**9FM0/3D: Decision Mathematics 01 Mark scheme**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1(a)** | 6 1 9 14 18 7 10 4 17 139 14 18 10 17 13 7 6 1 4 | M1 | 1.1b |
|  | 14 18 17 13 10 9 7 6 4 118 17 14 13 10 9 7 6 4 1  | A1 | 1.1b |
|  | 18 17 14 13 10 9 7 6 4 1 | A1 | 1.1b |
|  |  | **(3)** |  |
| **(b)** | Bin 1: 18 10 1Bin 2: 17 13Bin 3: 14 9 7Bin 4: 6 4 | M1A1 | 1.1b 1.1b |
|  |  | **(2)** |  |
| **(5 marks)** |
| **Notes:** |
| **(a)****M1:** quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot).**A1:** first two passes correct and correct pivots chosen for third pass**A1:** cso (correct solution only – all previous marks in this part must have been awarded) – must include a fourth pass **(b)****M1:** must be using ‘sorted’ list in descending order. First five items placed correctly and at least eight values placed in bins**A1:** cso (so no additional/repeated values)  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2(a)** | 7 | B1 | 2.2a |
|  |  | **(1)** |  |
| **(b)** | A semi-Eulerian graph requires exactly two odd nodes…  | B1 | 1.2 |
|  | …the graph has six odd nodes so only two arcs needs to be added to make the graph semi-Eulerian | B1 | 2.2a |
|  |  | **(2)** |  |
| **(c)** | Creates two lists of arcs | M1 | 2.1 |
|  | e.g. AB BF BE CE EF EG BG BD  | A1 | 1.1b |
|  | Since no arc appears in both lists, the graph is planar (or draws a planar version) | A1 | 2.4 |
|  |  | **(3)** |  |
| **(6 marks)** |
| **Notes:** |
| **(a)****B1:** cao**(b)****B1:** accurately recalls the fact that a semi-Eulerian graph contains exactly two odd nodes**B1:** dependent on previous B mark – cao**(c)****M1:** creates two list of arcs (with at least three arcs in each list) which contain no common arcs **A1:** cao **A1:** correct reasoning that no arc appears in both lists + so the graph is therefore planar |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3(i)(a)** | 22 | M1A1A1 | 1.1b1.1b1.1b |
|  | Length of quickest route from A to H is 47 minutes | A1ft | 2.2a |
|  |  | **(4)** |  |
| **(b)** | Shortest path from A to F via H: ABGEHF  | B1 | 1.1b |
|  | Length: 47 + 12 = 59 minutes  | B1ft | 2.2a |
|  |  | **(2)** |  |
| **(c)** | e.g. add 1 to each arc | M1 | 3.5c |
|  | except AB, AD, AC (or EH, GH, FH) | A1 | 2.3 |
|  |  | **(2)** |  |
| **(ii)(a)** | AB + EH = 13 + 10 = 23\*A(BG)E + B(GE)H = 37 + 34 = 71A(BGE)H + B(G)E = 47 + 24 = 71 | M1A1ftA1 | 2.11.1b1.1b |
|  | Length of the shortest route is 300 + 23 = 323 km | A1ft | 2.2a |
|  |  | **(4)** |  |
| **(b)** | Repeat arcs: AB, EH | B1 | 2.2a |
|  |  | **(1)** |  |
| **(13 marks)** |
| **Notes:****(i) (a)** **M1:** for a larger number replaced by a smaller one in the working values boxes at C, D, E, F or H **A1:** for all values correct (and in correct order) at A, B, G and C **A1:** for all values correct (and in correct order) at D, E, F and H **A1ft:** for 47 or ft their final value at H**(b)****B1:** cao**B1ft:** for 59 or ft their final at H + 12**(c)****M1:** valid general method – any mention of adding 1 to the weight of the arcs**A1:** cao – so adding 1 to each arc except {AB, AD, AC} or {EH, GH, FH}**(ii)(a)****M1:** correct three pairings of the required four odd nodes**A1ft:** at least two pairings and totals correct (ft their values from (a))**A1:** all three pairings and totals correct**A1ft:** for 323 or 300 + their shortest repeat**(b)****B1:** selecting the shortest pairing, and stating that these arcs should be repeated |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)****(i)** | G(10) | M1A1A1 | 2.11.1b1.1b |
| **(ii)** | Minimum completion time is 85 minutes | A1ft | 2.2a |
| **(iii)** | Critical activities are A, E and I | A1ft | 2.2a |
|  |  | **(5)** |  |
| **(b)** | e.g. | M1A1A1 | 1.1b1.1b1.1b |
|  |  | **(3)** |  |
| **(c)** | Currently five workers are required between time 20 and 40 and so activities F and H would have to be delayedIf F starts at 35 H could not begin until 55 but the latest start time for H is 40. Therefore the project cannot be completed in the minimum time with only four workers  | M1A1 | 2.42.2a |
|  |  | **(2)** |  |
| **(10 marks)** |
| **Notes:** |
| **(a)(i)****M1:** All boxes completed, number generally increasing L to R (condone one “rogue”) and decreasing R to L (condone one “rogue”)**A1:** Cao -Top boxes **A1:** Cao - Bottom boxes **(ii)****A1ft:** Deduction that result in diagram indicates that project can be completed in 85 minutes**(iii)****A1ft:** Deduction of correct critical activities (from their values at each event)**(b)****M1:** Plausible histogram with no holes or overhangs (must go to at least 70 on the time axis) **A1:** Histogram correct to time 40 **A1:** Histogram correct from time 40 to time 85**(c)****M1:** Explanation involving the need to delay activities F and H**A1:** Correct deduction that it is not possible to complete the project with only four workers in the minimum project completion time |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | Maximise   | B1 | 2.5 |
|  | Subject to   | M1A1B1 | 3.31.1b3.3 |
|  |  | **(4)** |  |
| **(b)** |

