GCSE
MATHEMATICS

H

Higher Tier　　Paper 2　Calculator
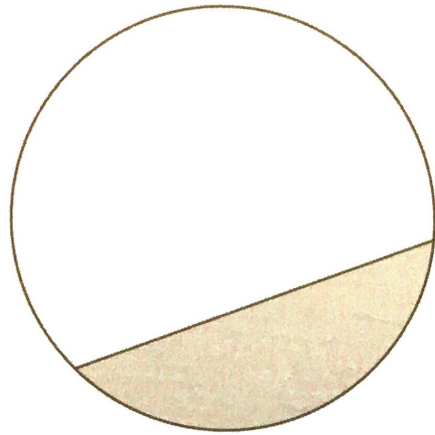
Thursday 7 June 2018　　Morning　　Time allowed: 1 hour 30 minutes

For Examiner's Use

1   Here is a circle.

Circle the word that describes the shaded part.

[1 mark]

(segment)   chord   sector   arc

2   Circle the number that is in standard form.

[1 mark]

0.25×10^4 (6×10^7) 38×10^{-3} $4 \times 10^{\frac{1}{2}}$

3 y is $1\frac{1}{2}$ times x.

Circle the ratio that is equivalent to $y : x$

1½ 1

[1 mark]

2 : 5   5 : 2   (3 : 2)   2 : 3

4 Work out 40 as a percentage of 10

Circle your answer.

[1 mark]

4%   25%   300%   (400%)

Turn over for the next question

5   Match each sequence to its description.

One has been done for you.

1 1 2 3 5 8		Arithmetic progression
1 2 4 8 16 32		Geometric progression
1 2 3 4 5 6		Fibonacci sequence
1 3 6 10 15 21		Triangular numbers
1 4 9 16 25 36		Cube numbers
1 8 27 64 125 216		Square numbers
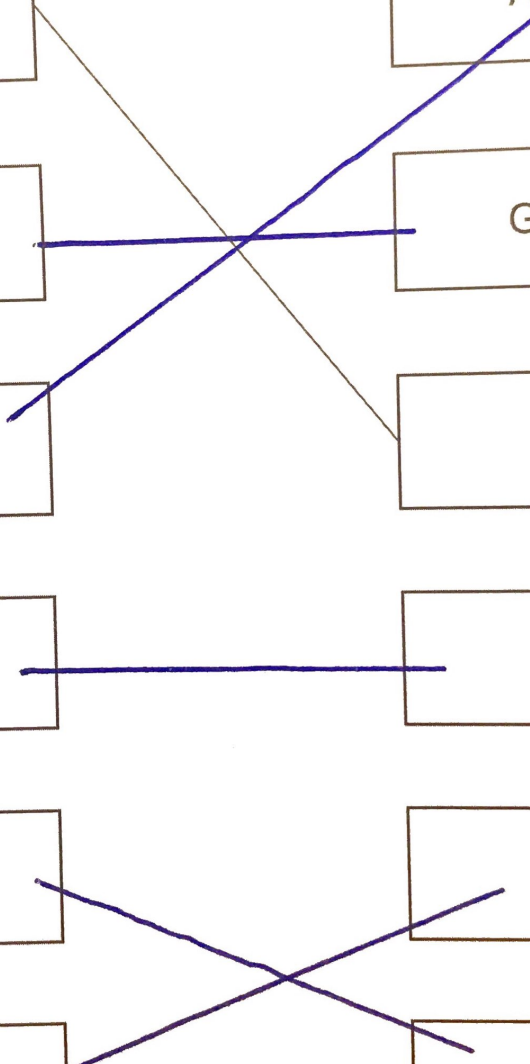
6 The table shows information about the population of a city.

Population in 2001	Population in 2011
420 000	480 000

Liam claims,

"From 2011 to 2021 the population of the city will increase by the same percentage as from 2001 to 2011"

He works out,

population increase from 2001 to 2011 = 480 000 – 420 000

= 60 000

population in 2021 = 480 000 + 60 000

= 540 000

Does the population of 540 000 match his claim?

You **must** show your working.

[3 marks]

$$\frac{60,000}{420,000} = 14.2857\%$$

No

$$\frac{60,000}{480,000} = 12.5\%$$

Needs to be more than 60,000 increase, as a percentage of a bigger number now.

Answer _____

Turn over for the next question

7 On three days, Ali throws darts at a target.

Here are his results.

	Number of throws	Number of hits	Number of misses
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Total	90	54	36

7 (a) Work out **two** different estimates for the probability of Ali hitting the target.

[2 marks]

15/20

Answer ___15/20___ and ___54/90___

7 (b) Which of your two answers is the better estimate for the probability of Ali hitting the target?

Give a reason for your answer.

[1 mark]

Answer ___54/90___

Reason ___Takes account of more data___

8

Theo starts with savings of £18
James starts with no savings.

