(2)

(c) Find $\mathrm{P}(Y > X)$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

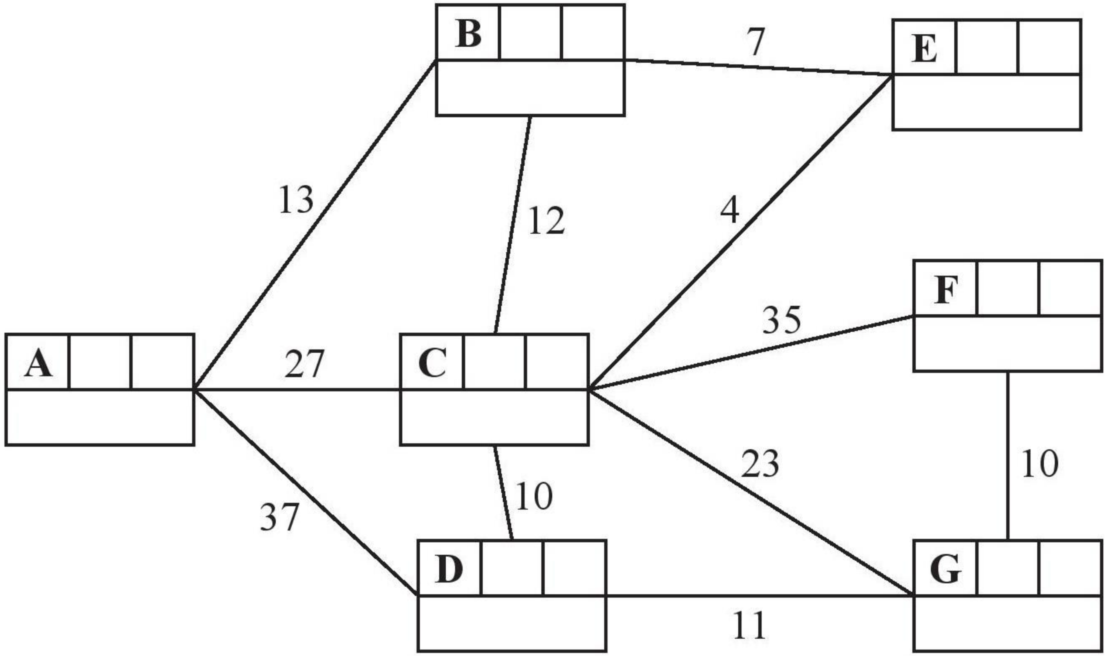
Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B. The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics question insert.

5.



Key:

Vertex	Order of labelling	Final value
Working value		

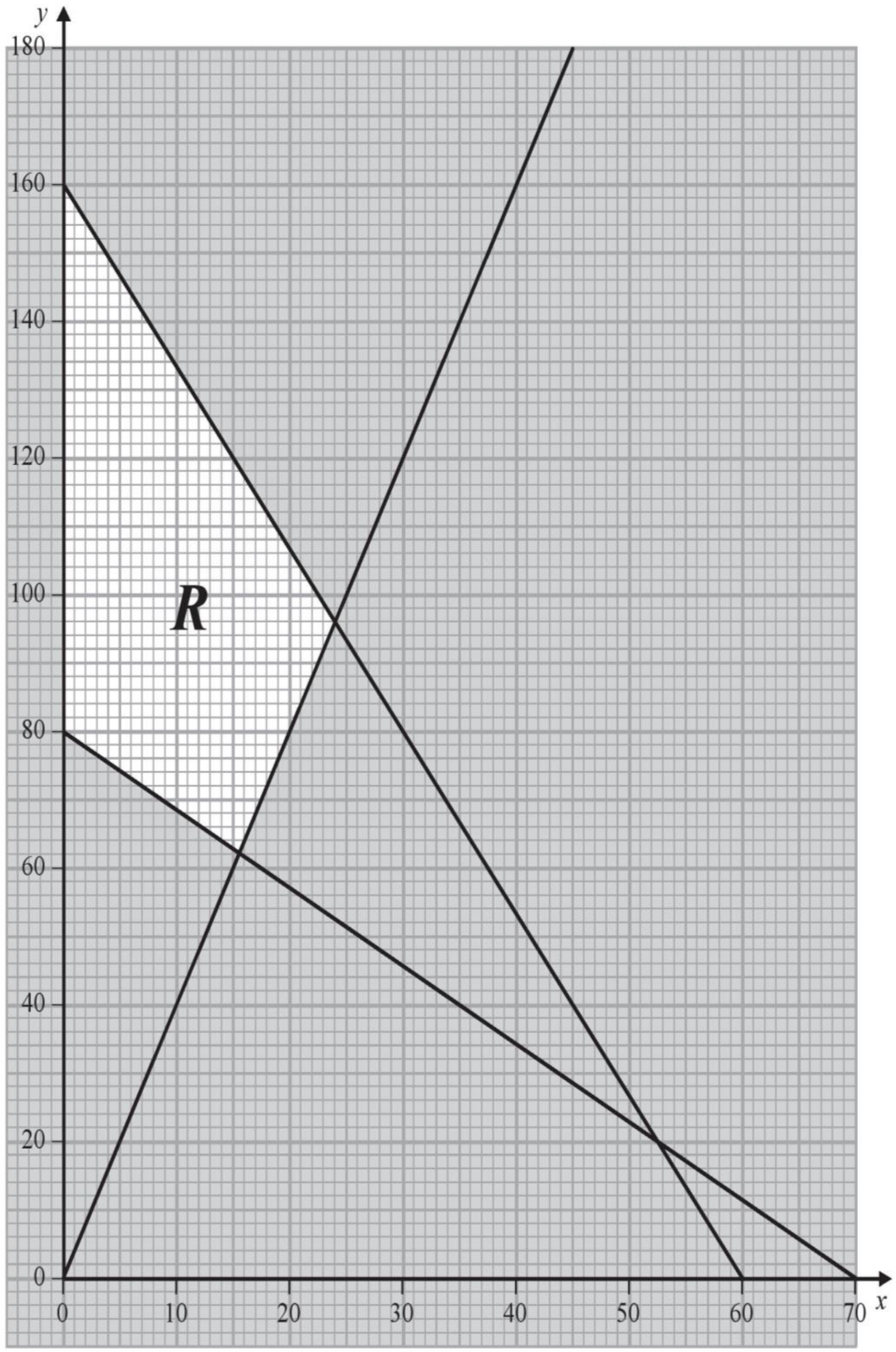
Shortest path: ....

Length of shortest path: ....

Question 5 continued

(Total for Question 5 is 12 marks)

6.



(Total for Question 6 is 12 marks)

7. (a) and (b)

(Total for Question 7 is 7 marks)

8.

(Total for Question 8 is 8 marks)

9.

(Total for Question 9 is 6 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE	
# Further Mathematics **Advance Subsidiary** **Paper 2: Further Mathematics options** **Option 2F: Section B Decision Mathematics**	
Sample assessment material for first teaching September 2017 **Questions 5 – 9**	Paper Reference(s) **8FM0/2F**
Do not return this document with the question paper.	

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.

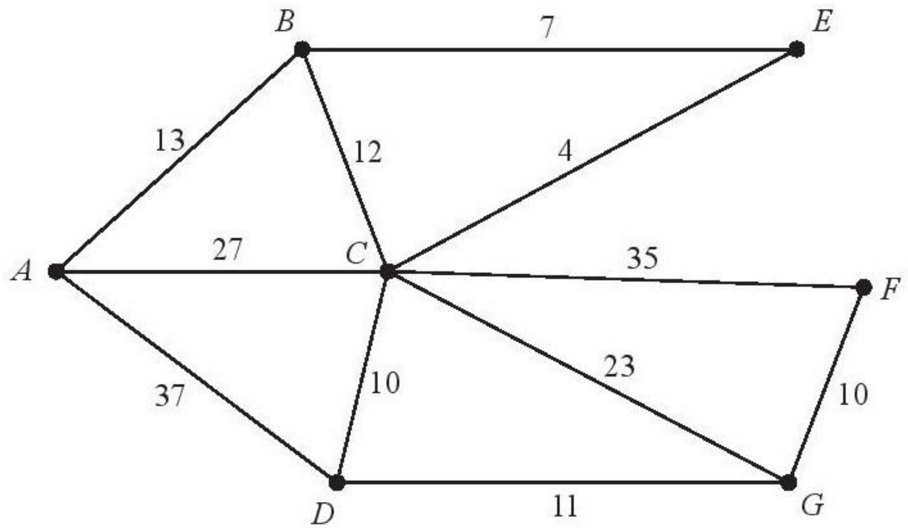


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 5 is 12 marks)

6.

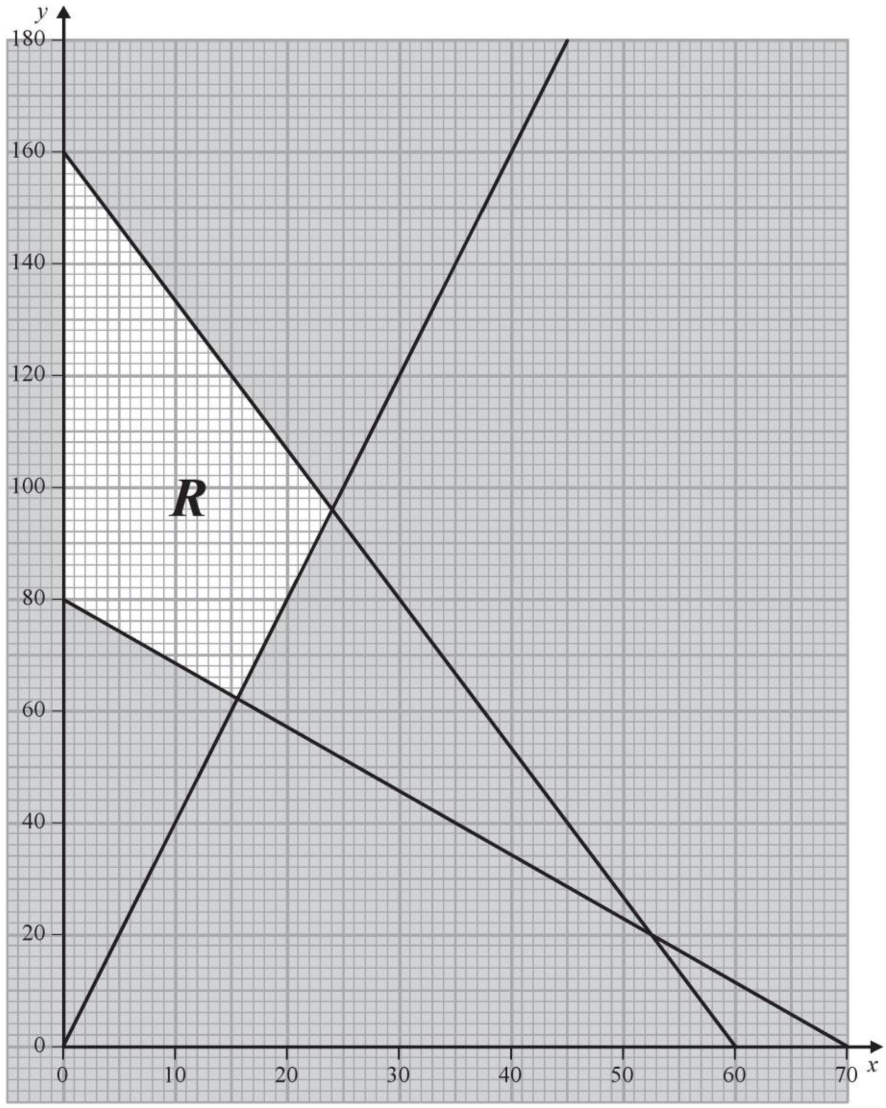


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) i) Show that the project can be completed in 21 days, showing your working.
ii) Identify the critical activities.

(4)

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x}-1,\ 2^{x},\ x+1,\ 2^{x+1}-3$ and $11-x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2F

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">1(a)</td><td>H_0: There is no association between language and gender.</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>$\frac{54 \times 85}{150} = 30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="4">(c)</td><td>
<table>
<tr><td colspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td></td><td></td><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \ldots + \frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>awrt <u>3.6/3.7</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td>(a)</td><td colspan="3">B1 for correct hypothesis in context</td></tr>
<tr><td>(b)</td><td colspan="3">B1* for a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td>(c)</td><td colspan="3">M1 for attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1 for applying $\sum \frac{(O-E)^2}{E}$
A1 awrt 3.6 or 3.7</td></tr>
<tr><td>(d)</td><td colspan="3">M1 for using degrees of freedom to set up a χ^2 model critical value
A1 for correct comparison and conclusion</td></tr>
<tr><td>(e)</td><td colspan="3">B1 for correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ [1]		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$ $2a + c = 0.45$ [2]	M1	2.1
	$2a + b + 2c = 1$ [3]	M1	1.1b
	$\begin{pmatrix}1&2&2\\2&0&1\\2&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1.2\\0.45\\1\end{pmatrix}\Rightarrow\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1&2&-2\\2&2&-3\\-2&-3&4\end{pmatrix}\begin{pmatrix}1.2\\0.45\\1\end{pmatrix}$ <u>or</u> e.g. [3] – [2] $\Rightarrow b + c = 0.55$ sub. $2(b+c)$ into [1] $\Rightarrow a = 0.1$ etc	M1	1.1b
	$a = 0.1$ $b = 0.3$ $c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\text{Var}(Y) = 5^2\text{Var}(X)$ implies$]$ $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \rightarrow \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes

(a)	1st M1 for using given information to find an expression for E(X) i.e. use of $\text{E}(Y) = 2 - 5\text{E}(X)$ 2nd M1 for use of $\sum x\text{P}(X = x) =$ '1.2' 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \text{P}(X = x) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values	1st M1 for using given information to find the probability distribution for Y leading to an expression for E(Y) 2nd M1 for use of $\sum y\text{P}(Y = y) = -4$ 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c 4th M1 for use of $\sum \text{P}(Y = y) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of a, b or c correct 2nd A1 for all 3 correct values
(b)	M1 for use of $\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$ (may be implied by a correct answer) A1 for use of $\text{Var}(aX) = a^2\,\text{Var}(X)$ to reach 2.36 or exact equivalent	
(c)	M1 for rearranging to the form $\text{P}(X < k)$ A1ft '0.1' + '025' (provided their a and c and their a+c are all probabilities)	M1 for comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ A1ft '0.1' + '025' (provided their a and c and their a+c are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) = P(X=2) × P(Y=2) = 0.25104…× 0.21685…	M1	3.3
	= 0.05444… awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow$ $W \sim \text{Po}(3.8)$	M1	3.4
	P(W = 3) = 0.20458…. awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	P(T < 9) = 0.64819… awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)(i)	Mean = np = **<u>2.4</u>**	B1	1.1b
(d)(ii)	Variance = $np(1 – p)$ = 2.3904 awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)(i)	[$D \sim \text{Po}(2.4)$ P($D \le 4$)]		
	= 0.9041… awrt **<u>0.904</u>**	B1	1.1b
(e)(ii)	Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes