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| b.v. |  |  |  |  |  |  | Value |
|  | 3 | 5 | 8 | 1 | 0 | 0 | 400 |
|   | 3 | 6 | 10 | 0 | 1 | 0 | 350 |
|   | 1 | 1.5 | 1.25 | 0 | 0 | 1 | 75 |
|  |   |   |   | 0 | 0 | 0 | 0 |

 | M1A1 | 3.41.1b |
|  |  | **(2)** |  |
| **(c)** |

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| b.v. |  |  |  |  |  |  | Value | Row Ops |
|  | 0.6 | 0.2 | 0 | 1 |   | 0 | 120 |   |
|  | 0.3 | 0.6 | 1 | 0 | 0.1 | 0 | 35 |   |
|   | 0.625 | 0.75 | 0 | 0 |   | 1 | 31.25 |   |
|  |   |   | 0 | 0 | 6.5 | 0 | 2275 |   |

 | M1A1ftA1B1ft | 2.11.1b1.1b2.4 |
|  |  | **(4)** |  |
| **(d)** |  so increasing  will decrease profit | B1 | 2.4 |
|  |  | **(1)** |  |
| **(e)** | (i) Make 50 lectern desks, 20 writing desks and no roll top desks | B1 | 3.2a |
|  | (ii) £3300 | B1 | 1.1b |
|  |  | **(2)** |  |
| **(f)** | The 90 is the value of the slack variable  which comes from the constraint  | B1 | 2.4 |
|  | Indicating that there is 90 m2 of wood still available | B1 | 3.2a |
|  |  | **(2)** |  |

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| --- | --- | --- | --- |
| **(g)** | e.g. there is no guarantee that all the desks will be sold | B1 | 3.5b |
|  |  | **(1)** |  |
| **(16 marks)** |
| **Notes:** |
| **(a)** **B1:** Correct objective function/expression (accept in pence rather than pounds e.g. 4000*x* + 5000*y* + 6500*z*) together with ‘maximise’**M1:** Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals **A1:** All three correct (non-trivial) inequalities **B1:** **(b)****M1:** Constructing all four rows including slack variables with at least one negative in *P* row (allow sign/numerical slips)**A1:** All four rows correct**(c)****M1:** Correct pivot located, attempt to divide row**A1ft:** Pivot row correct (including change of b.v.) and row operations used at least once, one of columns *x*, *y*,  or Value correct **A1:** Cao for values (ignore b.v. column and Row Ops)**B1ft:** The correct Row Operations (on the ft) explained either in terms of the ‘old’ or ‘new’ pivot rows**(d)****B1:** States correct objective function and mention of increasing  will decrease profit **(e)(i)****B1:** Cao – in context so not in terms of *x*, *y* and *z***(ii) B1:** Cao**(f)****B1:** Recognises that  and is linked to the wood constraint**B1:** Evaluates this value in context (so must see both units and mention of ‘wood’)**(g)****B1:** Cao – any suitable limitation to the solution in context |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** |  Distance table Route table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | - | 3 | 7 | 9 |  |
| **B** | 3 | - | 2 |  | 8 |
| **C** | 7 | 2 | - | 6 | 5 |
| **D** | 9 |  | 6 | - | 4 |
| **E** |  | 8 | 5 | 4 | - |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | A | B | C | D | E |
| **B** | A | B | C | D | E |
| **C** | A | B | C | D | E |
| **D** | A | B | C | D | E |
| **E** | A | B | C | D | E |