Each week from now,

Theo will save £4.50   and   James will save £4

In how many weeks will Theo and James have savings in the ratio 15 : 8 ?

[3 marks]

T	J
18	0
1 22.5	4
2 27	8
3 31.5	12
4 36	16
5 40.5	20
6 45	24

$40.5/20 = 2.025$

$15/8 = 1.875$

$45/24 = 1.875$

Answer _____ **6** _____

9 The length of each side of a regular pentagon is 8.4 cm to 1 decimal place.

9 (a) Complete the error interval for the length of one side.

_8.35_ cm ⩽ length < _8.45_ cm

9 (b) Complete the error interval for the perimeter.

× 5

_41.75_ cm ⩽ perimeter < _42.25_ cm

10

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

A container is a hemisphere of radius 30 cm
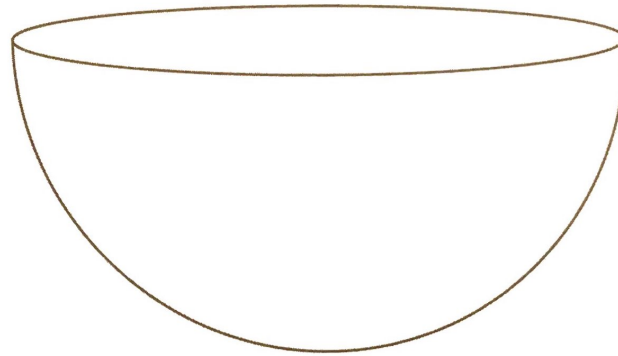
Sand fills the container at a rate of 4000 cm³ per minute.

Does it take **less than** a quarter of an hour to fill the container?

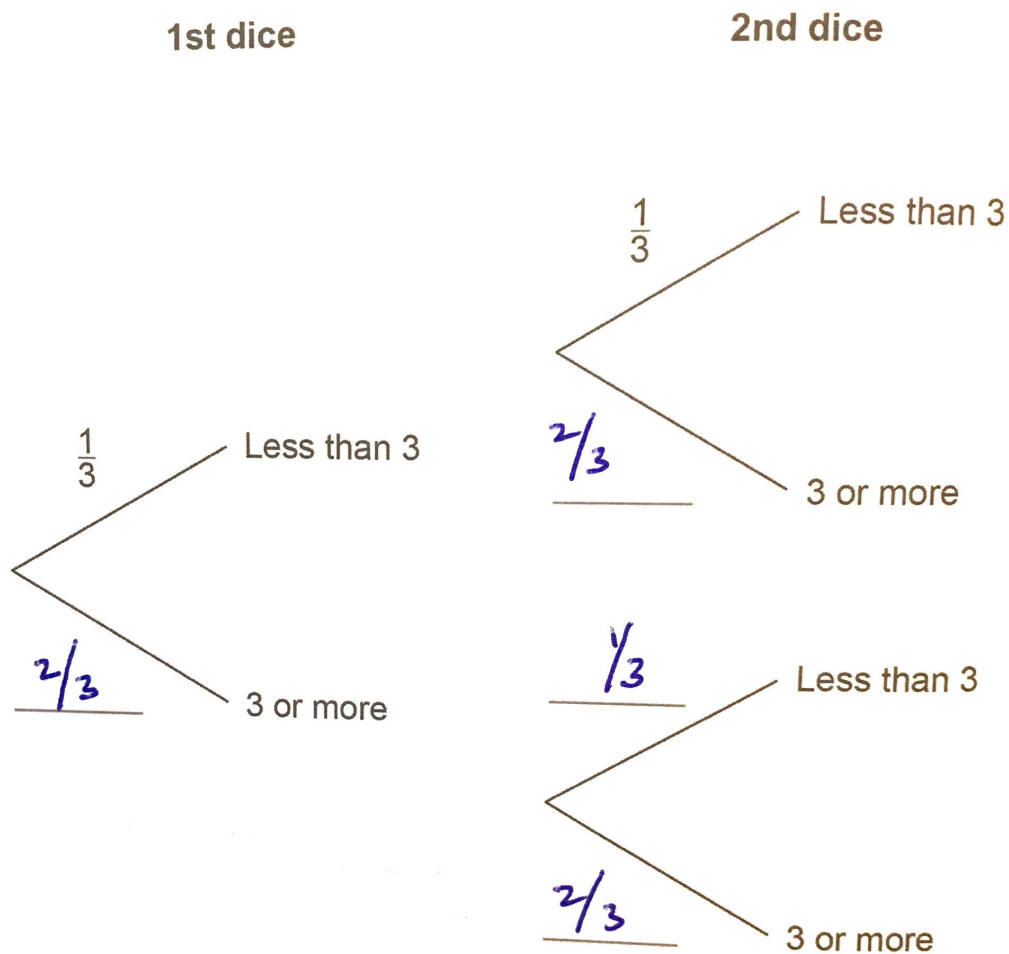You **must** show your working. **[3 marks]**

volume

$$\frac{\frac{4}{3} \times \pi \times 30^3}{2} = 56548.66776$$

$$4000 \times 15 = 60,000$$

11   Two ordinary fair dice are rolled.

11 (a)   Complete the tree diagram.