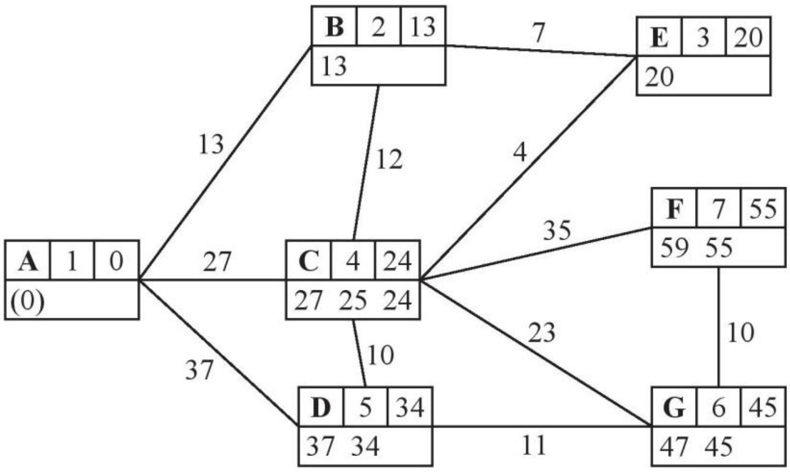
(a)	M1 for P(X=2)×P(Y=2) from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) A1 awrt 0.0544
(b)	M1 for combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1 awrt 0.205
(c)	M1 for setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1 awrt 0.648
(d)(i) **(d)(ii)**	B1 for 2.4 B1 for awrt 2.39
(e)(i) **(e)(ii)**	B1 for 0.904 B1 for a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)(i)	$P(X = 1) = 0.34523\ldots$ awrt **0.345**	B1	1.1b
(a)(ii)	$P(X \leqslant 4) = 0.98575\ldots$ awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times 10)+1\times 16+2\times 7+3\times 4+4\times 2+(5\times 0)+6\times 1}{40} = 1.4$ *	B1*cso	1.1b
		(1)	
(c)	$r = 40 \times$ '0.34523…' $s = 40 \times$ '1 – 0.986…'	M1	3.4
	$r =$ **13.81** $s =$ **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(10-9.86)^2}{9.86} + \ldots + \frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10 < \chi^2_{2,(0.05)} = 5.991$).The number of mortgages approved each week follows a Poisson distribution.	A1	3.5a
		(6)	
			(11 marks)

Notes

(a)(i) **(a)(ii)**	B1 awrt 0.345 B1 awrt 0.986
(b)	B1* for a fully correct calculation leading to given answer with no errors seen
(c)	M1 for attempt at r or s (may be implied by correct answers) A1ft for both values correct (follow through their answers to part (a))
(d)	1st B1 for both hypotheses correct (lambda should not be defined so correct use of the model) 1st M1 for understanding the need to combine cells before calculating the test statistic (may be implied) 2nd M1 for attempt to find the test statistic using $\chi^2 = \sum \frac{(O-E)^2}{E}$ 1st A1 awrt 1.1 2nd B1 for realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^{\,2}(0.05) = 5.991$ 2nd A1 concluding that a Poisson model is suitable for the number of mortgages approved each week

Decision Mathematics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)

M1: for a larger number replaced by a smaller one in the working values boxes at C, D, F or G

A1: for all values correct (and in correct order) at A, B, C and D

A1: for all values correct (and in correct order) at E, F & G

A1: for the correct path

A1ft: for 55 or ft their final value at F

(b)

M1: for 3 correct pairings of the four odd nodes (A,B, D & G)

A1: at least two pairings and totals correct

A2: all three pairings and totals correct

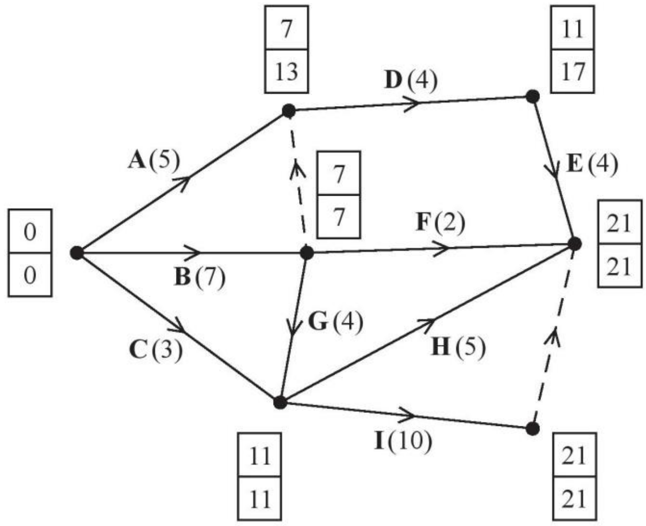
A3ft: selecting their shortest pairing, and stating that these arcs should be repeated

(c)	
B1ft:	for 213 or 189 + their shortest repeat
M1:	for translating the information in the question in to an equation involving x, 2x and 34
A1:	for a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
6	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4*x* and 8*x* + 7*y* = 560 to find the exact co-ordinate of the optimal point, must reach either *x* = or *y* =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15±1, 63±2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and *y* = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
	(7 marks)		

Notes:

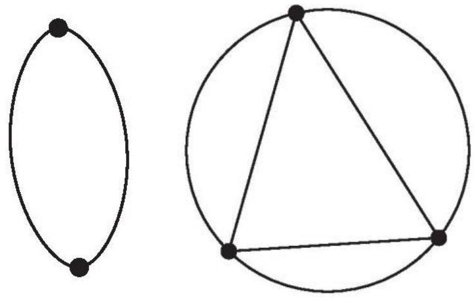
M1:	Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C
M1:	Solving simultaneous equations
A1:	cao
M1:	recognition that outcome from this model is non-integer and integer solutions are required –
testing	two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$
M1:	testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C
A1:	cao – deducing from tests which integer solution is both valid and optimal
A1ft:	interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
7(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
	The number(s) at the end of activity E indicate this project can be completed in 21 days Critical activities: B, G, I	M1 A1 A1 ft A1	2.1 1.1b 2.2a 1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: all boxes completed, number generally increasing L to R (condone one "rogue")
A1: all values cao
A1ft: deduction that result in diagram indicates that project can be completed in 21 days (or ft their repeated value at end of E)
A1: critical activities correct

Question	Scheme	Marks	AOs
8(a)	E.g. a graph cannot contain an odd number of odd nodes E.g. number of arcs $=\dfrac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1 A1	2.4 2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)
B1: explanation referring to need for an even number of odd nodes oe

(b)
M1: forming an equation involving the orders of the 5 odd nodes and $2(18)$
M1: simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: construct an argument involving the order of the 5 nodes
A1: explanation considering the number of odd nodes
A1: deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: interprets mathematical language to construct a disconnected graph
A1: deduce a correct graph

Question	Scheme	Marks	AOs
9	Minimise $(C =)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y \geqslant 150\ 000 \Rightarrow)\ 5x+8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: a correct objective function + minimise

B1: translate information in to a correct inequality

M1: for translating the information given into the LHS inequality

M1: for translating the information given in to the RHS inequality

A1: Simplifying to the correct inequalities

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2G: Further Statistics 1 and Further Statistics 2

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2G**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.
(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6
(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.
(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.
(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.
(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

5. In a gymnastics competition, two judges scored each of 8 competitors on the vault.

Competitor	A	B	C	D	E	F	G	H
Judge 1's scores	4.6	9.1	8.4	8.8	9.0	9.5	9.2	9.4
Judge 2's scores	7.8	8.8	8.6	8.5	9.1	9.6	9.0	9.3

(a) Calculate Spearman's rank correlation coefficient for these data.

(4)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.

(4)

(c) Give a reason to support the use of Spearman's rank correlation coefficient in this case.

(1)

The judges also scored the competitors on the beam.

Spearman's rank correlation coefficient for their ranks on the beam was found to be 0.952

(d) Compare the judges' ranks on the vault with their ranks on the beam.

(1)

Question 5 continued

(Total for Question 5 is 10 marks)

6. The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \frac{1}{18}(11-2x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $\mathrm{P}(X < 3)$

(2)

(b) State, giving a reason, whether the upper quartile of X is greater than 3, less than 3 or equal to 3

(1)

Given that $\mathrm{E}(X) = \dfrac{9}{4}$

(c) use algebraic integration to find $\mathrm{Var}(X)$

(3)

The cumulative distribution function of X is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}(11x - x^2 + c) & 1 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(d) Show that $c = -10$

(2)

(e) Find the median of X, giving your answer to 3 significant figures.

(3)

....

....

....

....

....

....

....

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7. A scientist wants to develop a model to describe the relationship between the average daily temperature, x °C, and a household's daily energy consumption, y kWh, in winter.

A random sample of the average temperature and energy consumption are taken from 10 winter days and are summarised below.

$$\sum x = 12 \quad \sum x^2 = 24.76 \quad \sum y = 251 \quad \sum y^2 = 6341 \quad \sum xy = 284.8$$

$$S_{xx} = 10.36 \qquad S_{yy} = 40.9$$

(a) Find the product moment correlation coefficient between y and x.

(2)

(b) Find the equation of the regression line of y on x in the form $y = a + bx$

(3)

(c) Use your equation to estimate the daily energy consumption when the average daily temperature is 2°C

(1)

(d) Calculate the residual sum of squares (RSS).

(2)

The table shows the residual for each value of x.

x	–0.4	–0.2	0.3	0.8	1.1	1.4	1.8	2.1	2.5	2.6
Residual	–0.63	–0.32	–0.52	–0.73	0.74	2.22	1.84	0.32	f	–1.88

(e) Find the value of f.

(2)

(f) By considering the signs of the residuals, explain whether or not the linear regression model is a suitable model for these data.

(1)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. The continuous random variable X is uniformly distributed over the interval [–3, 5].

(a) Sketch the probability density function $f(x)$ of X.

(2)

(b) Find the value of k such that $P(X < 2[k - X]) = 0.25$

(3)

(c) Use algebraic integration to show that $E(X^3) = 17$

(3)

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2G

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1(a)</td><td>H_0: There is no association between language and gender.</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$\frac{54 \times 85}{150} = 30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td colspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td></td><td></td><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + ... + \frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>awrt <u>3.6/3.7</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td>(a)</td><td colspan="3">B1 for correct hypothesis in context</td></tr>
<tr><td>(b)</td><td colspan="3">B1* for a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td>(c)</td><td colspan="3">M1 for attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1 for applying $\sum \frac{(O-E)^2}{E}$
A1 awrt 3.6 or 3.7</td></tr>
<tr><td>(d)</td><td colspan="3">M1 for using degrees of freedom to set up a χ^2 model critical value
A1 for correct comparison and conclusion</td></tr>
<tr><td>(e)</td><td colspan="3">B1 for correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ [1]		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$ $2a + c = 0.45$ [2]	M1	2.1
	$2a + b + 2c = 1$ [3]	M1	1.1b
	$\begin{pmatrix}1 & 2 & 2\\ 2 & 0 & 1\\ 2 & 1 & 2\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix} = \begin{pmatrix}1.2\\ 0.45\\ 1\end{pmatrix} \Rightarrow \begin{pmatrix}a\\ b\\ c\end{pmatrix} = \begin{pmatrix}1 & 2 & -2\\ 2 & 2 & -3\\ -2 & -3 & 4\end{pmatrix}\begin{pmatrix}1.2\\ 0.45\\ 1\end{pmatrix}$ <u>or</u> e.g. [3] – [2] $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into [1] $\Rightarrow a = 0.1$ etc	M1	1.1b
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	[$\text{Var}(Y) = 5^2\text{Var}(X)$ implies] $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \rightarrow \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes

(a)	1st M1 for using given information to find an expression for E(X) i.e. use of $\text{E}(Y) = 2 - 5\text{E}(X)$ 2nd M1 for use of $\sum x\text{P}(X = x) =$ '1.2' 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in *a* and *c* 4th M1 for use of $\sum \text{P}(X = x) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of *a*, *b* or *c* correct 2nd A1 for all 3 correct values	1st M1 for using given information to find the probability distribution for *Y* leading to an expression for E(Y) 2nd M1 for use of $\sum y\text{P}(Y = y) = -4$ 3rd M1 for use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in *a* and *c* 4th M1 for use of $\sum \text{P}(Y = y) = 1$ 5th M1 for solving their 3 linear equations (matrix or elimination) 1st A1 for any 2 of *a*, *b* or *c* correct 2nd A1 for all 3 correct values
(b)	M1 for use of $\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$ (may be implied by a correct answer) A1 for use of $\text{Var}(aX) = a^2\,\text{Var}(X)$ to reach 2.36 or exact equivalent	
(c)	M1 for rearranging to the form $\text{P}(X < k)$ A1ft '0.1' + '025' (provided their *a* and *c* and their *a*+*c* are all probabilities)	M1 for comparing distribution of *X* with distribution of *Y* to identify $X = -1$ and $X = 0$ A1ft '0.1' + '025' (provided their *a* and *c* and their *a*+*c* are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **0.0544**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow$ $W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **0.205**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **0.648**	A1	1.1b
		(2)	
(d)(i)	Mean = np = **2.4**	B1	1.1b
(d)(ii)	Variance = $np(1-p) = 2.3904$ awrt **2.39**	B1	1.1b
		(2)	
(e)(i)	$[D \sim \text{Po}(2.4)$ $\text{P}(D \leq 4)]$		
	$= 0.9041\ldots$ awrt **0.904**	B1	1.1b
(e)(ii)	Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes	
(a)	M1 for P(X=2)×P(Y=2) from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) A1 awrt 0.0544
(b)	M1 for combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1 awrt 0.205
(c)	M1 for setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1 awrt 0.648
(d)(i) **(d)(ii)**	B1 for 2.4 B1 for awrt 2.39
(e)(i) **(e)(ii)**	B1 for 0.904 B1 for a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)(i)	$P(X = 1) = 0.34523\ldots$ awrt **<u>0.345</u>**	B1	1.1b
(a)(ii)	$P(X \leqslant 4) = 0.98575\ldots$ awrt **<u>0.986</u>**	B1	1.1b
		(2)	
(b)	$\frac{(0\times 10)+1\times 16+2\times 7+3\times 4+4\times 2+(5\times 0)+6\times 1}{40}=1.4^{*}$	B1*cso	1.1b
		(1)	
(c)	$r = 40 \times$ '0.34523…' $s = 40 \times$ '1 – 0.986…'	M1	3.4
	r = **<u>13.81</u>** s = **<u>0.57</u>**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+\ldots+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **<u>1.1</u>**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10<\chi^2_{2,(0.05)}=5.991$).The number of mortgages approved each week follows a Poisson distribution.	A1	3.5a
		(6)	
			(11 marks)

Notes

(a)(i) **(a)(ii)**	B1 awrt 0.345 B1 awrt 0.986
(b)	B1* for a fully correct calculation leading to given answer with no errors seen
(c)	M1 for attempt at r or s (may be implied by correct answers) A1ft for both values correct (follow through their answers to part (a))
(d)	1st B1 for both hypotheses correct (lambda should not be defined so correct use of the model) 1st M1 for understanding the need to combine cells before calculating the test statistic (may be implied) 2nd M1 for attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$ 1st A1 awrt 1.1 2nd B1 for realising that there are 2 degrees of freedom leading to a critical value of $\chi_2{}^2(0.05)=5.991$ 2nd A1 concluding that a Poisson model is suitable for the number of mortgages approved each week

Further Statistics 2 Mark Scheme (Section B)

5(a)

Competitor	A	B	C	D	E	F	G	H	Marks	AOs
Judge 1's ranks	8	4	7	6	5	1	3	2	M1	1.1b
Judge 2's ranks	8	5	6	7	3	1	4	2		
$\boldsymbol{d^2}$	0	1	1	1	4	0	1	0	M1	1.1b

Question	Scheme	Marks	AOs
5(a)	$\sum d^2 = 8$		
	$r_s = 1 - \dfrac{6 \times 8}{8(64-1)}$	dM1	1.1b
	$r_s = 0.90476\ldots$ awrt **<u>0.905</u>**	A1	1.1b
		(4)	
(b)	H_0: $\rho_s = 0$ H_1: $\rho_s > 0$	B1	2.5
	Critical value $\rho_s = 0.8333$	B1	1.1b
	$r_s = 0.905$ lies in the critical region/reject H_0	M1	2.1
	The two judges are in agreement.	A1	2.2b
		(4)	
(c)	e.g. The data is unlikely to be from a bivariate normal distribution (competitor A)/The emphasis here is on the ranks and not the individual scores.	B1	2.4
		(1)	
(d)	Both show positive correlation, but the judges agree more on the beam (since 0.952 is closer to 1)	B1	2.2b
		(1)	
			(10 marks)

