NAME:

Date to be handed in:

MARK (out of 100):

PAPER M

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1 Prove by exhaustion that $1+2+3+...+n \circ \frac{n(n+1)}{2}$ for positive integers from 1 to 6 inclusive.

(3 marks)

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2 Solve $6\sin(\theta+60)=8\sqrt{3}\cos\theta$ in the range $0° \le \theta \le 360°$. Round your answer to 1 decimal place.

(4 marks)

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3 The temperature of a mug of coffee at time t can be modelled by the equation $\mathrm{T}(t)=T_R+(90-T_R)\mathrm{e}^{-\frac{1}{20}t}$, where $\mathrm{T}(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

a Using the equation for this model, explain why the initial temperature of the coffee is independent of the initial room temperature. **(2 marks)**

b Calculate the temperature of the coffee after 10 minutes if the room temperature is 20 °C. **(2 marks)**

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4 Given that $\int_a^4 (10-2x)^4 \, \mathrm{d}x = \frac{211}{10}$, find the value of a. **(5 marks)**

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5 Use proof by contradiction to show that there are no positive integer solutions to the statement $x^2 - y^2 = 1$ **(5 marks)**

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6 $\frac{18x^2-98x+78}{(x-4)^2(3x+1)} = \frac{A}{x-4}+\frac{B}{(x-4)^2}+\frac{C}{3x+1}, \quad x>4$

Find the values of the constants A, B and C. **(6 marks)**

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7 The functions p and q are defined by $\mathrm{p}: x \to x^2$ and $\mathrm{q}: x \to 5-2x$

a Given that pq(x) = qp(x), show that $3x^2-10x+10=0$ **(4 marks)**

b Explain why $3x^2-10x+10=0$ has no real solutions. **(2 marks)**

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8 The curve C has equation $y=x^3+6x^2-12x+6$

a Show that C is concave on the interval [–5, –3]. **(3 marks)**

b Find the coordinates of the point of inflection. **(3 marks)**

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9 For an arithmetic sequence $a_4 = 98$ and $a_{11} = 56$

a Find the value of the 20th term. **(4 marks)**

b Given that the sum of the first n terms is 78, find the value of n. **(4 marks)**

10 A stone is thrown from the top of a building. The path of the stone can be modelled using the parametric equations $x = 10t$, $y = 8t - 4.9t^2 + 10$, $t \geqslant 0$, where x is the horizontal distance from the building in metres and y is the vertical height of the stone above the level ground in metres.

a Find the horizontal distance the stone travels before hitting the ground. **(4 marks)**

b Find the greatest vertical height. **(5 marks)**

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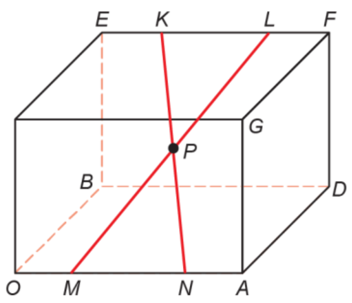
11 The diagram shows a cuboid whose vertices are *O*, *A*, *B*, *C*, *D*, *E*, *F* and *G*.

a, **b** and **c** are the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

The points *M* and *N* lie on *OA* such that $OM : MN : NA = 1:2:1$

The points *K* and *L* lie on *EF* such that $EK : KL : LF = 1:2:1$



Prove that the diagonals *KN* and *ML* bisect each other at *P*. **(10 marks)**

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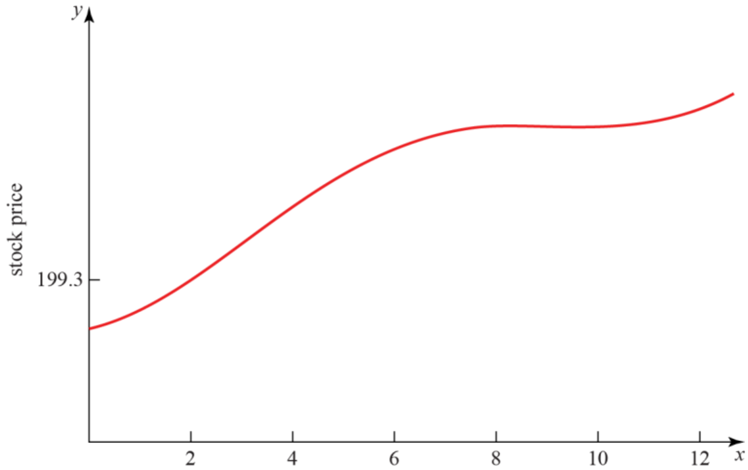
12 $\mathrm{f}(x) = \dfrac{21 - 14x}{(1-4x)(2x+3)}, x \neq \dfrac{1}{4}, x \neq -\dfrac{3}{2}$

a Given that $\mathrm{f}(x) = \dfrac{A}{1-4x} + \dfrac{B}{2x+3}$, find the values of the constants *A* and *B*. **(5 marks)**

b Find the exact value of $\int_{-1}^{0} \mathrm{f}(x)\,\mathrm{d}x$ **(5 marks)**

13 $\quad \mathrm{p}(t) = \frac{1}{10}\ln(t+1) - \cos\left(\frac{t}{2}\right) + \frac{1}{10}t^{\frac{3}{2}} + 199.3, \qquad 0 \le t \le 12$



a The diagram shows a graph of the price of a stock during a 12-hour trading window. The equation of the curve is given above.

Show that the price reaches a local maximum in the interval $8.5 < t < 8.6$ **(5 marks)**

b Figure 1 shows that the price reaches a local minimum between 9 and 11 hours after trading begins.

Using the Newton–Raphson procedure once and taking $t_0 = 9.9$ as a first approximation, find a second approximation of when the price reaches a local minimum. **(6 marks)**

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14 $\quad \frac{4x^2 - 4x - 9}{(2x+1)(x-1)} \circ A + \frac{B}{2x+1} + \frac{C}{x-1}$

a Find the values of the constants A, B and C. **(6 marks)**

b Hence, or otherwise, expand $\frac{4x^2 - 4x - 9}{(2x+1)(x-1)}$ in ascending powers of x, as far as the x^2 term.

(6 marks)

c Explain why the expansion is not valid for $x = \frac{3}{4}$ **(1 mark)**

(TOTAL: 100 MARKS)