[1 mark]

1st dice   **2nd dice**

$\frac{1}{3}$ Less than 3

$\frac{2}{3}$

$\frac{1}{3}$ Less than 3

$\frac{2}{3}$ 3 or more

$\frac{2}{3}$ 3 or more

$\frac{1}{3}$ Less than 3

$\frac{2}{3}$ 3 or more

11 (b)   Work out the probability that **both** dice land on a number less than 3

[1 mark]

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Answer $\frac{1}{9}$

11 (c) Work out the probability that **exactly one** of the dice lands on a number less than 3

[2 marks]
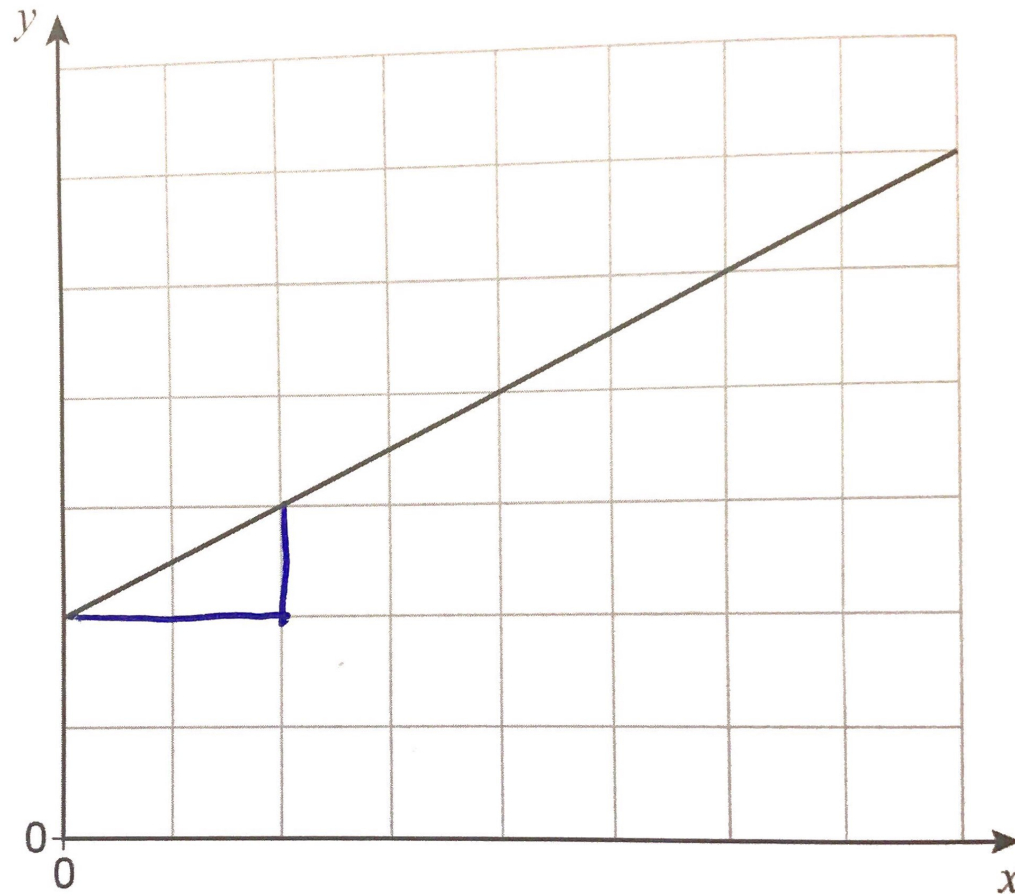
$1/3 \times 2/3 \quad \times 2 = 4/9$

Answer _____ $4/9$ _____

Turn over for the next question

12   A straight line is drawn on the centimetre grid.

Fay assumes that the scale is 1 cm represents 1 unit.

12 (a)   Use her assumption to work out the gradient of the line.

[1 mark]

$$\frac{\Delta y}{\Delta x} = \frac{1}{2}$$

12 (b)   In fact, the scale is 1 cm represents 2 units.

Which statement is correct?

Tick **one** box.

[1 mark]

☐   The answer to part (a) is too big

☑   The answer to part (a) stays the same

☐   The answer to part (a) is too small

Turn over for the next question

13 Show that, for $x \neq -1$

$$\frac{8x^2 - 8}{4x + 4}$$ simplifies to the form $ax + b$ where a and b are integers.