Notes

(a)	1st M1 for an attempt to rank at least one row (at least four correct) 2nd M1 for an attempt at d^2 row for their ranks 3rd M1 dependent on 1st M1 for use of $r_s = 1 - \dfrac{6 \times 8}{8(64-1)}$ with their $\sum d^2$ A1 for awrt 0.905
(b)	1st B1 both hypotheses stated in terms of ρ_s 2nd B1 for correct critical value M1 for comparing their '0.905' with their '0.8333' A1 for a correct contextual conclusion with no contradictions seen
(c)	B1 for a correct explanation to support the use of Spearman
(d)	B1 for a correct comparison of the correlation coefficients

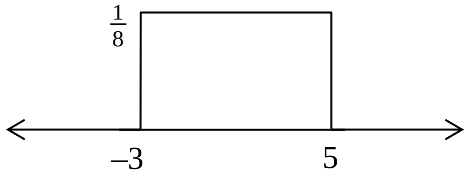
Question	Scheme	Marks	AOs
6(a)	$P(X < 3) = \int_1^3 \frac{1}{18}(11-2x)\mathrm{d}x$ <u>or</u> area of trapezium	M1	1.1a
	$= \left[\frac{1}{18}(11x - x^2)\right]_1^3$		
	$= \frac{7}{9}$	A1	1.1b
		(2)	
(b)	Since $P(X < 3) > 0.75$, the upper quartile is less than 3	B1ft	2.2a
		(1)	
(c)	$E(X^2) = \int_1^4 \frac{1}{18}x^2(11-2x)\mathrm{d}x \left[= \frac{23}{4}\right]$	M1	1.1b
	$\mathrm{Var}(X) = \frac{23}{4} - \left(\frac{9}{4}\right)^2$	M1	1.1b
	$= \frac{11}{16}$	A1	1.1b
		(3)	
(d)	$F(4) = 1 \rightarrow \frac{1}{18}(11(4) - 4^2 + c) = 1$ <u>or</u> $F(1) = 0 \rightarrow \frac{1}{18}(11(1) - 1^2 + c) = 0$	M1	2.1
	$c = -10$ *	A1*cso	1.1b
		(2)	
(e)	$F(m) = 0.5$	M1	1.2
	$\frac{1}{18}(11m - m^2 - 10) = 0.5 \rightarrow m^2 - 11m + 19 = 0$ and attempt to solve	M1	1.1b
	$m = \frac{11 \pm \sqrt{11^2 - 4(19)}}{2}$ $[= 2.1458 \text{ or } 8.8541\ldots]$		
	$m = 2.1458\ldots$ <u>**2.15**</u> (only)	A1	2.2a
		(3)	
			(11 marks)

Notes	
(a)	M1 for integrating $f(x)$ with correct limits <u>or</u> for finding area of trapezium A1 for $\frac{7}{9}$ (allow awrt 0.778)
(b)	B1ft for comparison of their (a) with 0.75 and concluding that the upper quartile is less than 3
(c)	1st M1 for an attempt to find $E(X^2)$ 2nd M1 for use of $\mathrm{Var}(X) = E(X^2) - \left(\frac{9}{4}\right)^2$ A1 for $\frac{11}{16}$ (allow awrt 0.688)(M1 marks may be implied by a correct answer)
(d)	M1 for use of $F(4) = 1$ or $F(1) = 0$ A1*cso for a fully correct solution leading to given answer with no errors seen
(e)	1st M1 for use of $F(m) = 0.5$ 2nd M1 for setting up quadratic and attempt to solve A1 for 2.15 and rejecting the other solution

Question	Scheme	Marks	AOs
7(a)	$r = \dfrac{284.4 - \frac{251(12)}{10}}{\sqrt{10.36 \times 40.9}}$	M1	1.1b
	$r = -0.79671...$ awrt **$\underline{-0.797}$**	A1	1.1b
		(2)	
(b)	$b = \dfrac{'-16.4'}{10.36}$	M1	3.3
	$a = \dfrac{251}{10} - 'b'\dfrac{12}{10}$	M1	1.1b
	$y = 27.0 - 1.58x$	A1	1.1b
		(3)	
(c)	$y = [27.0 - 1.58(2)] = 23.84$ awrt **$\underline{23.8}$**	B1ft	3.4
		(1)	
(d)	$\text{RSS} = 40.9 - \dfrac{(-16.4)^2}{10.36}$	M1	1.1b
	$\text{RSS} = 14.938...$ awrt **$\underline{14.9}$**	A1	1.1b
		(2)	
(e)	$\sum$residuals = 0 → $-0.63 + (-0.32) + \ldots + f + (-1.88) = 0$	M1	3.1a
	$f = \underline{\mathbf{-1.04}}$	A1	1.1b
		(2)	
(f)	The residuals should be randomly scattered above and below zero so linear model may not be appropriate	B1	3.5b
		(1)	
			(11 marks)

Notes

(a)	M1 for a complete correct method for finding r A1 for awrt –0.797
(b)	1st M1 for use of a correct model i.e. a correct expression for b (ft their S_{xy}) 2nd M1 for use of a correct model i.e. a correct (ft) expression for a A1 for $y = 27.0 - 1.58x$ [a correct answer here can imply both method marks]
(c)	B1ft for awrt 23.8 (evaluating their model found in part (b) with $x = 2$)
(d)	M1 for a correct expression for RSS A1 for awrt 14.9
(e)	M1 for use of $\sum$residuals = 0 [Use of regression equation needs correct sign] A1 for –1.04
(f)	B1 for identifying that the residuals are not randomly scattered above and below zero and concluding the linear regression model may not be appropriate.

Question	Scheme	Marks	AOs
8(a)		B1 (shape) B1 (labels)	1.1b 1.1b
		(2)	
(b)	$\mathrm{P}(X < 2(k - X)) = \mathrm{P}(X < \frac{2}{3}k)$	M1	3.1a
	$\dfrac{\frac{2}{3}k - (-3)}{5 - (-3)} = 0.25$	M1	1.1b
	$k = -\dfrac{3}{2}$	A1	1.1b
		(3)	
(c)	$\mathrm{E}(X^3) = \displaystyle\int_{-3}^{5} \frac{1}{5-(-3)} x^3 \mathrm{d}x$	M1	2.1
	$= \left[\dfrac{1}{32}x^4\right]_{-3}^{5} = \frac{1}{32}(5^4 - (-3)^4)$	dM1	1.1b
	$= 17*$	A1*cso	1.1b
		(3)	
			(8 marks)

Notes

(a)	1st B1 for correct shape 2nd B1 for correct labels	
(b)	1st M1 for simplifying to $\mathrm{P}(X < \frac{2}{3}k)$ 2nd M1 for equating probability expression to 0.25 A1 for $-\dfrac{3}{2}$	1st M1 for understanding $2[k - x] = -1$ and $x = -1$ 2nd M1 for substitution and attempt to solve A1 for $-\dfrac{3}{2}$
(c)	1st M1 for integrating $x^3\mathrm{f}(x)$ 2nd M1 for use of correct limits (dependent on previous M1) A1*cso for fully correct solution leading to the given answer with no errors seen	

Pearson Edexcel Level 3 GCE
Further Mathematics **Advanced Subsidiary** **Paper 2: Further Mathematics options** **Option 2H: Further Mechanics 1 and Decision Maths 1**

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2H**

You must have:
Decision Mathematics question insert
Mathematical Formulae and Statistical Tables
Calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided - *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for each question are shown in brackets - *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A
Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of $25\,\mathrm{m\,s^{-1}}$. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use g =10 m s*$^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\ \mathrm{m\,s^{-1}}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\mathrm{m\,s^{-1}}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\mathrm{m\,s^{-1}}$.

(5)

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

Question 4 continued

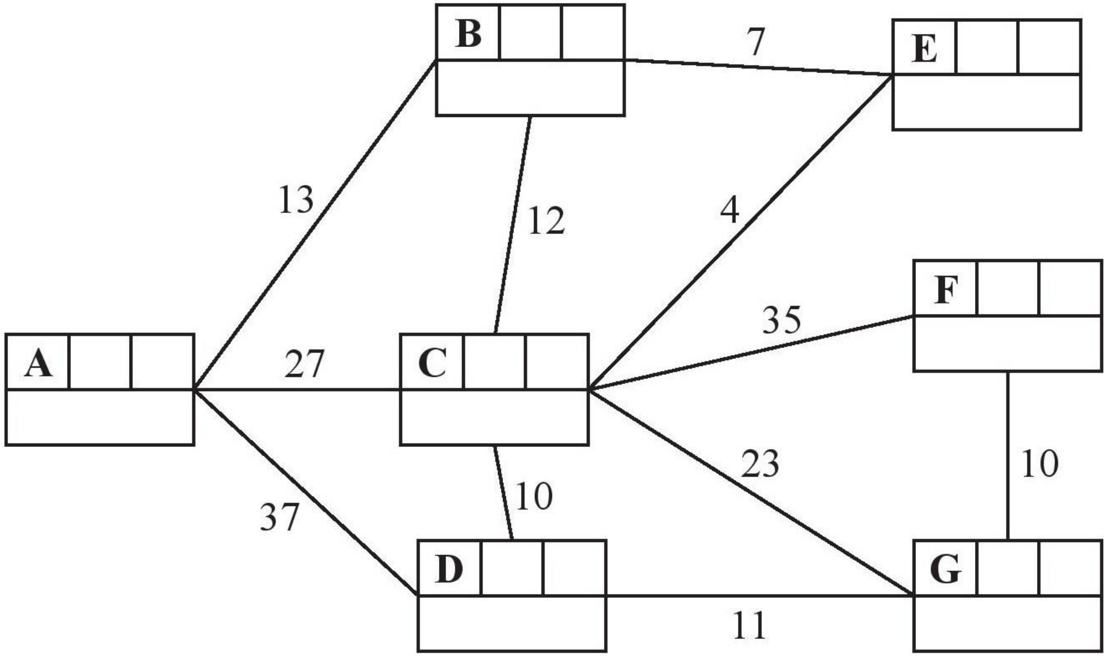
Question 4 continued

(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B. The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics question insert.

5.



Key:

Vertex	Order of labelling	Final value
Working value		

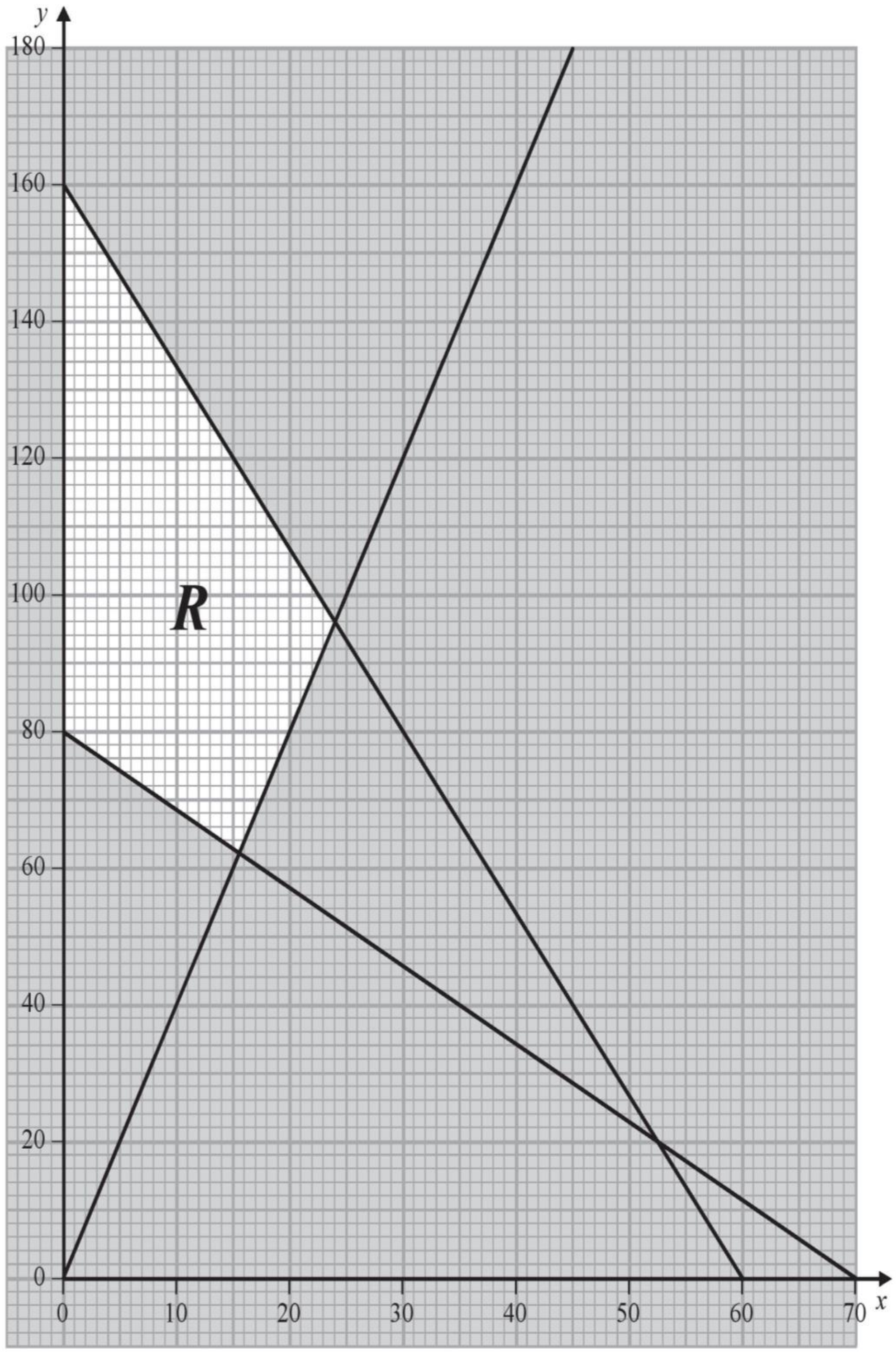
Shortest path: ....

Length of shortest path: ....

Question 5 continued

(Total for Question 5 is 12 marks)

6.



(Total for Question 6 is 12 marks)

7. (a) and (b)

(Total for Question 7 is 7 marks)

8.

(Total for Question 8 is 8 marks)

9.

(Total for Question 9 is 6 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE	
# Further Mathematics **Advance Subsidiary** **Paper 2: Further Mathematics options** **Option 2H: Section B Decision Mathematics**	
Sample assessment material for first teaching September 2017 **Questions 5 – 9**	Paper Reference(s) **8FM0/2H**
Do not return this document with the question paper.	

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.

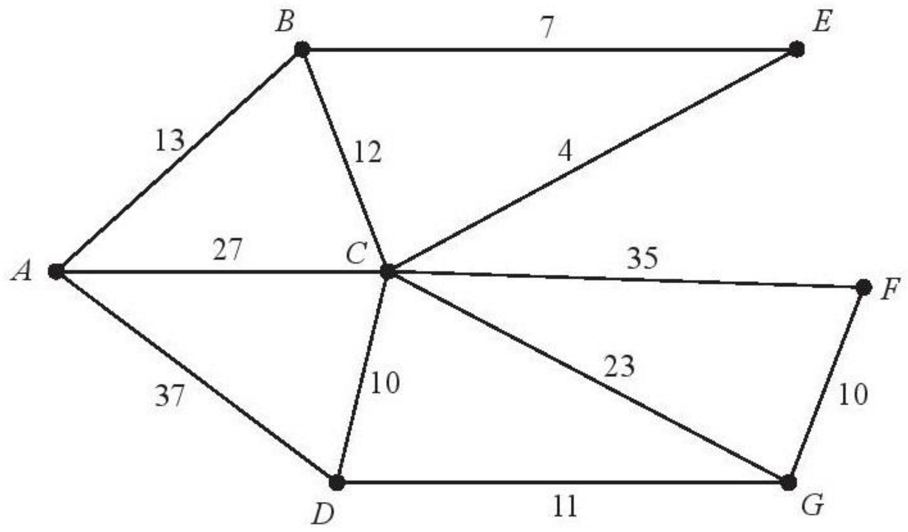


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 5 is 12 marks)

6.