 | B1B1 | 1.1b1.1b |
|  |  | **(2)** |  |
| **(b)** | 1st iteration: Distance table Route table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | - | 3 | 7 | 9 |  |
| **B** | 3 | - | 2 | 12 | 8 |
| **C** | 7 | 2 | - | 6 | 5 |
| **D** | 9 | 12 | 6 | - | 4 |
| **E** |  | 8 | 5 | 4 | - |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | A | B | C | D | E |
| **B** | A | A | C | A | E |
| **C** | A | B | C | D | E |
| **D** | A | A | C | D | E |
| **E** | A | B | C | D | E |

  | M1A1 | 1.1b1.1b |
|  | 2nd iteration: Distance table Route table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | -  | 3 | 5 | 9 | 11 |
| **B** | 3 | - | 2 | 12 | 8 |
| **C** | 5 | 2 | - | 6 | 5 |
| **D** | 9 | 12 | 6 | - | 4 |
| **E** | 11 | 8 | 5 | 4 | - |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** |
| **A** | A | B | B | D | B |
| **B** | A | B | C | A | E |
| **C** | B | B | C | D | E |
| **D** | A | A | C | D | E |
| **E** | B | B | C | D | E |

 | M1A1ft | 1.1b1.1b |
|  |  | **(4)** |  |
| **(c)** |  | M1A1 | 1.1b1.1b |
|  |  | **(2)** |  |
| **(d)(i)** | NNA: E – D – C – B – A – E  | B1 | 1.1b |
| **(ii)** | 4 + 6 + 2 + 3 + 10 = 25 km | B1 | 1.1b |
|  |  | **(2)** |  |
| **(e)** | E – D – C – B – A – B – C – E  | B1 | 3.2a |
|  |  | **(1)** |  |
| **(f)** | Prim’s algorithm on reduced network starting at B: BC, CE, DE | B1 | 1.1b |
|  | Lower bound = 11 + 3 + 5 = 19 km | B1ft | 2.2a |
|  |  | **(2)** |  |
| **(g)** |  | M1 | 2.2b |
|  |   | A1 | 1.1b |
|  |  | **(2)** |  |
| **(15 marks)** |
| **Notes:** |
| **(a)****B1:** Correct distance table**B1:** Correct route table**(b)****M1:** No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed **A1:** cao **M1:** No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed **A1ft:** Correct second iteration follow through from the candidate’s first iteration**(c)****M1:** K5 drawn with at least one shortest distance from the final distance table present**A1:** cao**(d)(i)****B1:** cao**(ii)****B1:** cao**(e)****B1:** cao**(f)****B1:** correct RMST starting at any node (except A)**B1ft:** length of their RMST + 3 + 5**(g)****M1:** Their numbers correctly used, accept any inequalities or any indication of interval from their 19 to their 25 (so 19 – 25 can score this mark). Please note that UB > LB for this mark**A1:** cao (no follow through on their values) including correct inequalities or equivalent set notation (but condone 19 $<$ optimal  25 ) |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7** | Objective line   | B1 | 3.4 |
|  |    | B1 | 3.4 |
|  | Line through (0, 12) and (8, 0) is   | M1 | 1.1b |
|  | Line through (5, 0) and (10, 10) is   | M1 | 1.1b |
|  |      | M1A1ftA1 | 2.11.1b1.1b |
|  |     | M1 | 2.2a |
|  | e.g.

|  |  |  |  |  |  |  |  |  |  |
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| b.v. |   |   |   |   |   |   |   |   | Value |
|   | 2 |   | 1 | 0 | 0 | 0 | 0 | 0 | 10 |
|  | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 10 |
|  | 1 | 0 | 0 | 0 |   | 0 | 1 | 0 | 4 |
|  | 3 | 2 | 0 | 0 | 0 |   | 0 | 1 | 24 |
|   |   |   | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| *A*  |  |   | 0 | 0 | 1 | 1 | 0 | 0 |   |

 | M1A1 | 2.12.2a |
| **(10 marks)** |
| **Notes:** |
| **B1:** cao for objective function (oe e.g. *P* – 3*x* – 4*y* = *k*)**B1:** cao **M1:** correct method for finding the equation of the line through (0, 12) and (8, 0)**M1:** correct method for finding the equation of the line through (5, 0) and (10, 10)**M1:** translate all 4 inequalities into equations **–** must include all three types of variables (slack, surplus and artificial)**A1ft:** two correct equations following their inequalities**A1:** all four correct equations**M1:** setting up the new objective and substituting for  and **M1:** setting up tableau – all six lines with four basic variables**A1:** cao (oe e.g. consistent *P* line with their objective equation) |