[3 marks]

$$\frac{8(x^2 - 1)}{4(x + 1)} = \frac{\overset{2}{\cancel{8}}(\cancel{x+1})(x-1)}{\cancel{4}(\cancel{x+1})}$$

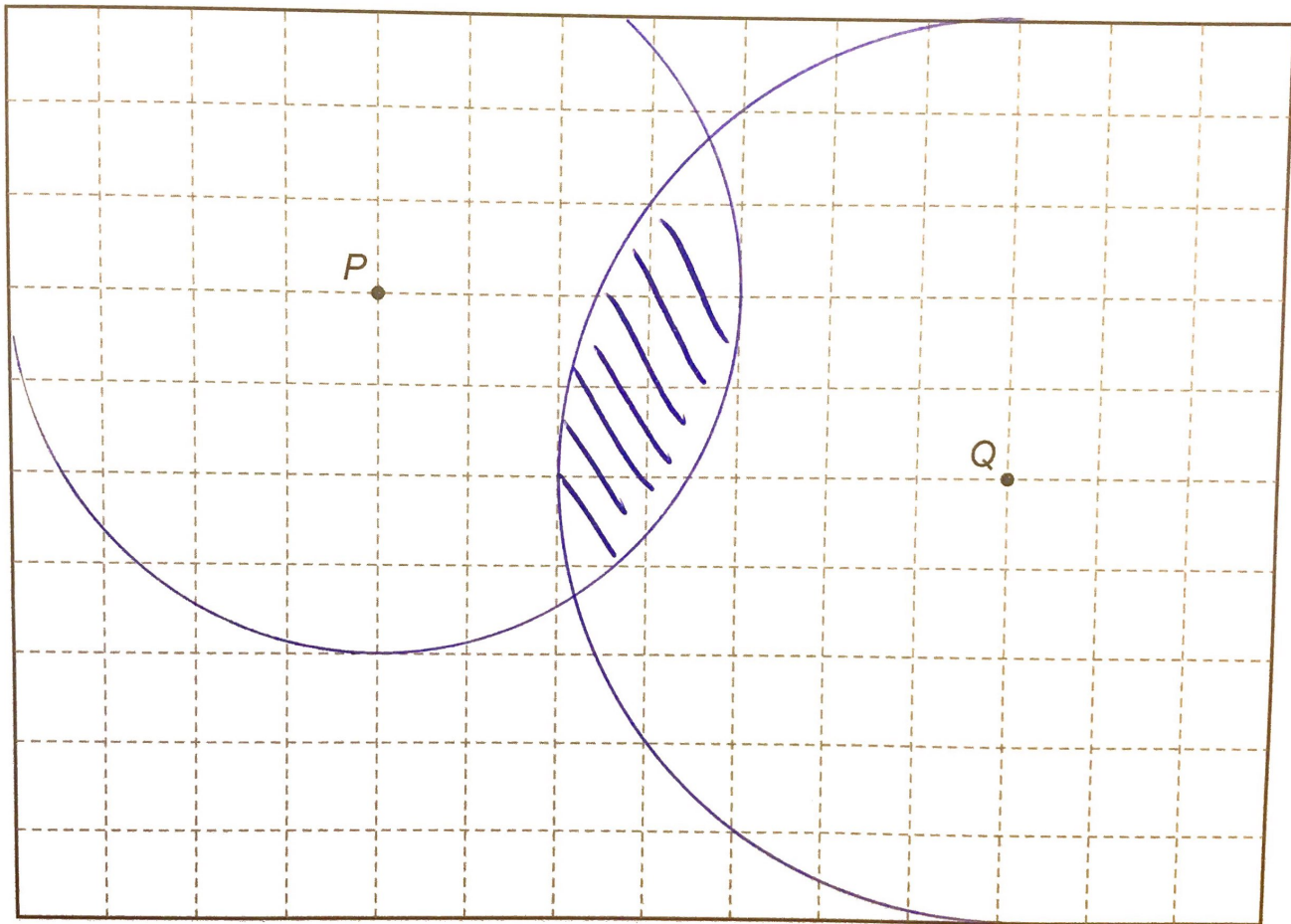$$= 2(x - 1) = 2x - 2$$

$$a = 2$$
$$b = -2$$

14 The scale drawing represents a garden.

Water from a sprinkler at *P* reaches up to 20 metres from *P*.

Water from a sprinkler at *Q* reaches up to 25 metres from *Q*.

Scale: 1 cm represents 5

Using a pair of compasses,

show the region that water from **both** sprinklers reaches.

[2 marks]

Turn over for the next question

15 100 men and 100 women took a test.

Scores

	Median	Interquartile range	Range
Men	28	7.5	31
Women	30	9	37

Using this data, which statement **must** be true?

Tick **one** box.