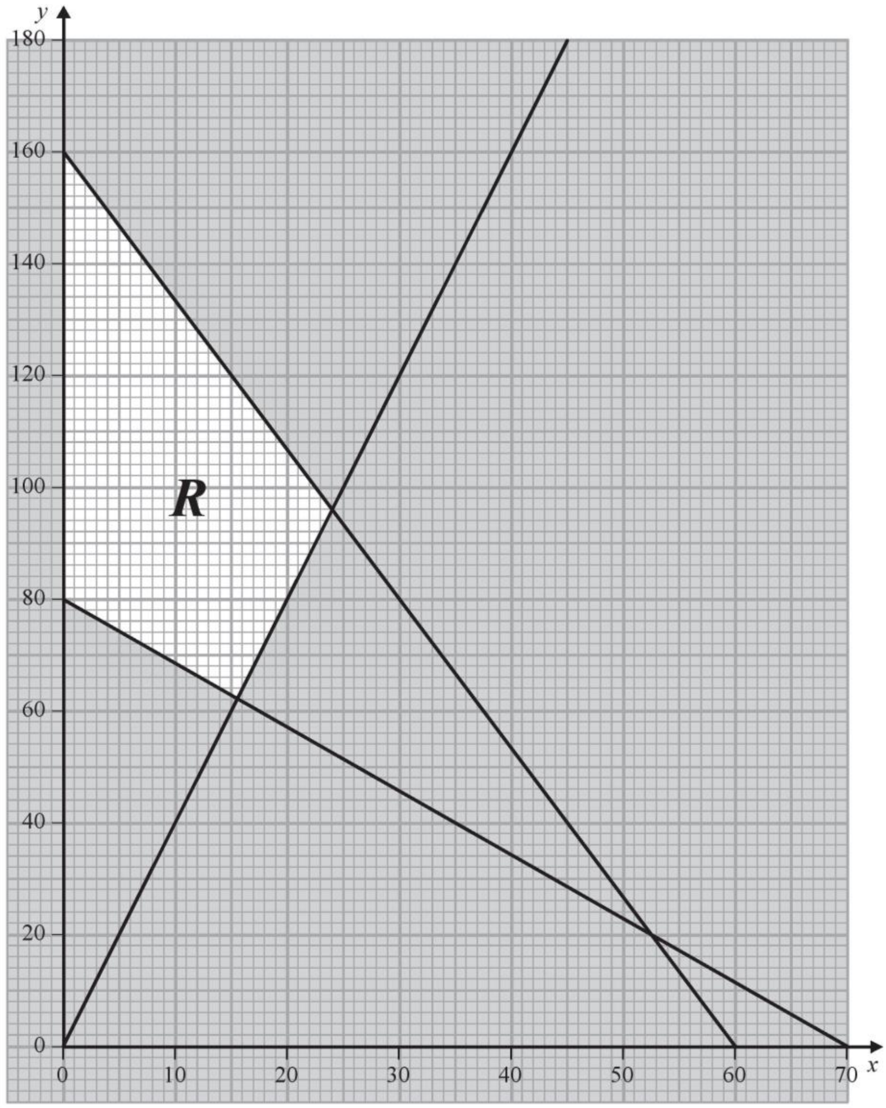


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) i) Show that the project can be completed in 21 days, showing your working.
ii) Identify the critical activities.

(4)

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x}-1$, 2^x, $x+1$, $2^{x+1}-3$ and $11-x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2H

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g$ ´ $2.4 = 4.8g$ => $v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g$ ´ 0.6 => $u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1 *	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25(1.2g) - 2gh$ => $h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one.	B1	2.2b
		(1)	

(10 marks)

Notes:

(a)

M1: for a complete method to find *v*

A1: for a correct value (may be numerical)

M1: for a complete method to find *u*

A1: for a correct value (may be numerical)

M1: for finding <u>both</u> *v* and *u* and use of Newton's Law of Restitution

A1*: for the given answer

(b)

M1: for use of Newton's Law of Restitution to find rebound speed

M1: for a complete method to find *h*

A1: for 0.15 (m) oe

(c)

B1: for a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1 **ft**	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)

M1: for a difference in KE and PE

A1: for a correct expression

A1: for either 58 (2SF) or 58.3(3SF)

(b)

M1: for use of work-energy principle

A1ft: for either 2.9 (2SF) or 2.91 (3SF) follow through on their answer to (a)

(c)

B1: for variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force ´ Speed = 30 ´ 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force ´ Speed = 70 ´ 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: for force = 30 seen

M1: for use of $P = Fv$

A1ft: for 120 (W), follow through on their '30'

(b)

M1: for resolving parallel to the slope with correct no. of terms and 60g resolved

A1: for a correct equation

A1: for $F = 70$

M1: for use of $P = Fv$

A1ft: for 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \le 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = \frac{-u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

(a)

M1: for use of CLM, with correct no. of terms, condone sign errors

A1: for a correct equation

M1: for use of Newton's Law of Restitution, with e on the correct side

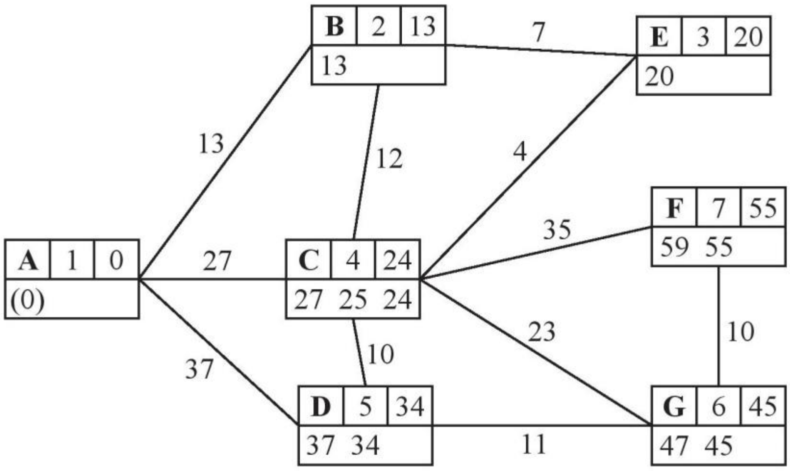
A1: for a correct equation

M1: for setting *two* equations and solving their equations for v

A1: for a correct expression for v

(b) **M1:** for use of an appropriate inequality **A1:** for a complete range of values of e
(c) **M1:** for solving their equations for w **A1:** for the given answer
(d) **M1:** for substituting $e = 5/9$ into their v and w **A1:** for correct expressions for v and w **M1:** for use of Newton's Law of Restitution, with e on the correct side **M1:** for use of appropriate inequality **A1:** for a correct inequality **A1:** for a correct range

Decision Mathematics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)
M1: for a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: for all values correct (and in correct order) at A, B, C and D
A1: for all values correct (and in correct order) at E, F & G
A1: for the correct path
A1ft: for 55 or ft their final value at F

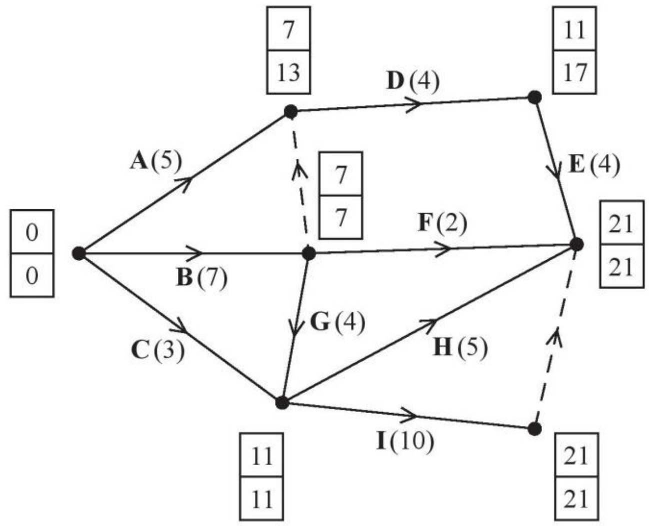
(b)
M1: for 3 correct pairings of the four odd nodes (A,B, D & G)
A1: at least two pairings and totals correct
A2: all three pairings and totals correct
A3ft: selecting their shortest pairing, and stating that these arcs should be repeated

(c)	
B1ft:	for 213 or 189 + their shortest repeat
M1:	for translating the information in the question in to an equation involving x, 2x and 34
A1:	for a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
6	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15±1, 63±2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

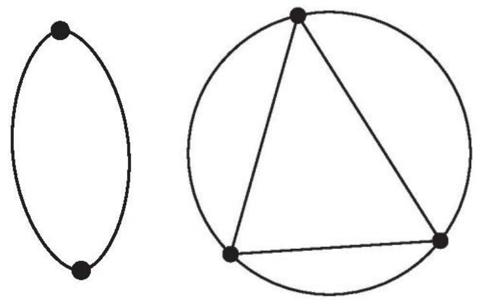
M1:	Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C
M1:	Solving simultaneous equations
A1:	cao
M1:	recognition that outcome from this model is non-integer and integer solutions are required –
testing	two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$
M1:	testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C
A1:	cao – deducing from tests which integer solution is both valid and optimal
A1ft:	interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
7(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
	The number(s) at the end of activity E indicate this project can be completed in 21 days Critical activities: B, G, I	M1 A1 A1 ft A1	2.1 1.1b 2.2a 1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: all boxes completed, number generally increasing L to R (condone one "rogue")
A1: all values cao
A1ft: deduction that result in diagram indicates that project can be completed in 21 days (or ft their repeated value at end of E)
A1: critical activities correct

Question	Scheme	Marks	AOs
8(a)	E.g. a graph cannot contain an odd number of odd nodes E.g. number of arcs $= \dfrac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2(18)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0 \Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0 \Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1 A1	2.4 2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)
B1: explanation referring to need for an even number of odd nodes oe

(b)
M1: forming an equation involving the orders of the 5 odd nodes and $2(18)$
M1: simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: construct an argument involving the order of the 5 nodes
A1: explanation considering the number of odd nodes
A1: deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: interprets mathematical language to construct a disconnected graph
A1: deduce a correct graph

Question	Scheme	Marks	AOs
9	Minimise $(C =)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y \geqslant 150\ 000 \Rightarrow)\ 5x+8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: a correct objective function + minimise

B1: translate information in to a correct inequality

M1: for translating the information given into the LHS inequality

M1: for translating the information given in to the RHS inequality

A1: Simplifying to the correct inequalities

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2J: Further Mechanics 1 and Further Mechanics 2

Sample assessment material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference(s) **8FM0/2J**
You must have: Mathematical Formulae and Statistical Tables, calculator	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of $25\,\text{m}\,\text{s}^{-1}$. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use g =10 m s*$^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of 4 $\mathrm{m\,s^{-1}}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\mathrm{m\,s^{-1}}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\mathrm{m\,s^{-1}}$.

(5)

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

5. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geq 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

Question 5 continued

(Total for Question 5 is 8 marks)

6. A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

(i) the tension in AP is $\frac{4m}{25}(9a\omega^2 + 5g)$

(ii) the tension in BP is $\frac{3m}{25}(16a\omega^2 - 5g)$.

(10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

(c) State how in your calculations you have used the assumption that the string is light.

(1)

....

....

....

....

....

....

....

....

....

....

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

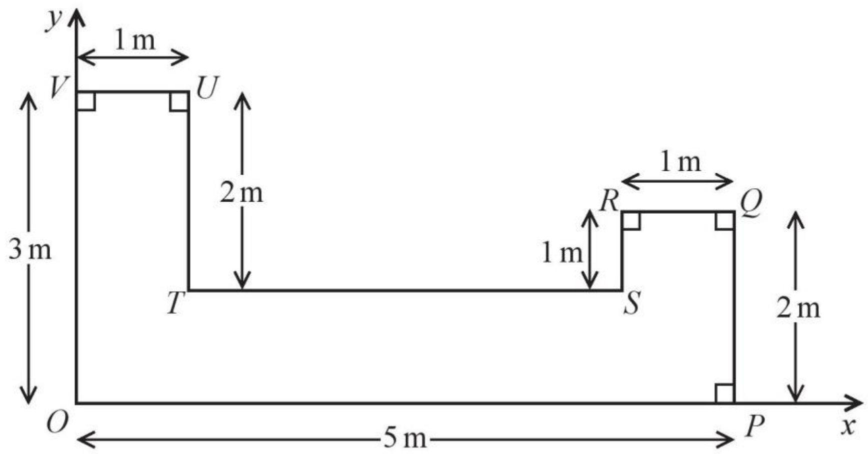


Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.
$OP = 5$ m , $PQ = 2$ m , $QR = 1$ m , $RS = 1$ m , $TU = 2$ m , $UV = 1$ m , $VO = 3$ m.
Figure 1 also shows a coordinate system with O as origin and the x-axis and y-axis along OP and OV respectively. The unit of length on both axes is the metre.
The centre of mass of the template has coordinates $(\bar{x}, \bar{y})$.

(a) (i) Show that $\bar{y} = 1$
(ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius r metres, are removed from the template which is shown in Figure 1, to form a new template L. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, a) where a is a constant.

(b) Find the value of r.

(4)

(c) (i) Explain how symmetry can be used to find the value of a.
(ii) Find the value of a.

(2)

The template L is now freely suspended from the point U and hangs in equilibrium.

