OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			Differentiate to produce form $k_1x(x+2)^m + k_2x^2(x+2)^n$	*M1	For positive integers k_1, k_2, m, n; allow M1 if slip to, for example, $(x+3)$ in both brackets
			Obtain $6x(x+2)^6 + 18x^2(x+2)^5$	A1	Or unsimplified equiv
			Substitute $x=-1$ to obtain value 12	A1	From correct work only
			Attempt equation of tangent (not normal) through point $(-1, 3)$	M1	Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates
			Obtain $y = 12x + 15$	A1 **[5]**	Answer required in $y = mx + c$ form
2	i		Expand to produce form $k_1 + \frac{k_2}{x} + \frac{k_3}{x^2}$	M1	For non-zero constants k_1, k_2, k_3; allow if middle term appears as two, so far, unsimplified terms
			Obtain $4x - 4\ln x - \frac{1}{x}$ or $4x - 4\ln x - x^{-1}$	A1	Condoning absence of modulus signs but A0 if expression involves $\lvert\ln x\rvert$ or $\lvert 4\ln x\rvert$
	ii		Integrate to obtain form $k(4x+1)^{\frac{4}{3}}$	M1	Any non-zero constant k
			Obtain $\frac{3}{16}(4x+1)^{\frac{4}{3}}$	A1	With coefficient simplified
			Include $\ldots + c$ or $\ldots + k$ at least once anywhere in answer to question 2	B1 **[5]**	Even if associated with incorrect integral

Question			Answer	Marks	Guidance	
3	i		Obtain 128 for value corresponding to 10	B1	Allow any value rounding to 128	
			Obtain 65.5 for value corresponding to 25	B1	Allow any value rounding to 65 or 66; whether obtained using powers of 0.8 or by use of formula	
				[2]		
	ii		Attempt to find formula for *m* of form $200e^{kt}$ or $200\times r^{\lambda t}$	M1	Whether attempted in part (i) or (ii)	If formula attempted in part (i), marks earned must be recorded in part (ii)
			Obtain $200e^{(0.2\ln 0.8)t}$ or $200e^{-0.0446t}$ or $200\times 0.8^{0.2t}$ or 200×0.956^{t}	A1	Or equiv	
			Show correct process for solving equation of form $200e^{kt}=50$ or $200r^{\lambda t}=50$	M1		
			Obtain 31	A1	Or greater accuracy rounding to 31; ignore any units given; second M1 is implied by correct answer	Special case: no formula anywhere and answer 31 (or greater accuracy) given, award B2 (i.e. 2/4 for part (ii))
				[4]		
4			Use identity $\sec^2 A = 1+\tan^2 A$	B1		
			Attempt solution of three-term quadratic equation to obtain two values of $\tan A$	M1	Implied if correct values obtained; allow M1 for incorrect factorisation provided expansion would give correct first and third terms; allow M1 for incorrect use of formula if only one error present	
			Obtain $\tan A=-3$ and $\tan A=4$	A1	And no others; implied by $A=\tan^{-1}-3$ and $\tan^{-1}4$;	$A=-3, 4$ is A0 here unless subsequent work shows values used correctly
			Use correct identities to produce equation in $\tan B$ only	M1	Equation might be $t^3=27$ …	… or $t^5+t^3-27t^2-27=0$
			State $\tan B=3$	A1	And no others	

Question			Answer	Marks	Guidance	
			Substitute at least one pair of non-zero numerical values into $\dfrac{\tan A - \tan B}{1 + \tan A \tan B}$	M1	Must be the correct identity	
			Obtain one of $\frac{1}{13}$ and $\frac{3}{4}$ or exact equiv	A1		
			Obtain the other exact value or equiv	A1 **[8]**	And no others	
5	i		Either State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		Verifying by substitution of $\ln 2$ in each equation earns B0B0
			Obtain $e^{x} = 2$ and hence $x = \ln 2$	B1	AG; necessary detail needed	
			Or 1 State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		
			State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
			Or 2 State $e^{2x} = 8e^{-x}$ and $2x = \ln 8 - x$	B1		
			State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1 **[2]**	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
	ii		Integrate to obtain $k_1 e^{-x}$ and $k_2 e^{2x}$	M1	Any non-zero constants k_1 and k_2	
			Obtain correct $-8e^{-x} - \frac{1}{2}e^{2x}$ or, if done separately, $-8e^{-x}$ and $\frac{1}{2}e^{2x}$	A1		
			Apply limits 0 and $\ln 2$ correctly to their integral(s)	M1	Condone one sign slip; earned by sight of $-8e^{-\ln 2} - \frac{1}{2}e^{2\ln 2} + 8 + \frac{1}{2}$ (or equivs if integrals treated separately)	M1 also implied by sight only of $-4 - 2 + 8 + \frac{1}{2}$ (or equivs …)
			Obtain at least $-4 - 2 + 8 + \frac{1}{2}$ (or equivs)	*A1		
			Obtain $\frac{5}{2}$ or equiv	A1 **[5]**	Final A1 dependent on *A1	