☐ Men had a higher average score than women

☑ Men had more consistent scores than women

☐ A woman had the highest score

☐ A man had the lowest score

16 Some concrete has volume 3.8 m³

16 (a) The density of the concrete is 2400 kg/m³

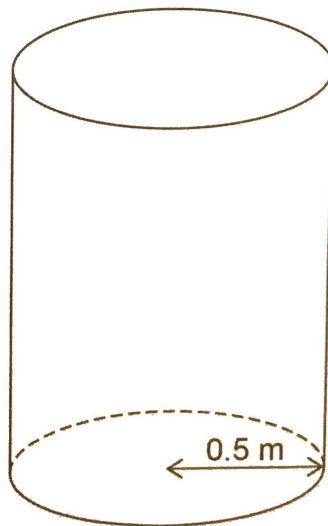Work out the mass of the concrete.

[2 marks]

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$2400 = \frac{\text{Mass}}{3.8}$$

$$\text{Mass} = 2400 \times 3.8$$

Answer __9120__ kg

16 (b) The 3.8 m³ of concrete is made into the shape of a cylinder.

The base has radius 0.5 metres.

0.5 m

$$\text{volume} = \pi r^2 h$$

Work out the height of the cylinder.

[2 marks]

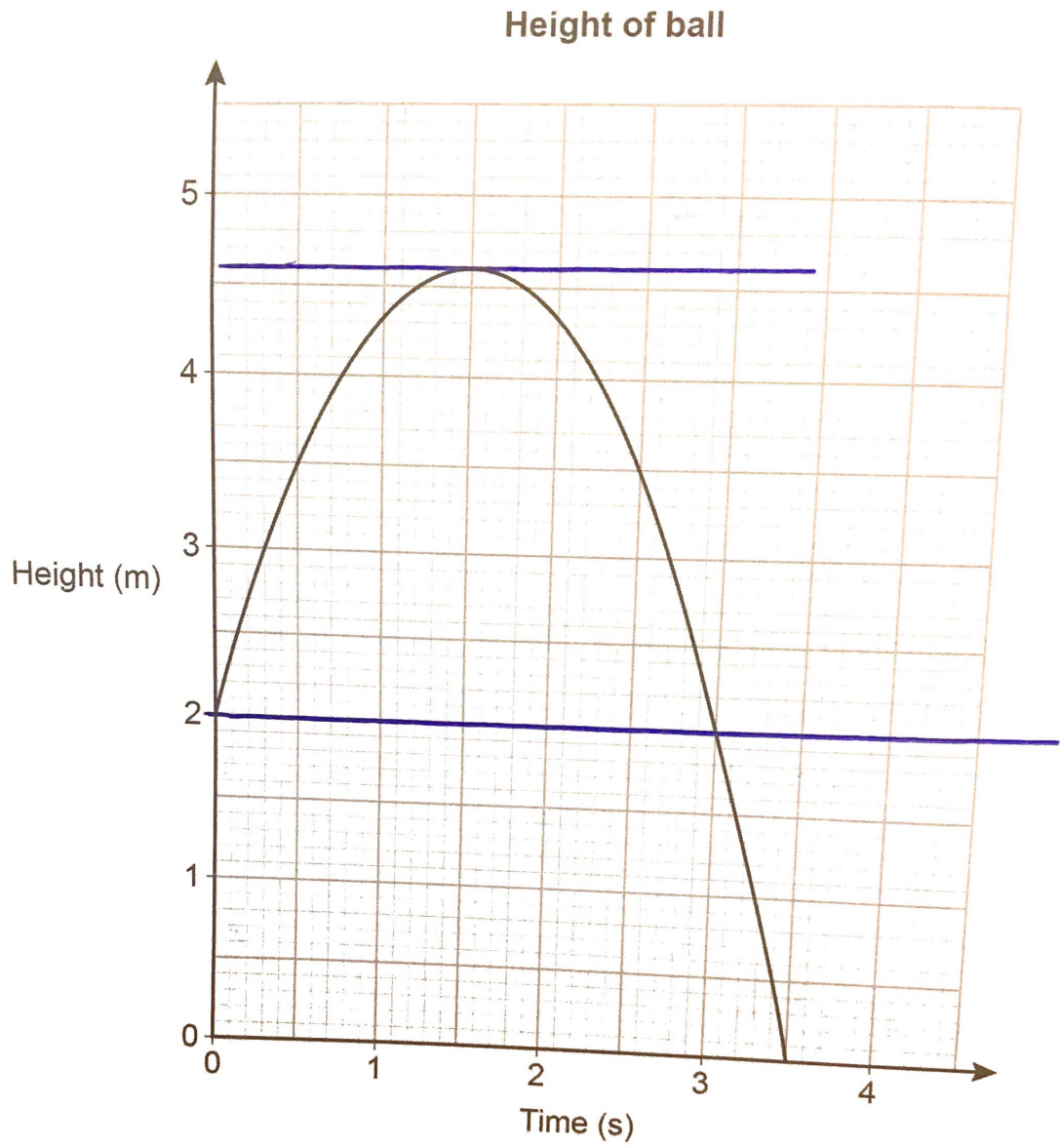$$3.8 = \pi \times 0.5^2 \times h$$

$$h = 4.83831027$$

Answer __4.8__ m

5

17 A ball is thrown vertically upwards.

The graph shows the height of the ball above the ground after it is thrown.