(d) Find the size of the angle between the line TU and the horizontal.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2J

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g$ ´ $2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g$ ´ $0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1 *	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one.	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)

M1: for a complete method to find v

A1: for a correct value (may be numerical)

M1: for a complete method to find u

A1: for a correct value (may be numerical)

M1: for finding <u>both</u> v and u and use of Newton's Law of Restitution

A1*: for the given answer

(b)

M1: for use of Newton's Law of Restitution to find rebound speed

M1: for a complete method to find h

A1: for 0.15 (m) oe

(c)

B1: for a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	$= 58.25 = 58$ (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1 **ft**	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)

M1: for a difference in KE and PE

A1: for a correct expression

A1: for either 58 (2SF) or 58.3(3SF)

(b)

M1: for use of work-energy principle

A1ft: for either 2.9 (2SF) or 2.91 (3SF) follow through on their answer to (a)

(c)

B1: for variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force ´ Speed = 30 ´ 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force ´ Speed = 70 ´ 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: for force = 30 seen

M1: for use of $P = Fv$

A1ft: for 120 (W), follow through on their '30'

(b)

M1: for resolving parallel to the slope with correct no. of terms and $60g$ resolved

A1: for a correct equation

A1: for $F = 70$

M1: for use of $P = Fv$

A1ft: for 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in *u* and *v* and solving for *v*	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e$ £ 1	A1	1.1b
		(2)	
(c)	Solving for *w*	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = \frac{-u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

(a)

M1: for use of CLM, with correct no. of terms, condone sign errors

A1: for a correct equation

M1: for use of Newton's Law of Restitution, with *e* on the correct side

A1: for a correct equation

M1: for setting *two* equations and solving their equations for *v*

A1: for a correct expression for *v*

(b)	
M1:	for use of an appropriate inequality
A1:	for a complete range of values of *e*
(c)	
M1:	for solving their equations for *w*
A1:	for the given answer
(d)	
M1:	for substituting $e = 5/9$ into their *v* and *w*
A1:	for correct expressions for *v* and *w*
M1:	for use of Newton's Law of Restitution, with *e* on the correct side
M1:	for use of appropriate inequality
A1:	for a correct inequality
A1:	for a correct range

Further Mechanics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5 (a)	Multiply out and differentiate wrt t	M1	1.1b
	$v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$	A1	1.1b
		(2)	
(b)	Multiply out and integrate wrt t	M1	1.1b
	$s = \int 3t^2 - 16t + 20\mathrm{d}t = t^3 - 8t^2 + 20t(+C)$	A1	1.1b
	$t = 0, s = 0$ => $C = 0$ $t = 2, s = 8 - 32 + 40 = 16$	A1	1.1b
		(3)	
(c)	$s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$	M1	2.1
	Explanation to show that $t^2 - 8t + 20 > 0$ for all t.	M1	2.4
	So $s = 0$ has no non-zero solutions, so s is never zero again, so never returns to O *	A1*	3.2a
		(3)	
			(8 marks)

Notes:

(a)

M1: for multiplying out and differentiating (powers decreasing by 1)

A1: for a correct expression for a

(b)

M1: for multiplying out and integrating (powers increasing by 1)

A1: for a correct expression for s with or without C

A1: for $C = 0$ and correct final answer

(c)

M1: for equating their s to 0 and producing a quadratic

M1: for clear explanation that $t^2 - 8t + 20 > 0$ for all t (e.g. completing the square or another complete method)

A1*: for a correct conclusion in context

Question	Scheme	Marks	AOs
6(a)	$\cos a = \frac{4}{5}$ or $\sin a = \frac{3}{5}$	B1	1.1b
	$r = 4a\sin a$	B1	1.1b
	Resolving vertically	M1	3.1b
	$T_1\cos a - T_2\sin a = mg$	A1	1.1b
	Resolving horizontally	M1	3.1b
	$T_1\sin a + T_2\cos a = mrW^2$	A1	1.1b
	$T_1\sin a + T_2\cos a = mrW^2$	A1	1.1b
	Solving for either tension	M1	2.1
	$T_1 = \frac{4m}{25}(9aW^2 + 5g)$ *	A1*	1.1b
	$T_2 = \frac{3m}{25}(16aW^2 - 5g)$ *	A1*	1.1b
		(10)	
(b)	$\frac{4m}{25}(9aW^2 + 5g) < 4mg$	M1	2.1
	$\frac{3m}{25}(16aW^2 - 5g) > 0$	M1	2.1
	$w > \sqrt{\frac{5g}{16a}}$ or $w < \sqrt{\frac{20g}{9a}}$	A1	2.2a
	$S = \frac{2p}{w}$	M1	1.1b
	$3p\sqrt{\frac{a}{5g}} < S < 8p\sqrt{\frac{a}{5g}}$ *	A1*	1.1b
		(5)	
(c)	String being light implies that the tension is constant in both portions of the string	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)

B1: for correct trig. ratio seen

B1: for a correct radius expression seen

M1: for resolving vertically with correct no. of terms and tensions resolved

A1:	for a correct equation
M1:	for resolving horizontally with correct no. of terms and tensions resolved
A1:	for a correct equation
M1:	for solving their two equations to find either tension
A1*:	for the given answer
A1*:	for the given answer
(b)	
M1:	for use of $T_1 < 4mg$
M1:	for using $T_2 > 0$
A1:	for a correct inequality (either) for w
M1:	for use of $S = \frac{2p}{w}$ with either critical value
A1*:	for given answer
(c)	
B1:	for a clear explanation

Question	Scheme	Marks	AOs
7 (a)	Rel. Mass: 2 5 1 8	B1	1.2
	y : 2 0.5 1.5 $\overline{y}$	B1	1.2
	x : 0.5 2.5 4.5 $\overline{x}$	B1	1.2
	(2 ´ 2) + (5 ´ 0.5) + (1 ´ 1.5) = 8$\overline{y}$	M1	2.1
	$\overline{y}$ = 1 *	A1*	1.1b
	(2 ´ 0.5) + (5 ´ 2.5) + (1 ´ 4.5) = 8$\overline{x}$	M1	2.1
	$\overline{x}$ = 2.25	A1	1.1b
		(7)	
(b)	Use of correct strategy to solve the problem by use of 'moments equation'	M1	3.1b
	(8 ´ 2.25) - ($2\pi r^2$ ´ 0.5) = (8 - $2\pi r^2$)2.5	A1**ft**	1.1b
	Solving for r	M1	1.1b
	$r = \frac{1}{\sqrt{2p}} = 0.399$	A1	1.1b
(c)	Since $\overline{y}$ for original plate is 1, holes must be symmetrically placed about the line y = 1	B1	2.4
	a = 1.5	B1	2.2a
		(2)	
(d)	Use of tan from an appropriate triangle	M1	1.1a
	$\tan a = \frac{2}{1.5} = \frac{4}{3}$	A1 **ft**	1.1b
	$\alpha = 53.1^\circ$	A1	1.1b
		(3)	
		(16 marks)	

Notes:

3(a)

B1: for correct relative masses

B1: for correct y values

B1: for correct x values

M1: for a moments equation, correct no. of terms, condone sign errors

A1*: for a correct given answer (1)

M1: for a moments equation, correct no. of terms

A1: for 2.25

(b)

M1: for a moments equation, correct no. of terms, condone sign errors

A1ft:	for a correct equation, follow through on their $\overline{x}$
M1:	for solving for r
A1:	for 0.399 or 0.40
(c)	
B1:	for consideration of symmetry about $y = 1$
B1:	for $a = 1.5$
(d)	
M1:	for use of tan from an appropriate triangle
A1ft:	for a correct equation, follow through on their a
A1:	for a correct angle

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 2: Further Mathematics options
Option 2K: Decision Maths 1 and Decision Maths 2

Sample assessment material for first teaching September 2017	Paper Reference(s)
Time: 1 hour 40 minutes	**8FM0/2K**

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics
- answer book provided.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

SECTION A

Answer ALL questions. Write your answers in the answer book provided.

1.

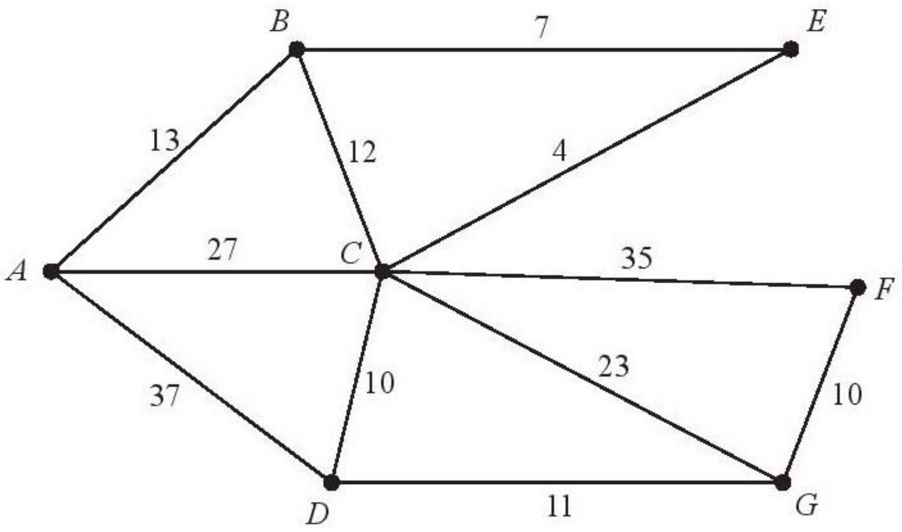


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 1 is 12 marks)

2.

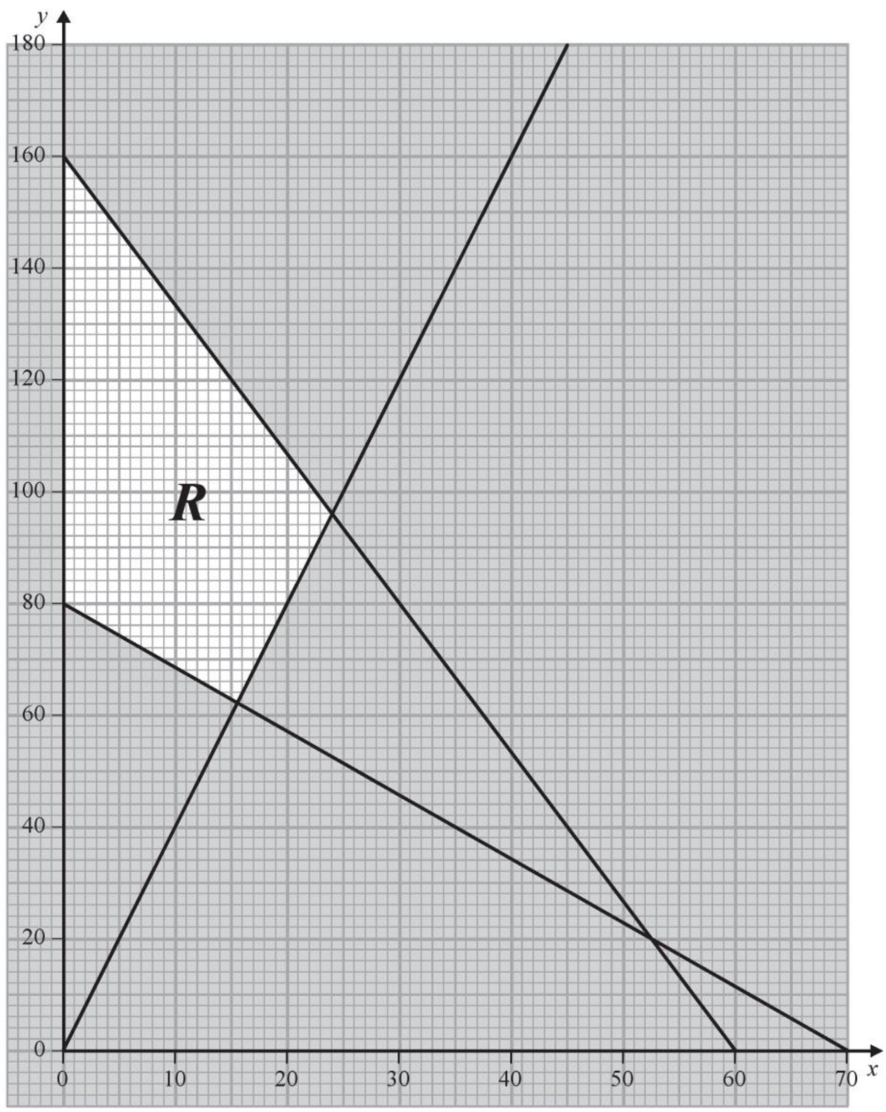


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 2 is 7 marks)

3.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) i) Show that the project can be completed in 21 days, showing your working.
ii) Identify the critical activities.

(4)

(Total for Question 3 is 7 marks)

4. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 4 is 9 marks)

5. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

Total for Question 6 is 9 marks)

7. In two-dimensional space, lines divide a plane into a number of different regions.

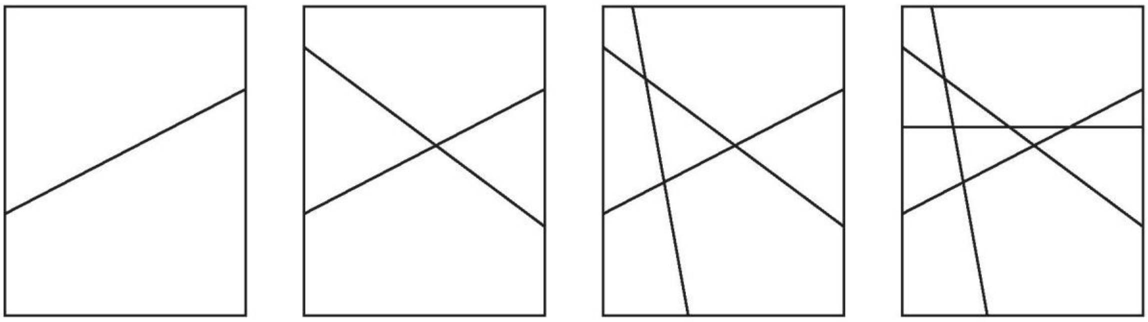


Figure 1 **Figure 2** **Figure 3** **Figure 4**

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n+1)$ lines where $n \geq 1$

(1)

(c) (i) Solve the recurrence relation for u_n

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 7 is 5 marks)

8.

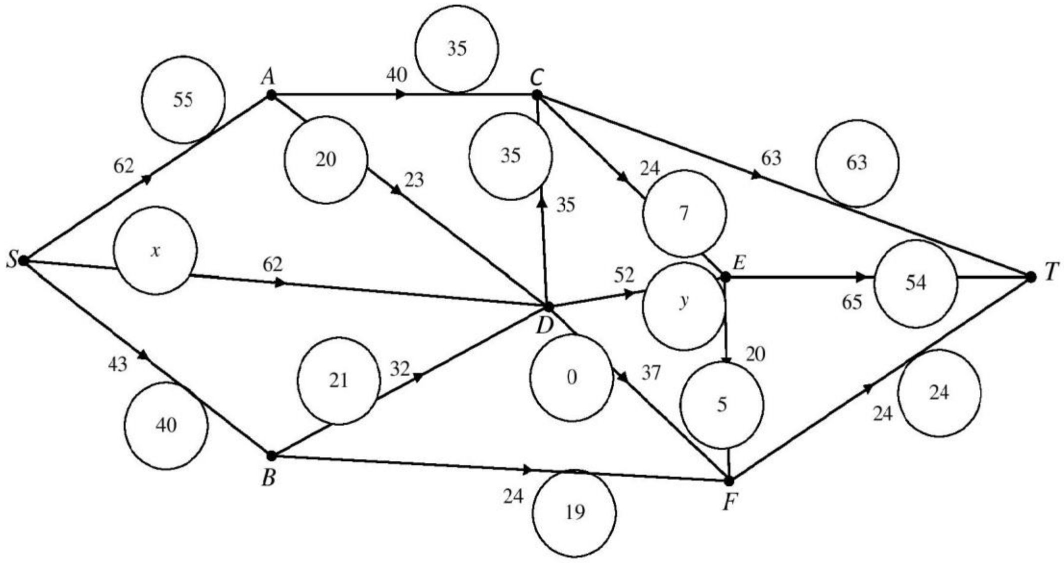


Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11am on Friday morning, all students leave the hall (S) after assembly and travel to the cybercafé (T). The numbers in circles represent the initial flow of students recorded at 11am one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

(b) Find the value of x and the value of y, explaining your reasoning.

(3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from S to T that increase the flow by at least 5

(ii) State which route the new students should take, giving a reason for your answer.

(3)

(d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to part (d) is optimal.

(3)

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

(i) which corridor they should widen,

(ii) the resulting increase of flow through the network.

(3)

(Total for Question 8 is 14 marks)

9. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	4	1	2
A plays 2	2	4	3

(a) Verify that there is no stable solution.

(3)

(b) (i) Find the best strategy for player *A*.

(ii) Find the value of the game to her.