Question			Answer	Marks	Guidance	
6			State, at some stage, $a(4+b)^{\frac{1}{2}}=18$	B1		
			Obtain derivative $\dfrac{4}{4x-7}$ for C_1	B1		
			Obtain derivative $kx(x^2+b)^{-\frac{1}{2}}$ for C_2	M1	Any non-zero constant k	
			Obtain correct $ax(x^2+b)^{-\frac{1}{2}}$	A1		
			Equate derivatives with $x=2$	M1		
			Attempt values of a and b from two equations involving a and $(4+b)^{\frac{1}{2}}$	M1	Using correct process Correct equations are $a(4+b)^{\frac{1}{2}}=18$ and $2a(4+b)^{-\frac{1}{2}}=4$	
			Obtain $a=6$ Obtain $b=5$	A1 A1		
				[8]		
7	i		Draw more or less correct sketch of $y=\cos^{-1}x$ existing in first and second quadrants	*B1	Ignore any curve outside $0\le y\le\pi$; condone no or wrong intercepts on axes	
			Draw U-shaped parabola passing through origin and showing minimum point	*B1	Curve must exist in first and third quadrants	
			Indicate one intersection in first quadrant by blob or reference in words or …	B1	Dep *B *B	
				[3]		

Question			Answer	Marks	Guidance	
	ii		Obtain correct first iterate showing at least 4 s.f. rounded or truncated	B1		
			Show iterative process to produce at least three iterates in all showing at least 3 s.f.	M1	Implied by incorrect values apparently converging	0.25
			Obtain at least four correct iterates in all showing at least 4 s.f.	A1	Allowing recovery after error	0.23965… 0.24250…
			Conclude with value 0.242	A1	Answer to be clearly indicated by underlining final value in sequence or by separate statement; answer required to precisely 3 s.f.; allow final A1 even if iterates have been shown to only 3 s.f.; answer only earns 0/4	0.24172… 0.24193…
				[4]		
	iii		State $y=-\cos^{-1}(-x)$ or $y=\cos^{-1}x-\pi$	B1		
			State $y=x(-2x+5)$ or equiv	B1	Allow $y=--x(2(-x)+5)$ or similar; condone missing $y=$ in each case	
			State -0.242 for x-coordinate	B1 FT	Following their answer to **(ii)**; allow greater accuracy here	
			State -1.33 for y-coordinate	B1	Allow value rounding to -1.33	
				[4]		
8	i		State range of f is $\mathrm{f}(x)\geq 3a$ or $y\geq 3a$	B1	Allow $\mathrm{f}\geq 3a$ or equiv expression in words but $3a$ to be included	
			State range of g is all real numbers or equiv such as $y\in\mathrm{R}$ (real numbers)	B1		
				[2]		

Question			Answer	Marks	Guidance	
	ii		State function is not 1 – 1 or different x-values give same y-value or equiv	B1	no credit for 'no inverse due to modulus' nor for 'cannot be reflected across $y = x$'	
			Obtain form $k(y+4a)$ or $k(x+4a)$	M1	for non-zero constant k	
			Obtain $\frac{1}{5}(x+4a)$ or $\frac{1}{5}x+\frac{4}{5}a$	A1 **[3]**	Must finally be in terms of x	
	iii		Either Attempt composition of functions the right way round	M1	Earned for 5(what they think f(x) is) – 4a	
			Obtain $5\lvert 2x+a\rvert+11a=31a$ or equiv	A1		
			Or Apply their g^{-1} to $31a$	M1		
			Obtain $\lvert 2x+a\rvert+3a=7a$ or equiv	A1		
			Either Solve $2x+a=4a$ and obtain $\frac{3}{2}a$	B1 FT	Following their $\lvert 2x+a\rvert=ka$	
			Solve linear equation in which signs of (their) $2x$ and (their) $4a$ are different	M1	Condone other sign slips	
			Obtain $-\frac{5}{2}a$	A1	And no others; obtaining $-\frac{5}{2}a$ and then concluding $\frac{5}{2}a$ is A0	
			Or Square both sides and obtain $4x^2+4ax-15a^2=0$	B1 FT	Following their $\lvert 2x+a\rvert=ka$	
			Solve 3-term quadratic equation to obtain two values	M1	Allow M1 if factorisation wrong but expansion gives correct first and third terms; allow M1 if incorrect use of formula involves only one error	
			Obtain $-\frac{5}{2}a$, $\frac{3}{2}a$	A1 **[5]**	And no others; continuing from two correct answers to conclude $\frac{5}{2}a$, $\frac{3}{2}a$ is A0	

Question			Answer	Marks	Guidance	
9	i		Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		Note that going directly from $2\sin^2\theta + 2\cos^2\theta$ to 2 is M0 but $2(\sin^2\theta + \cos^2\theta)$ to 2 is M1A1
			State $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ or $\tan\theta + \frac{1}{\tan\theta}$	B1	Perhaps as part of expression	
			Simplify using correct identities	M1		
			Obtain 2 correctly	A1	AG; necessary detail needed	
				[4]		
	ii	a	Obtain expression involving at least one of $\sin\frac{1}{6}\pi$ and $\sin\frac{1}{4}\pi$	M1		
			Obtain $\frac{2}{\sin\frac{1}{6}\pi} + \frac{2}{\sin\frac{1}{4}\pi}$	A1	Or equiv involving cosecant	
			Obtain $4 + 2\sqrt{2}$ or exact equiv	A1	Answer only is 0/3	
				[3]		
		b	Use $\sin 4\theta = 2\sin 2\theta\cos 2\theta$	B1		
			Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2\theta = \frac{5}{8}$ or $\sin^2\theta = \frac{3}{8}$	B1		
			Obtain 0.659 or 0.66	B1	Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π; answer only earns 0/3	
				[3]		
		c	Express in form $k_1\sin^4\theta \times \frac{k_2}{\sin^3\theta}$	M1		
			Obtain $4\sin^4\theta \times \frac{8}{\sin^3\theta}$ and hence $32\sin\theta$	A1	A0 if $(-2\sin^2\theta)^2$ involved in simplification	
				[2]		

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