Height of ball

Height (m)

Time (s)

17 (a) For how many seconds is the ball at a height of **more than** 2 metres?

[1 mark]

Answer _____3_____ s

17 (b) After how many seconds is the ball at instantaneous rest when it is in the air?

[1 mark]

Answer _____1·5_____ s

17 (c) Work out the average speed of the ball when it is moving downwards.

[2 marks]

$$\text{Average Speed} = \frac{\text{distance}}{\text{time}} = \frac{4.6}{2}$$

Answer _____ **2.3** _____ m/s

18 The solution of $3^x = 300$ lies between two consecutive integers.
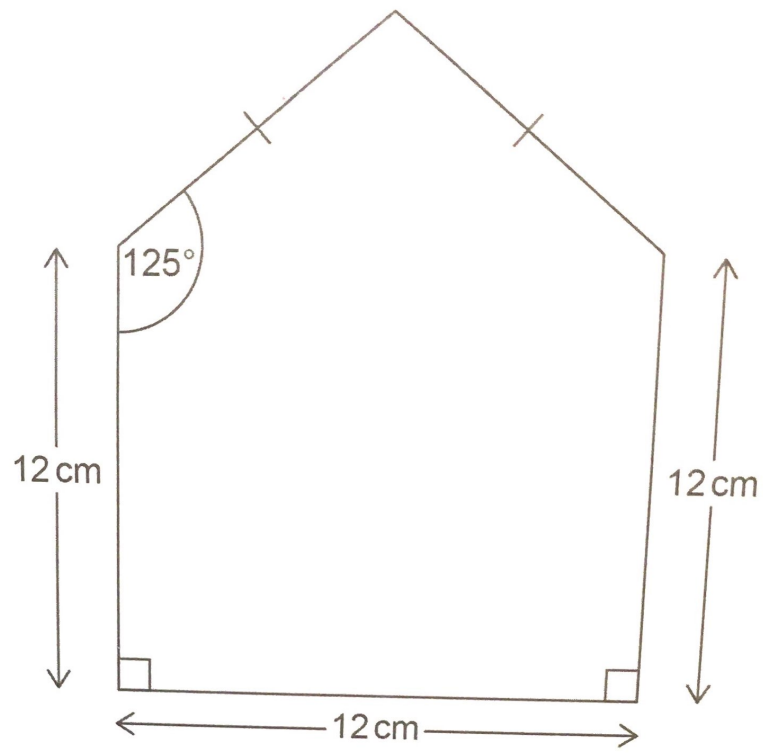
Work out the two integers.

[1 mark]

$$3^{10} = 59049 \qquad 3^6 = 729$$

$$3^5 = 243$$

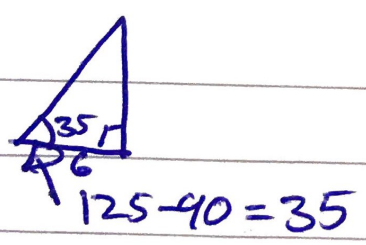Answer _____ **5** _____ and _____ **6** _____

19   A pentagon is made from a square and an isosceles triangle.

Not drawn accurately

125°

12 cm

12 cm

12 cm

Work out the perimeter of the pentagon.

[4 marks]

35
6

$125 - 40 = 35$

SOH $\left(\text{CAH}\right)$ TOA

$\cos \theta = \dfrac{A}{H}$
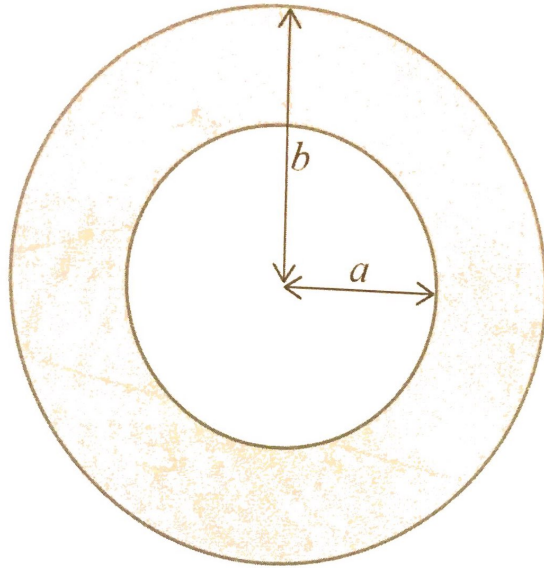
$\cos 35 = \dfrac{6}{H}$

$H \cos 35 = 6$

$H = \dfrac{6}{\cos 35} = 7.324647533$

$12 + 12 + 12 + H \times 2 =$

Answer ___50.64929507___ cm

Here is an inflated swimming ring with dimensions in centimetres.