(9)

(Total for Question 9 is 12 marks)

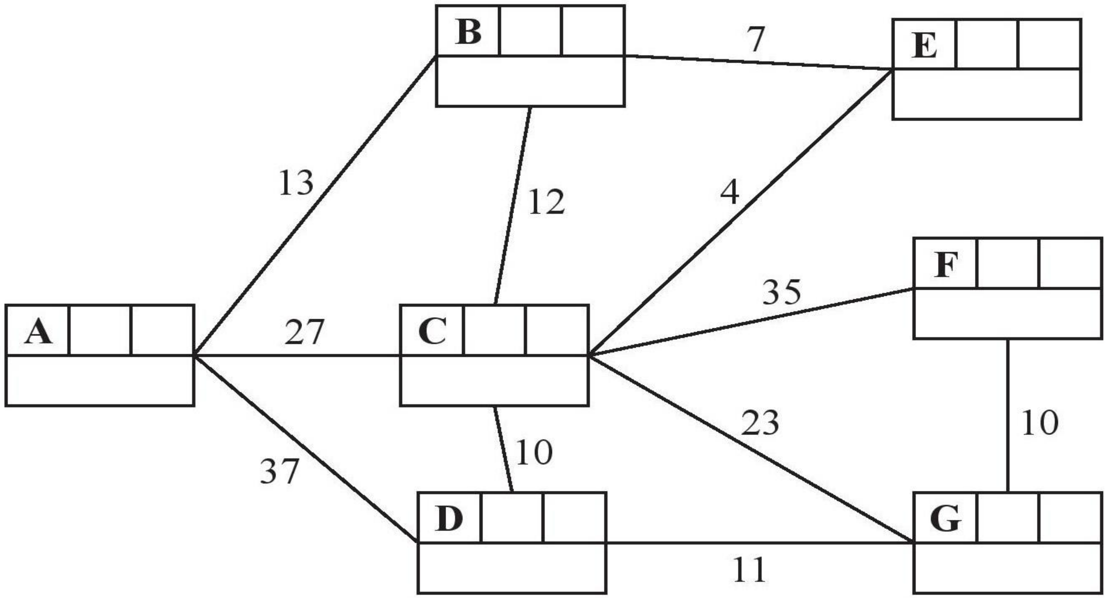
TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE	
Further Mathematics **Advanced Subsidiary** **Paper 2: Further Mathematics options** **Option 2K: Decision Maths 1 and Decision Maths 2**	
Sample assessment material for first teaching September 2017	Paper Reference(s) **8FM0/2K**
Answer Book Do not return the question paper with the answer book.	

SECTION A

1.



Key:

Vertex	Order of labelling	Final value
Working value		

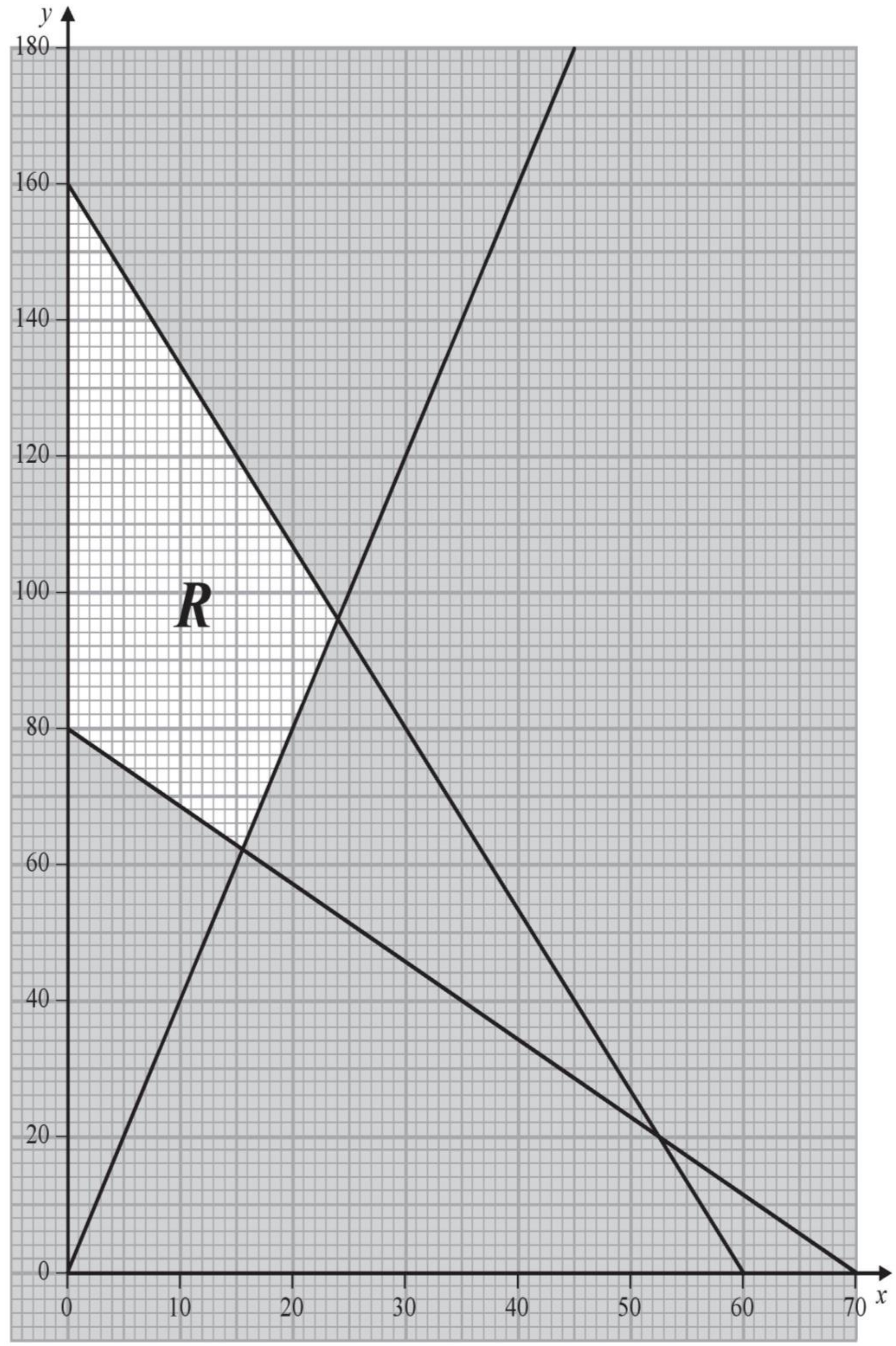
Shortest path: ....

Length of shortest path: ....

Question 1 continued

(Total for Question 1 is 12 marks)

2.



Question 2 continued

(Total for Question 2 is 12 marks)

3. (a) and (b)

....

....

....

....

....

(Total for Question 3 is 7 marks)

4.

(Total for Question 4 is 8 marks)

5.

(Total for Question 5 is 6 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

6.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

(Total for Question 6 is 9 marks)

7. (a)

Number of Lines	1	2	3	4	5	6	7
Maximum Number of Regions	2	4	7	11			

(Total for Question 7 is 5 marks)

8.

Question 8 continued

(d)

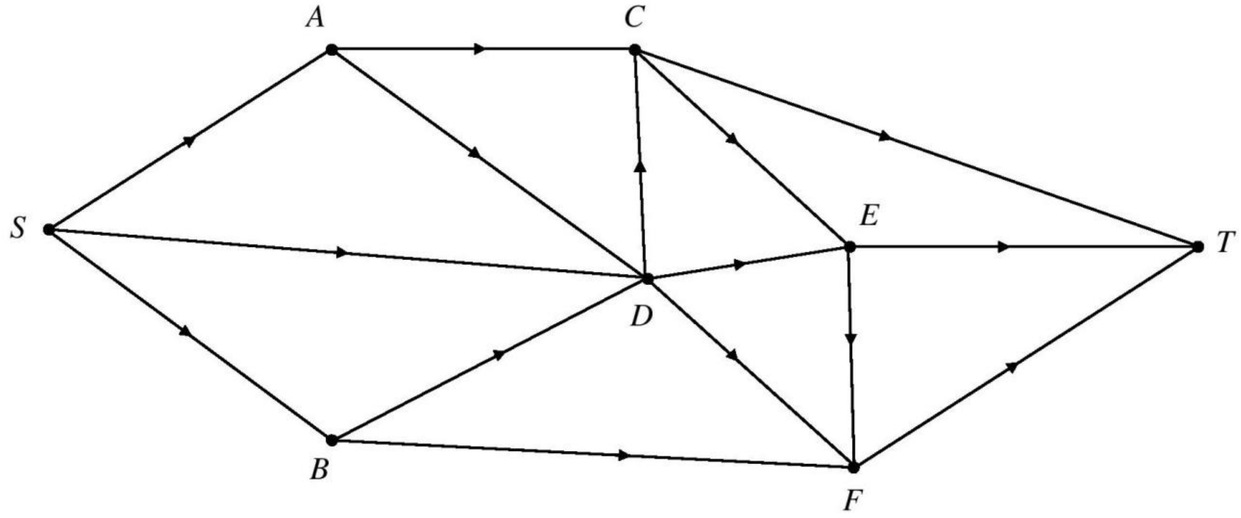


Diagram 1

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(Total for Question 8 is 14 marks)

9.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	4	1	2
A plays 2	2	4	3

Question 9 continued

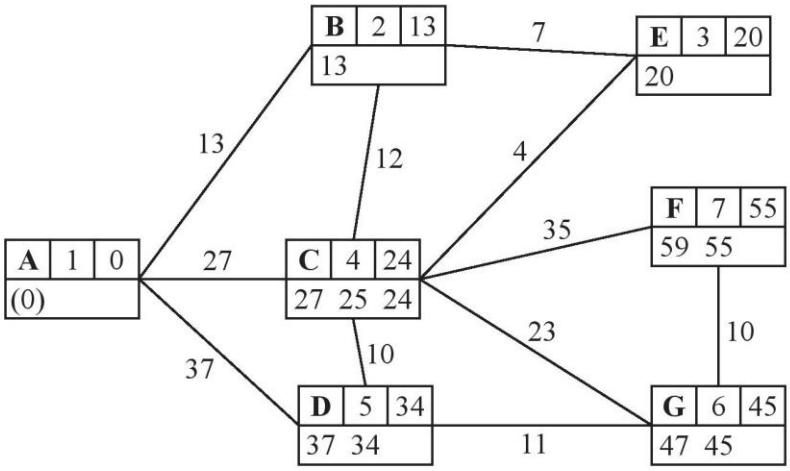
(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS

TOTAL FOR PAPER IS 80 MARKS

AS Paper 2 Option 2K

Decision Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)
M1: for a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: for all values correct (and in correct order) at A, B, C and D
A1: for all values correct (and in correct order) at E, F & G
A1: for the correct path
A1ft: for 55 or ft their final value at F

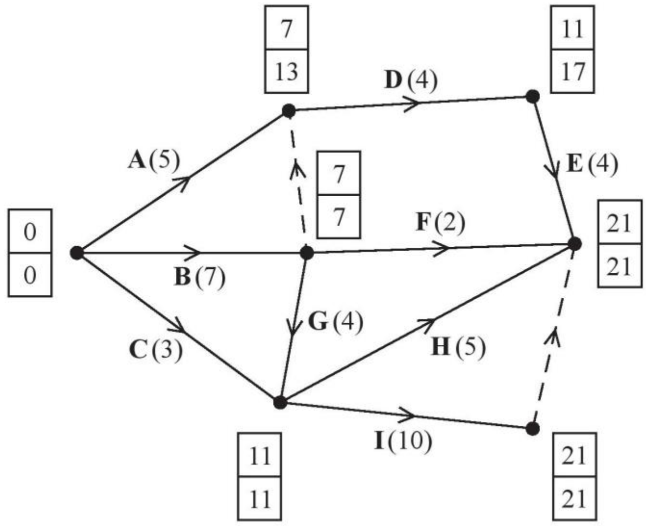
(b)
M1: for 3 correct pairings of the four odd nodes (A,B, D & G)
A1: at least two pairings and totals correct
A2: all three pairings and totals correct
A3ft: selecting their shortest pairing, and stating that these arcs should be repeated

(c)	
B1ft:	for 213 or 189 + their shortest repeat
M1:	for translating the information in the question in to an equation involving x, 2x and 34
A1:	for a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
2	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15±1, 63±2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

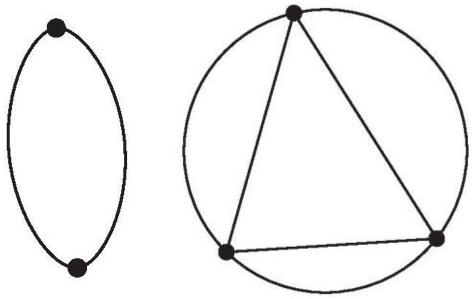
M1:	Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C
M1:	Solving simultaneous equations
A1:	cao
M1: testing	recognition that outcome from this model is non-integer and integer solutions are required – two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$
M1:	testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C
A1:	cao – deducing from tests which integer solution is both valid and optimal
A1ft:	interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
3(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
	The number(s) at the end of activity E indicate this project can be completed in 21 days Critical activities: B, G, I	M1 A1 A1 ft A1	2.1 1.1b 2.2a 1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: all boxes completed, number generally increasing L to R (condone one “rogue”)
A1: all values cao
A1ft: deduction that result in diagram indicates that project can be completed in 21 days (or ft their repeated value at end of E)
A1: critical activities correct

Question	Scheme	Marks	AOs
4(a)	E.g. a graph cannot contain an odd number of odd nodes E.g. number of arcs $= \frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$(2^{2x}-1)+(2^x) + (x+1) + (2^{x+1}-3)+(11-x)=2(18)$	M1	1.1b
	$2^{2x}+3(2^x)-28=0 \Rightarrow x=...$	M1	1.1b
	$(2^x+7)(2^x-4)=0 \Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1 A1	2.4 2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)
B1: explanation referring to need for an even number of odd nodes oe

(b)
M1: forming an equation involving the orders of the 5 odd nodes and $2(18)$
M1: simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: construct an argument involving the order of the 5 nodes
A1: explanation considering the number of odd nodes
A1: deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: interprets mathematical language to construct a disconnected graph
A1: deduce a correct graph

Question	Scheme	Marks	AOs
5	Minimise $(C =)\ 25x + 35y$	B1	3.3
	Subject to: $(500x + 800y \geqslant 150\ 000 \Rightarrow)\ 5x + 8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
	(5 marks)		

Notes:

B1: a correct objective function + minimise
B1: translate information in to a correct inequality
M1: for translating the information given into the LHS inequality
M1: for translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Decision Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6	$\begin{pmatrix} & P & Q & R & S & T & X \\ A & 32 & 32 & 35 & 34 & 33 & 40 \\ B & 28 & 35 & 31 & 37 & 40 & 40 \\ C & 35 & 29 & 33 & 36 & 35 & 40 \\ D & 36 & 30 & 34 & 33 & 35 & 40 \\ E & 30 & 31 & 29 & 37 & 36 & 40 \\ F & 29 & 28 & 32 & 31 & 34 & 40 \end{pmatrix}$	B1	1.1b
	Reducing rows and then columns $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 2 & 1 & 8 \\ B & 0 & 7 & 3 & 9 & 12 & 12 \\ C & 6 & 0 & 4 & 7 & 6 & 11 \\ D & 6 & 0 & 4 & 3 & 5 & 10 \\ E & 1 & 2 & 0 & 8 & 7 & 11 \\ F & 1 & 0 & 4 & 3 & 6 & 12 \end{pmatrix}$ then $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 3 & 7 & 11 & 4 \\ C & 6 & 0 & 4 & 5 & 5 & 3 \\ D & 6 & 0 & 4 & 1 & 4 & 2 \\ E & 1 & 2 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 4 & 1 & 5 & 4 \end{pmatrix}$	M1 A1	1.1b 1.1b
	e.g. augment by 1 then augment by 1 $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 1 & 1 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 2 & 6 & 10 & 3 \\ C & 6 & 0 & 3 & 4 & 4 & 2 \\ D & 6 & 0 & 3 & 0 & 4 & 1 \\ E & 2 & 3 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 3 & 0 & 4 & 3 \end{pmatrix}$ followed by $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 2 & 2 & 3 & 1 & 0 & 0 \\ B & 0 & 7 & 1 & 6 & 9 & 2 \\ C & 6 & 0 & 2 & 4 & 3 & 1 \\ D & 6 & 0 & 2 & 0 & 3 & 0 \\ E & 3 & 4 & 0 & 7 & 6 & 3 \\ F & 1 & 0 & 2 & 0 & 3 & 2 \end{pmatrix}$	M1 A1ft M1 A1ft A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A – T, B – P, C – Q, (D –), E – R, F – S	A1	2.2a
			(9 marks)