The volume of the ring, V cm^3, is given by

$$V = 0.25\pi^2(b-a)^2(b+a)$$

Work out the volume when $a = 20$ and $b = 30$
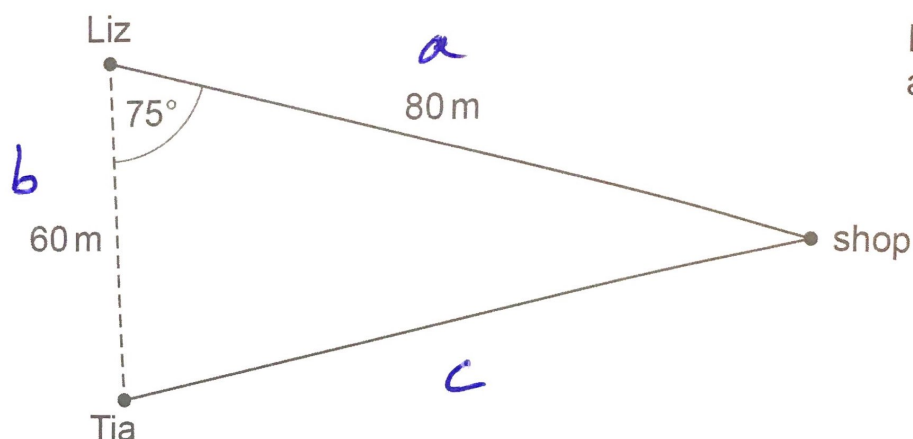
Give your answer to 3 significant figures.

[3 marks]

$0.25 \times \pi^2 \times 10^2 \times 50$

Answer $\underline{12337.0055}$ cm^3

12300 cm^3 to 3 SF

Turn over for the next question

21 Liz and Tia are walking towards a shop along different straight paths.

The diagram shows their positions at 2 pm

Liz
a
Not drawn accurately
75°
80 m
b
60 m
shop
c
Tia

21 (a) Assume they walk at the same speed.

Who will arrive at the shop first?

You **must** show your working.

[3 marks]

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 80^2 + 60^2 - 2 \times 80 \times 60 \times \cos 75$

$c = 86.691044 33$

So   Liz

Answer _____

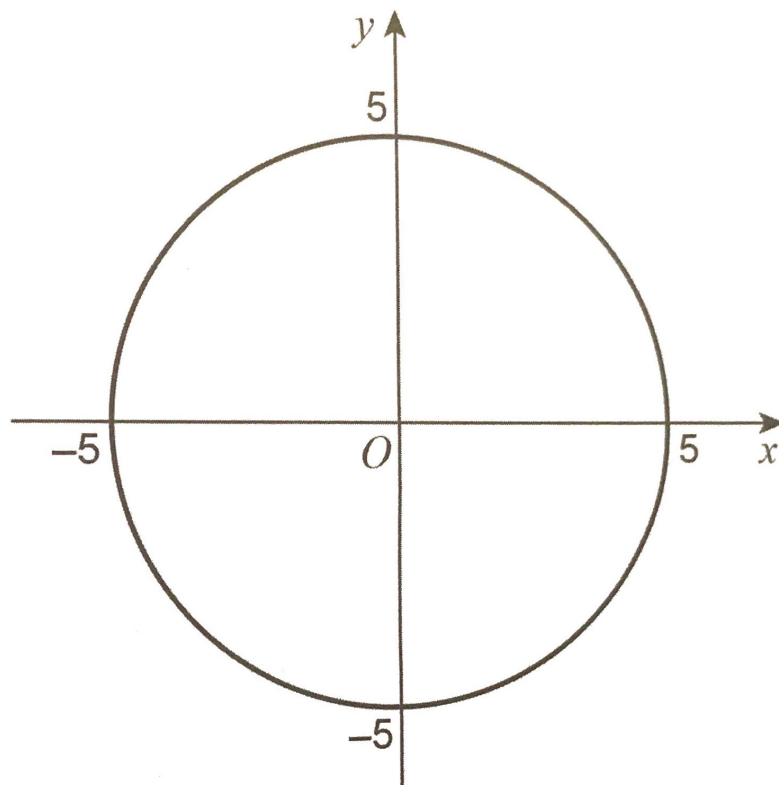**21 (b)** In fact, Liz walks at a faster speed than Tia.

How does this affect the answer to part (a)?

[1 mark]

Not ~~does~~ make any difference as Liz there first.

A circle, centre O, passes through $(5, 0)$.

What is the equation of the circle?

Circle your answer.

[1 mark]

$x^2 + y^2 = 25$ $x^2 + y^2 = 5$ $x^2 + y^2 = 10$ $x^2 + y^2 = 100$

23 Solids X and Y are similar.

X has volume 64 cm^3

Y has volume 343 cm^3

The surface area of X is 176 cm^2

Work out the surface area of Y.

[3 marks]

Volume SF   X $\rightarrow$ Y $\dfrac{343}{64}$

length   SF $\sqrt[3]{\dfrac{343}{64}} = \dfrac{7}{4}$

Area   SF $\left(\sqrt[3]{\dfrac{343}{64}}\right)^2 = \dfrac{49}{16}$

$176 \times \dfrac{49}{16} =$

Answer   539   cm^2

24 A tank is a cuboid measuring 50 cm by 35 cm by 20 cm

All lengths are to the **nearest centimetre**.

A container has a capacity of **exactly** 34 litres.

1 litre = 1000 cm^3

Which has the greater capacity?

Tick **one** box.

☐ Tank ☐ Container ☑ Cannot tell

Show working to support your answer.

[4 marks]

Min   49.5 × 34.5 × 19.5 = ~~34266375~~

33301.125

33.3 ℓ

Max   50.5 × 35.5 × 20.5 = 36751.375

36.75 ℓ
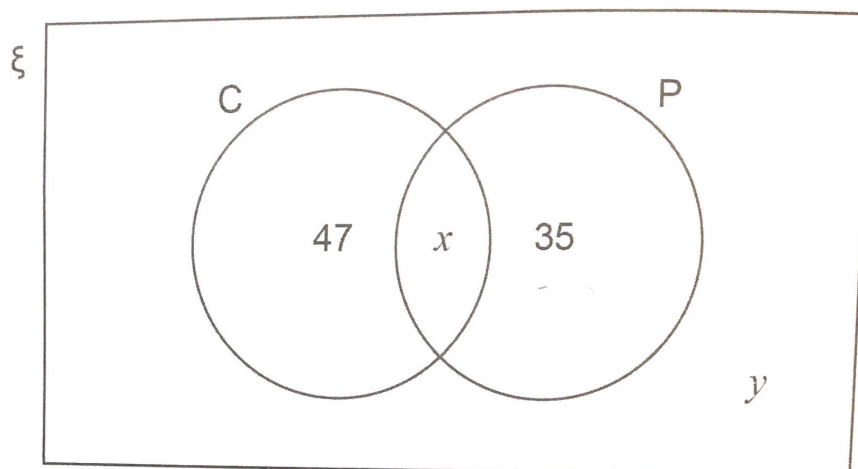
So not possible to tell

Turn over for the next question

25 The Venn diagram shows some information about 150 students.

ξ = 150 students

C = students who study Chemistry

P = students who study Physics

The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$

One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

[4 marks]

$$\frac{x}{35+x} = \frac{5}{12}$$

$$x \times 12 = 5(35+x)$$

$$12x = 175 + 5x$$

$$175 = 7x$$

$$x = 25$$

$$150 - 47 - 25 - 35 = 43$$

Answer ____ $\frac{43}{150}$ ____

A curve has equation $\quad y = 4x^2 + 5x + 3$

A line has equation $\quad y = x + 2$

Show that the curve and the line have **exactly** one point of intersection.

Do **not** use a graphical method.

[4 marks]

Sub ② in ①

$$x + 2 = 4x^2 + 5x + 3$$
$$0 = 4x^2 + 4x + 1$$
$$0 = (2x + 1)(2x + 1)$$
$$x = -\tfrac{1}{2} \text{ only place}$$

of intersect, with $y = -\tfrac{1}{2} + 2 = 1\tfrac{1}{2}$

27 Prove algebraically that $2.7\dot{5}$ converts to the fraction $\dfrac{124}{45}$

[3 marks]

let

$$10x = 27.5\dot{5}$$
$$x = 2.7\dot{5}$$
$$9x = 24.8$$
$$x = \frac{24.8}{9} = \frac{124}{45}$$

28 $f(x) = 5 - x$ and $g(x) = 3x + 7$

28 (a)   Simplify $f(2x) + g(x - 1)$

[3 marks]

$f(2x) = 5 - 2x$

$g(x-1) = 3(x-1) + 7 = 3x - 3 + 7 = 3x + 4$

$f(2x) + g(x-1) = x + 9$

Answer _____

28 (b)   Solve $g^{-1}(x) = 2x$

[3 marks]

let $y = 3x + 7$

$y - 7 = 3x$

$\dfrac{y-7}{3} = x$ so $g^{-1}(x) = \dfrac{x-7}{3}$

$\dfrac{x-7}{3} = 2x$ $x - 7 = 6x$

                              $-7 = 5x$

                              $x = \dfrac{-7}{5}$

$x =$ _____ $-7/5$ _____

END OF QUESTIONS