Notes

B1: cao – introducing a dummy task and appropriate value

M1: Simplifying the initial matrix by reducing rows **and then** columns

A1: CAO

M1: Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed

A1ft: Follow through on their previous table – no errors

M1: Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)

A1ft: Follow through on their previous table – no errors

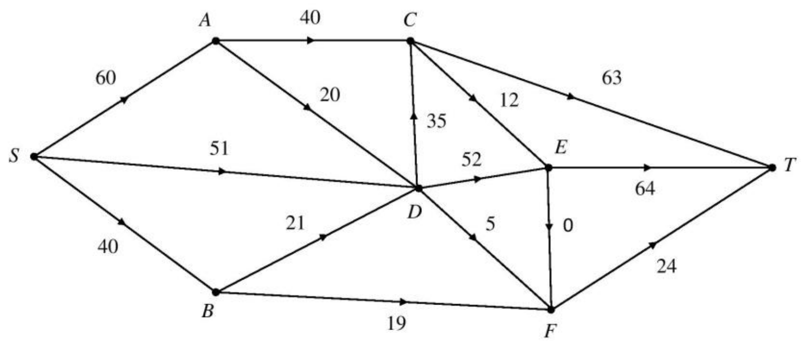
A1: CSO on final table (so must have scored all previous marks)

A1: CAO – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table

Question	Scheme	Marks	AOs
7(a)	16, 22, 29	B1	1.1b
		(1)	
(b)	$u_{n+1} = u_n + n + 1$	B1	3.3
		(1)	
(c)	As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$	M1	1.1b
	$u_n = \frac{1}{2}n(n+1)+1$ 20 101 (regions)	A1 A1ft	1.1b 1.1b
		(3)	
			(5 marks)

Notes:

B1: cao
B1: translating problem to mathematical model - correct recurrence relation needed
M1: an attempt to solve the recurrence relation to determine maximum number of regions
1A1: cao
A1ft: substitution of n = 200 into their quadratic u_n expression

Question	Scheme	Marks	AOs
8(a)	Corridors must be one-way	B1	3.4
		(1)	
(b)	e.g. $55 + x + 40 = 63 + 54 + 24$ or $7 + y = 54 + 5$	M1	2.4
	$x = 46$ $y = 52$	A1 A1	1.1b 1.1b
		(3)	
(c)	(i) SACET (= 5) SDFET (= 5)	M1 A1	1.1b 1.1b
	(ii) Students must choose SACET, as they cannot travel from F to E.	A1	2.2a
		(3)	
(d)		B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem,	M1	2.1
	Identification of cut through AC, DC, DE, (EF), FT = 151 value of flow = 151	A1	3.1a
	Therefore it follows that flow is optimal	A1	2.2a
		(3)	
(f)	Consider increasing capacity of arcs in minimum cut	B1	2.1
	Explanation based on a valid argument, such as: • Increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152 • Increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)	B1	2.4
	Therefore school should choose to widen FT, which could increase the flow through the network by 16	B1	2.2a
		(3)	
		(14 marks)	

Notes:	
(a)	
B1:	explanation of assumption to use this model
(b)	
M1:	either a correct equation, or explanation that flow in = flow out
A1:	cao
A1:	cao
(c)	
M1:	one flow augmenting route found from S to T
A1:	two correct flow augmenting routes 5+
A1:	deduce that SACET must be used as students cannot travel from F to E as route is one-way
(d)	
B1:	a consistent flow pattern = 151
(e)	
M1:	constructing argument based on max-flow min-cut theorem
A1:	use appropriate process of finding a minimum cut – cut + value correct
A1:	correct deduction that the flow is maximal
(f)	
B1	constructing an argument based on arcs in the minimum cut
B1	detailed explanation as to why choosing anything other than FT does not help
B1	correct deduction and correct increase in flow of 16

Question	Scheme	Marks	AOs
9(a)	Row minima: 1, 2 max is 2 Column maxima: 4, 4, 3 min is 3	M1 A1	1.1b 1.1b
	Row maximin (2) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(b)	Let A play strategy 1 with probability p and strategy 2 with probability 1- p, and using this to get at least one equation in p	M1	3.3
	Then if B plays strategy 1, A's gains are $4p + 2(1- p) = 2p + 2$ If B plays strategy 2, A's gains are $p + 4(1- p) = 4 - 3p$ If B plays strategy 3, A's gains are $2p + 3(1- p) = 3 - p$	A1 A1	1.1b 1.1b
	Graph (see handwritten diagram – follow style of June 2015 Q2e MS)	M1 A1	3.1a 1.1b
	Intersection of $2p + 2$ and $3 – p$ occurs where $p = \frac{1}{3}$	dM1 A1ft	1.1b 1.1b
	Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time.	A1ft	3.2a
	The value of the game to player A is $2\frac{2}{3}$	A1	1.1b
		(9)	

(12 marks)

Notes:

(a)

M1: Finding row minimums and column maximums – condone one error

A1: row minima and column maxima correct

A1: explanation involving $2 \neq 3$ and a conclusion

(b)

M1: Translating situation into model by defining variables and constructing at least one equation

A1: one row correct

A1: all three rows correct

M1: axes correct, at least one line correctly drawn for their expression

A1: correct graph

M1: using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as p =

A1ft: follow through on their optimal intersection

A1ft: interpret their value of p in the context of the question – must refer to play, player A

A1: cao

Pearson Edexcel Level 3 Advanced Subsidiary and Advanced GCE Mathematics and Further Mathematics

Mathematical formulae and statistical tables

For first certification from June 2018 for:
Advanced Subsidiary GCE in Mathematics (8MA0)
Advanced GCE in Mathematics (9MA0)
Advanced Subsidiary GCE in Further Mathematics (8FM0)
For first certification from June 2019 for:
Advanced GCE in Further Mathematics (9FM0)

Contents

1 Introduction

The formulae in this booklet have been arranged by qualification. Students sitting AS or A Level Further Mathematics papers may be required to use the formulae that were introduced in AS or A Level Mathematics papers.

It may also be the case that students sitting Mechanics and Statistics papers will need to use formulae introduced in the appropriate Pure Mathematics papers for the qualification they are sitting.

2 AS Level in Mathematics

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ \dots \ + \binom{n}{r} a^{n-r}b^r + \ \dots \ + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Statistics

Probability

$P(A') = 1 - P(A)$

Standard deviation

Standard deviation = √(Variance)

Interquartile range = IQR = $Q_3 - Q_1$

For a set of n values $x_1, x_2, \dots x_i, \dots x_n$

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\dfrac{S_{xx}}{n}}$ or $\sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$

Statistical tables

The following statistical tables are required for A Level Mathematics:

Binomial Cumulative Distribution Function (see page 25)

Random Numbers (see page 34)

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2} at^2$

$s = vt - \frac{1}{2} at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2} (u + v)t$

3 A Level in Mathematics

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (|x|<1,\ n\in\mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x\ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Differentiation

First Principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

f(*x*)	**f ′(*x*)**
$\tan kx$	$k \sec^2 kx$
$\sec kx$	$k \sec kx \tan kx$
$\cot kx$	$-k \operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k \operatorname{cosec} kx \cot kx$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\,\mathrm{g}(x) - \mathrm{f}(x)\,\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Integration (+ constant)

f(*x*)	$\int \mathrm{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln \lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln \lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\tfrac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\tfrac{1}{2}kx + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Statistics

Probability

$$\mathrm{P}(A') = 1 - \mathrm{P}(A)$$

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A)$$

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\,\mathrm{P}(A)}{\mathrm{P}(B \mid A)\,\mathrm{P}(A) + \mathrm{P}(B \mid A')\,\mathrm{P}(A')}$$

For independent events *A* and *B*,

$$\mathrm{P}(B \mid A) = \mathrm{P}(B),\ \mathrm{P}(A \mid B) = \mathrm{P}(A),$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\ \mathrm{P}(B)$$

Standard deviation

Standard deviation = √(Variance)

Interquartile range = IQR = $Q_3 - Q_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\frac{S_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

Discrete distributions

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

Statistical tables

The following statistical tables are required for A Level Mathematics:

Binomial Cumulative Distribution Function (see page 25)

Percentage Points Of The Normal Distribution (30)

Critical Values for Correlation Coefficients: Product Moment Coefficient (see page 33)

Random Numbers (see page 34)

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + ½ at^2$

$s = vt - ½ at^2$

$v^2 = u^2 + 2as$

$s = ½ (u + v)t$

4 AS Level in Further Mathematics

Students sitting a AS Level Further Mathematics paper may also require those formulae listed for A Level Mathematics in Section 3.

Pure Mathematics

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Matrix transformations

Anticlockwise rotation through θ about *O*: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Area of a sector

$A = \dfrac{1}{2}\int r^2 \, \mathrm{d}\theta$ (polar coordinates)

Complex numbers

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$\mathrm{f}(x)=\mathrm{f}(0)+x\,\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\ldots+\frac{x^r}{r!}\mathrm{f}^{(r)}(0)+\ldots$$

$$\mathrm{e}^x=\exp(x)=1+x+\frac{x^2}{2!}+\ldots+\frac{x^r}{r!}+\ldots \quad \text{for all } x$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\ \ldots\ +(-1)^{r+1}\frac{x^r}{r}+\ \ldots \qquad (-1<x\leqslant 1)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^r\frac{x^{2r+1}}{(2r+1)!}+\ldots \quad \text{for all } x$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots+(-1)^r\frac{x^{2r}}{(2r)!}+\ldots \quad \text{for all } x$$

$$\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\ \ldots+\ (-1)^r\frac{x^{2r+1}}{2r+1}+\ \ldots \qquad (-1\leqslant x\leqslant 1)$$

Vectors

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{pmatrix}$

$$\mathbf{a}.(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\mathbf{b}.(\mathbf{c}\times\mathbf{a})=\mathbf{c}.(\mathbf{a}\times\mathbf{b})$$

If A is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\ (=\lambda)$$

The plane through A with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$n_1x+n_2y+n_3z+d=0$ where $d=-\mathbf{a.n}$

The plane through non-collinear points *A*, *B* and *C* has vector equation

$$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$$

The perpendicular distance of (α,β,γ) from $n_1x+n_2y+n_3z+d=0$ is $\dfrac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\operatorname{arcosh} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \qquad (x \geq 1)$

$\operatorname{arsinh} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$

$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$

Differentiation

f(*x*)	**f′(*x*)**
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\frac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\operatorname{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Statistics

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $P(X = x_i)$

Expectation (mean): $E(X) = \mu = \Sigma x_i P(X = x_i)$

Variance: $Var(X) = \sigma^2 = \Sigma(x_i - \mu)^2 P(X = x_i) = \Sigma x_i^2 P(X = x_i) - \mu^2$

Discrete distributions

Standard discrete distributions:

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $E(X) = \mu = \int x f(x) dx$

Variance: $Var(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x) f(x) dx$

Cumulative distribution function: $F(x_0) = P(X \leqslant x_0) = \int_{-\infty}^{x_0} f(t) \, dt$

Standard continuous distribution:

Distribution of X	P.D.F.	Mean	Variance
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is:

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Residual Sum of Squares (RSS) = $S_{yy} - \frac{\left(S_{xy}\right)^2}{S_{xx}} = S_{yy}\left(1 - r^2\right)$

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_\nu^2$

Statistical tables

The following statistical tables are required for AS Level Further Mathematics:

Binomial Cumulative Distribution Function (see page 25)

Poisson Cumulative Distribution Function (see page 31)

Percentage Points of the χ2 Distribution (see page 32)

Critical Values for Correlation Coefficients: Product Moment Coefficient and Spearman's Coefficient (see page 33)

Random Numbers (see page 34)

Mechanics

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r, angle at centre 2α : $\dfrac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α : $\dfrac{2r \sin \alpha}{3\alpha}$ from centre

5 A Level in Further Mathematics

Students sitting a A Level Further Mathematics paper may also require those formulae listed for A Level Mathematics in Section 3.

Pure Mathematics

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4} n^2 (n+1)^2$$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Area of a sector

$A = \dfrac{1}{2}\displaystyle\int r^2 \, \mathrm{d}\theta$ (polar coordinates)

Complex numbers

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ \ldots \ + (-1)^{r+1}\frac{x^r}{r} + \ \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ \ldots + \ (-1)^r \frac{x^{2r+1}}{2r+1} + \ \ldots \quad (-1 \leqslant x \leqslant 1)$$

Vectors

Vector product: $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b}.(\mathbf{c}\times\mathbf{a}) = \mathbf{c}.(\mathbf{a}\times\mathbf{b})$$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} \ (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$

The plane through non-collinear points *A*, *B* and *C* has vector equation
$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b}-\mathbf{a}) + \mu(\mathbf{c}-\mathbf{a}) = (1-\lambda-\mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation
$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$ The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is

$$\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\text{arcosh } x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$

$\text{arsinh } x = \ln\{x + \sqrt{x^2 + 1}\}$

$\text{artanh } x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(x)	f′(x)
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\dfrac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\text{arcosh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\text{arsinh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\text{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$$ (cartesian coordinates)

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$ (parametric form)

$$s = \int \sqrt{r^2 + \left(\frac{\mathrm{dr}}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$$ (polar form)

Surface area of revolution

$S_x = 2\pi \int y \sqrt{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)} \, \mathrm{d}x$ (cartesian coordinates)

$S_x = 2\pi \int y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t$ (parametric form)

$S_x = 2\pi \int r \sin\theta \sqrt{r^2 + \left(\frac{\mathrm{dr}}{\mathrm{d}\theta}\right)^2} \, \mathrm{d}\theta$ (polar form)

Statistics

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $\mathrm{P}(X = x_i)$

Expectation (mean): $\mathrm{E}(X) = \mu = \Sigma x_i \, \mathrm{P}(X = x_i)$

Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma(x_i - \mu)^2 \, \mathrm{P}(X = x_i) = \Sigma \, x_i^2 \, \mathrm{P}(X = x_i) - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \Sigma \mathrm{g}(x_i) \, \mathrm{P}(X = x_i)$

The probability generating function of X is $\mathrm{G}_X(t) = \mathrm{E}\left(t^X\right)$ and

$$\mathrm{E}(X) = \mathrm{G}'_X(1) \text{ and } \mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \left[\mathrm{G}'_X(1)\right]^2$$

For $Z = X + Y$, where X and Y are independent: $\mathrm{G}_Z(t) = \mathrm{G}_X(t) \times \mathrm{G}_Y(t)$

Discrete distributions

Standard discrete distributions:

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance	P.G.F.
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pt)^n$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda} \dfrac{\lambda^x}{x!}$	λ	λ	$\mathrm{e}^{\lambda(t-1)}$
Geometric $\mathrm{Geo}(p)$ on $1, 2, \ldots$	$p(1-p)^{x-1}$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$	$\dfrac{pt}{1-(1-p)t}$
Negative binomial on $r, r+1, \ldots$	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$	$\dfrac{r}{p}$	$\dfrac{r(1-p)}{p^2}$	$\left(\dfrac{pt}{1-(1-p)t}\right)^r$

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $\mathrm{E}(X) = \mu = \int x \, \mathrm{f}(x) \, \mathrm{d}x$

Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x - \mu)^2 \, \mathrm{f}(x) \, \mathrm{d}x = \int x^2 \, \mathrm{f}(x) \, \mathrm{d}x - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x) \, \mathrm{f}(x) \, \mathrm{d}x$

Cumulative distribution function: $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \int_{-\infty}^{x_0} \mathrm{f}(t) \, \mathrm{d}t$

Standard continuous distribution:

Distribution of X	P.D.F.	Mean	Variance
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$\mathrm{S}_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$\mathrm{S}_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$\mathrm{S}_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{\mathrm{S}_{xy}}{\sqrt{\mathrm{S}_{xx}\mathrm{S}_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{\mathrm{S}_{xy}}{\mathrm{S}_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Residual Sum of Squares (RSS) = $\mathrm{S}_{yy} - \frac{\left(\mathrm{S}_{xy}\right)^2}{\mathrm{S}_{xx}} = \mathrm{S}_{yy}\left(1 - r^2\right)$

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

Expectation algebra

For independent random variables X and Y

$$\mathrm{E}(XY) = \mathrm{E}(X)\,\mathrm{E}(Y), \quad \mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$$

Sampling distributions

(i) Tests for mean when σ is known

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2:

$\bar{X}$ is an unbiased estimator of μ, with $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \dfrac{\Sigma(X_i - \bar{X})^2}{n-1}$

For a random sample of n observations from $\mathrm{N}(\mu,\ \sigma^2)$, $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,\ 1)$

For a random sample of n_x observations from $\mathrm{N}(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $\mathrm{N}(\mu_y, \sigma_y^2)$, $\dfrac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0,1)$

(ii) Tests for variance and mean when σ is not known

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$ (also valid in matched-pairs situations)

For a random sample of n_x observations from $\mathrm{N}(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $\mathrm{N}(\mu_y, \sigma_y^2)$

$$\frac{S_x^2/\sigma_x^2}{S_y^2/\sigma_y^2} \sim F_{n_x-1,\, n_y-1}$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$\dfrac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\dfrac{1}{n_x}+\dfrac{1}{n_y}\right)}} \sim t_{n_x+n_y-2}$ where $S_p^2 = \dfrac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_\nu^2$ **Statistical tables**

The following statistical tables are required for A Level Further Mathematics:

Binomial Cumulative Distribution Function (see page 25)

Percentage Points Of The Normal Distribution (see page 30)

Poisson Cumulative Distribution Function (see page 31)

Percentage Points of the χ^2 *Distribution* (see page 32)

Critical Values for Correlation Coefficients: Product Moment Coefficient and Spearman's Coefficient (see page 33)

Random Numbers (see page 34)

Percentage Points of Student's t Distribution (see page 35)

Percentage Points of the F Distribution (see page 36

Mechanics

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r, angle at centre 2α : $\dfrac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α : $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Solid cone or pyramid of height h: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

6 Statistical Tables

Binomial Cumulative Distribution Function

The tabulated value is $P(X \leqslant x)$, where X has a binomial distribution with index n and parameter p.

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 5, x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
$n = 6, x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
$n = 7, x = 0$	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
$n = 8, x = 0$	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
$n = 9, x = 0$	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
$n = 10, x = 0$	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 12, x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
$n = 15, x = 0$	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20, x = 0$	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 25, x = 0$	0.2774	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000
3	0.9659	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001
4	0.9928	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005
5	0.9988	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020
6	0.9998	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073
7	1.0000	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216
8	1.0000	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539
9	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148
10	1.0000	1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122
11	1.0000	1.0000	0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450
12	1.0000	1.0000	1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878
15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9971	0.9868	0.9560	0.8852
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9957	0.9826	0.9461
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9942	0.9784
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 30, x = 0$	0.2146	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000
5	0.9967	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002
6	0.9994	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007
7	0.9999	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026
8	1.0000	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081
9	1.0000	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214
10	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494
11	1.0000	1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002
12	1.0000	1.0000	0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808
13	1.0000	1.0000	1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923
14	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278
15	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722
16	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9955	0.9788	0.9286	0.8192
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9917	0.9666	0.8998
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9971	0.9862	0.9506
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

p =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
n = 40, x = 0	0.1285	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.3991	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.6767	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.8619	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000
4	0.9520	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000
5	0.9861	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000
6	0.9966	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000
7	0.9993	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000
8	0.9999	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001
9	1.0000	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003
10	1.0000	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011
11	1.0000	0.9996	0.9880	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032
12	1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083
13	1.0000	1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192
14	1.0000	1.0000	0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403
15	1.0000	1.0000	0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769
16	1.0000	1.0000	1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341
17	1.0000	1.0000	1.0000	0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148
18	1.0000	1.0000	1.0000	0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179
19	1.0000	1.0000	1.0000	1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373
20	1.0000	1.0000	1.0000	1.0000	0.9998	0.9976	0.9827	0.9256	0.7870	0.5627
21	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9925	0.9608	0.8669	0.6821
22	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9970	0.9811	0.9233	0.7852
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989	0.9917	0.9595	0.8659
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9804	0.9231
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9914	0.9597
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9808
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9917
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9968
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 50, x = 0$	0.0769	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.2794	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.5405	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7604	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8964	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.9622	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000
6	0.9882	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000
7	0.9968	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000
8	0.9992	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000
9	0.9998	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000
10	1.0000	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000
11	1.0000	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000
12	1.0000	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002
13	1.0000	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005
14	1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013
15	1.0000	1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033
16	1.0000	1.0000	0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077
17	1.0000	1.0000	0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164
18	1.0000	1.0000	0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325
19	1.0000	1.0000	1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595
20	1.0000	1.0000	1.0000	0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013
21	1.0000	1.0000	1.0000	0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611
22	1.0000	1.0000	1.0000	1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399
23	1.0000	1.0000	1.0000	1.0000	0.9996	0.9944	0.9604	0.8438	0.6134	0.3359
24	1.0000	1.0000	1.0000	1.0000	0.9999	0.9976	0.9793	0.9022	0.7160	0.4439
25	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9900	0.9427	0.8034	0.5561
26	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9955	0.9686	0.8721	0.6641
27	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9840	0.9220	0.7601
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9924	0.9556	0.8389
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9966	0.9765	0.8987
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9986	0.9884	0.9405
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9947	0.9675
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9836
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9923
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9967
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Percentage Points Of The Normal Distribution

The values z in the table are those which a random variable $Z \sim \mathrm{N}(0, 1)$ exceeds with probability p; that is, $\mathrm{P}(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Poisson Cumulative Distribution Function

The tabulated value is $P(X \leq x)$, where X has a Poisson distribution with parameter λ.

$\lambda =$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$x = 0$	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\lambda =$	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$x = 0$	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
2	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997

Percentage Points of the χ^2 Distribution

The values in the table are those which a random variable with the χ^2 distribution on ν degrees of freedom exceeds with the probability shown.

ν	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.000	0.000	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.580	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.088	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672

Critical Values for Correlation Coefficients

These tables concern tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

Product Moment Coefficient					Sample Size, n	Spearman's Coefficient		
		Level					Level	
0.10	0.05	0.025	0.01	0.005		0.05	0.025	0.01
0.8000	0.9000	0.9500	0.9800	0.9900	4	1.0000	-	-
0.6870	0.8054	0.8783	0.9343	0.9587	5	0.9000	1.0000	1.0000
0.6084	0.7293	0.8114	0.8822	0.9172	6	0.8286	0.8857	0.9429
0.5509	0.6694	0.7545	0.8329	0.8745	7	0.7143	0.7857	0.8929
0.5067	0.6215	0.7067	0.7887	0.8343	8	0.6429	0.7381	0.8333
0.4716	0.5822	0.6664	0.7498	0.7977	9	0.6000	0.7000	0.7833
0.4428	0.5494	0.6319	0.7155	0.7646	10	0.5636	0.6485	0.7455
0.4187	0.5214	0.6021	0.6851	0.7348	11	0.5364	0.6182	0.7091
0.3981	0.4973	0.5760	0.6581	0.7079	12	0.5035	0.5874	0.6783
0.3802	0.4762	0.5529	0.6339	0.6835	13	0.4835	0.5604	0.6484
0.3646	0.4575	0.5324	0.6120	0.6614	14	0.4637	0.5385	0.6264
0.3507	0.4409	0.5140	0.5923	0.6411	15	0.4464	0.5214	0.6036
0.3383	0.4259	0.4973	0.5742	0.6226	16	0.4294	0.5029	0.5824
0.3271	0.4124	0.4821	0.5577	0.6055	17	0.4142	0.4877	0.5662
0.3170	0.4000	0.4683	0.5425	0.5897	18	0.4014	0.4716	0.5501
0.3077	0.3887	0.4555	0.5285	0.5751	19	0.3912	0.4596	0.5351
0.2992	0.3783	0.4438	0.5155	0.5614	20	0.3805	0.4466	0.5218
0.2914	0.3687	0.4329	0.5034	0.5487	21	0.3701	0.4364	0.5091
0.2841	0.3598	0.4227	0.4921	0.5368	22	0.3608	0.4252	0.4975
0.2774	0.3515	0.4133	0.4815	0.5256	23	0.3528	0.4160	0.4862
0.2711	0.3438	0.4044	0.4716	0.5151	24	0.3443	0.4070	0.4757
0.2653	0.3365	0.3961	0.4622	0.5052	25	0.3369	0.3977	0.4662
0.2598	0.3297	0.3882	0.4534	0.4958	26	0.3306	0.3901	0.4571
0.2546	0.3233	0.3809	0.4451	0.4869	27	0.3242	0.3828	0.4487
0.2497	0.3172	0.3739	0.4372	0.4785	28	0.3180	0.3755	0.4401
0.2451	0.3115	0.3673	0.4297	0.4705	29	0.3118	0.3685	0.4325
0.2407	0.3061	0.3610	0.4226	0.4629	30	0.3063	0.3624	0.4251
0.2070	0.2638	0.3120	0.3665	0.4026	40	0.2640	0.3128	0.3681
0.1843	0.2353	0.2787	0.3281	0.3610	50	0.2353	0.2791	0.3293
0.1678	0.2144	0.2542	0.2997	0.3301	60	0.2144	0.2545	0.3005
0.1550	0.1982	0.2352	0.2776	0.3060	70	0.1982	0.2354	0.2782
0.1448	0.1852	0.2199	0.2597	0.2864	80	0.1852	0.2201	0.2602
0.1364	0.1745	0.2072	0.2449	0.2702	90	0.1745	0.2074	0.2453
0.1292	0.1654	0.1966	0.2324	0.2565	100	0.1654	0.1967	0.2327

Random Numbers

86	13	84	10	07	30	39	05	97	96	88	07	37	26	04	89	13	48	19	20
60	78	48	12	99	47	09	46	91	33	17	21	03	94	79	00	08	50	40	16
78	48	06	37	82	26	01	06	64	65	94	41	17	26	74	66	61	93	24	97
80	56	90	79	66	94	18	40	97	79	93	20	41	51	25	04	20	71	76	04
99	09	39	25	66	31	70	56	30	15	52	17	87	55	31	11	10	68	98	23
56	32	32	72	91	65	97	36	56	61	12	79	95	17	57	16	53	58	96	36
66	02	49	93	97	44	99	15	56	86	80	57	11	78	40	23	58	40	86	14
31	77	53	94	05	93	56	14	71	23	60	46	05	33	23	72	93	10	81	23
98	79	72	43	14	76	54	77	66	29	84	09	88	56	75	86	41	67	04	42
50	97	92	15	10	01	57	01	87	33	73	17	70	18	40	21	24	20	66	62
90	51	94	50	12	48	88	95	09	34	09	30	22	27	25	56	40	76	01	59
31	99	52	24	13	43	27	88	11	39	41	65	00	84	13	06	31	79	74	97
22	96	23	34	46	12	67	11	48	06	99	24	14	83	78	37	65	73	39	47
06	84	55	41	27	06	74	59	14	29	20	14	45	75	31	16	05	41	22	96
08	64	89	30	25	25	71	35	33	31	04	56	12	67	03	74	07	16	49	32
86	87	62	43	15	11	76	49	79	13	78	80	93	89	09	57	07	14	40	74
94	44	97	13	77	04	35	02	12	76	60	91	93	40	81	06	85	85	72	84
63	25	55	14	66	47	99	90	02	90	83	43	16	01	19	69	11	78	87	16
11	22	83	98	15	21	18	57	53	42	91	91	26	52	89	13	86	00	47	61
01	70	10	83	94	71	13	67	11	12	36	54	53	32	90	43	79	01	95	15

Percentage Points of Student's t Distribution

The values in the table are those which a random variable with Student's t distribution on ν degrees of freedom exceeds with the probability shown.

ν	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.435	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.369	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617

Percentage Points of the *F* Distribution

The values in the table are those which a random variable with the *F* distribution on ν_1 and ν_2 degrees of freedom exceeds with probability 0.05 or 0.01.

Probability	ν_1 \ ν_2	1	2	3	4	5	6	8	10	12	24	∞
0.05	1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	241.9	243.9	249.1	254.3
	2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.40	19.41	19.46	19.50
	3	10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.79	8.74	8.64	8.53
	4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.96	5.91	5.77	5.63
	5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.74	4.68	4.53	4.37
	6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.06	4.00	3.84	3.67
	7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.64	3.57	3.41	3.23
	8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.35	3.28	3.12	2.93
	9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.14	3.07	2.90	2.71
	10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.98	2.91	2.74	2.54
	11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.85	2.79	2.61	2.40
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.85	2.75	2.69	2.51	2.30
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.60	2.53	2.35	2.13
	16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.49	2.42	2.24	2.01
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.41	2.34	2.15	1.92
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.35	2.28	2.08	1.84
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.34	2.24	2.16	1.96	1.71
	30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.16	2.09	1.89	1.62
	40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.08	2.00	1.79	1.51
	60	4.00	3.15	2.76	2.53	2.37	2.25	2.10	1.99	1.92	1.70	1.39
	120	3.92	3.07	2.68	2.45	2.29	2.18	2.02	1.91	1.83	1.61	1.25
	∞	3.84	3.00	2.60	2.37	2.21	2.10	1.94	1.83	1.75	1.52	1.00
0.01	1	4052.	5000.	5403.	5625.	5764.	5859.	5982.	6056.	6106.	6235.	6366.
	2	98.50	99.00	99.17	99.25	99.30	99.33	99.37	99.40	99.42	99.46	99.50
	3	34.12	30.82	29.46	28.71	28.24	27.91	27.49	27.23	27.05	26.60	26.13
	4	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.55	14.37	13.93	13.45
	5	16.26	13.27	12.06	11.39	10.97	10.67	10.29	10.05	9.89	9.47	9.02
	6	13.70	10.90	9.78	9.15	8.75	8.47	8.10	7.87	7.72	7.31	6.88
	7	12.20	9.55	8.45	7.85	7.46	7.19	6.84	6.62	6.47	6.07	5.65
	8	11.30	8.65	7.59	7.01	6.63	6.37	6.03	5.81	5.67	5.28	4.86
	9	10.60	8.02	6.99	6.42	6.06	5.80	5.47	5.26	5.11	4.73	4.31
	10	10.00	7.56	6.55	5.99	5.64	5.39	5.06	4.85	4.17	4.33	3.91
	11	9.65	7.21	6.22	5.67	5.32	5.07	4.74	4.54	4.40	4.02	3.60
	12	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.30	4.16	3.78	3.36
	14	8.86	6.51	5.56	5.04	4.70	4.46	4.14	3.94	3.80	3.43	3.00
	16	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.69	3.55	3.18	2.75
	18	8.29	6.01	5.09	4.58	4.25	4.01	3.71	3.51	3.37	3.00	2.57
	20	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.37	3.23	2.86	2.42
	25	7.77	5.57	4.68	4.18	3.86	3.63	3.32	3.13	2.99	2.62	2.17
	30	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.98	2.84	2.47	2.01
	40	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.80	2.66	2.29	1.80
	60	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.63	2.50	2.12	1.60
	120	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.47	2.34	1.95	1.38
	∞	6.63	4.61	3.78	3.32	3.02	2.80	2.51	2.32	2.18	1.79	1.00

If an *upper* percentage point of the *F* distribution on ν_1 and ν_2 degrees of freedom is *f* , then the corresponding *lower* percentage point of the *F* distribution on ν_2 and ν_1 degrees of freedom is 1/ *f*.

April 2017

For information about Edexcel, BTEC or LCCI qualifications visit
qualifications.pearson.com

Pearson Education Limited. Registered in England and Wales No. 872828
Registered Office: 80 Strand, London WC2R 0RL.
VAT Reg No GB 278 537121